AMDG

Name Ms. Kresovic

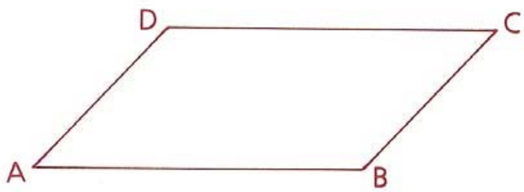
Adv Geo - 5.6: Proving that a quadrilateral is a parallelogram

Objective

After studying this section, you will be able to

- Prove that a quadrilateral is a parallelogram

PROVING A QUADRILATERAL IS A PARALLELOGRAM



Any one of the following methods might be used to prove that quadrilateral ABCD is a parallelogram.

1 If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram (reverse of the definition). Both prs opp sds $\parallel \Rightarrow$ ▱

2 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram (converse of a property). Both prs opp sds $\cong \Rightarrow$ ▱

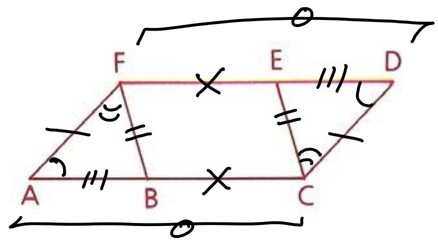
3 If one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram. 1 pr sds $\cong$ & $\parallel \Rightarrow$ ▱

4 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram (converse of a property). diags bis eachother $\Rightarrow$ ▱

5 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram (converse of a property). Both prs opp $\angle$s $\cong \Rightarrow$ ▱

Class Examples

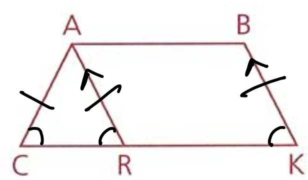
Problem 1 Given: ACDF is a ▱.
$\angle AFB \cong \angle ECD$
Prove: FBCE is a ▱.



Proof

1 ACDF is a ▱.	1 Given
2 $\angle A \cong \angle D$	2 ▱ ⇒ opp ∠s ≅
3 $\overline{AF} \cong \overline{DC}$	3 ▱ ⇒ opp sds ≅
4 $\angle AFB \cong \angle ECD$	4 Given
5 $\triangle AFB \cong \triangle DCE$	5 ASA (2 3 4)
6 $\overline{FB} \cong \overline{EC}$	6 cpctc
7 $\overline{AB} \cong \overline{ED}$	7 cpctc
8 $\overline{AC} \cong \overline{FD}$	8 ▱ ⇒ opp sds ≅
9 $\overline{BC} \cong \overline{FE}$	9 Subtract
10 FBCE is a ▱.	10 Both prs opp sds ≅ ⇒ ▱ (6 & 9)

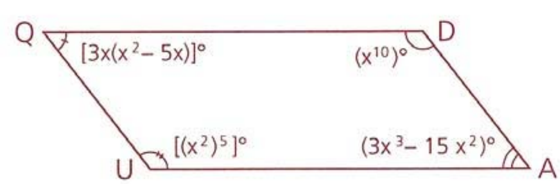
Problem 2 Given: $\triangle CAR$ is isosceles, with base $\overline{CR}$.
$\overline{AC} \cong \overline{BK}$,
$\angle C \cong \angle K$
Prove: BARK is a ▱.



Proof

1 $\triangle CAR$ is isos., with base $\overline{CR}$.	1 Given
2 $\overline{AC} \cong \overline{AR}$	2 isos ⇒ ≅ sds
3 $\overline{AC} \cong \overline{BK}$	3 Given
4 $\overline{AR} \cong \overline{BK}$	4 transitive
5 $\angle C \cong \angle ARC$	5 ≅ sds △ ⇒ ≅ ∠s △
6 $\angle C \cong \angle K$	6 Given
7 $\angle ARC \cong \angle K$	7 trans.
8 $\overleftrightarrow{AR} \parallel \overleftrightarrow{BK}$	8 corr ∠s ≅ ⇒ ∥
9 BARK is a ▱.	9 1 pr opp sds ≅ & ∥ ⇒ ▱

Problem 3 Given: Quadrilateral QUAD, with angles as shown
Show that QUAD is a ▱.



Solution If both pairs of opp ∠s ≅ ⇒ ▱

$\angle Q = \angle A$ & $\angle U = \angle D$

$3x(x^2 - 5x) = 3x^3 - 15x^2$ $\quad (x^2)^5 = x^{10}$

$3x^3 - 15x^2 = 3x^3 - 15x^2$ ✓ $\quad x^{10} = x^{10}$ ✓

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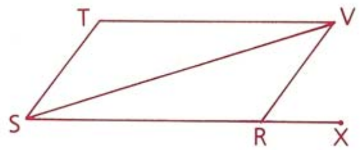
HOMEWORK

2 Given: $\angle XRV \cong \angle RST$,
$\angle RSV \cong \angle TVS$

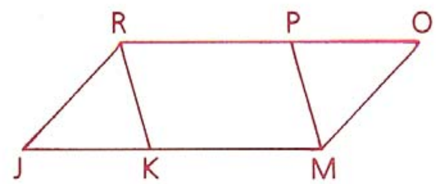
Conclusion: RSTV is a ▱.



4 Given: RKMP is a ▱.
$\angle JRK \cong \angle PMO$

Prove: RJMO is a ▱.

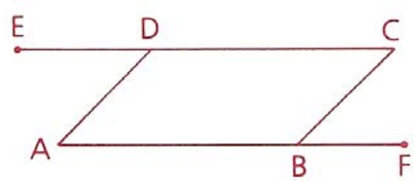
Supply each missing reason.



1 RKMP is a ▱.	1
2 $\overleftrightarrow{RO} \parallel \overleftrightarrow{JM}$	2
3 $\overline{RK} \cong \overline{PM}$	3
4 $\angle RKM \cong \angle MPR$	4
5 $\angle JKR$ supp. $\angle RKM$	5
6 $\angle OPM$ supp. $\angle MPR$	6
7 $\angle JKR \cong \angle OPM$	7
8 $\angle JRK \cong \angle PMO$	8
9 $\triangle JRK \cong \triangle OMP$	9
10 $\overline{JK} \cong \overline{PO}$	10
11 $\overline{RP} \cong \overline{KM}$	11
12 $\overline{RO} \cong \overline{JM}$	12
13 RJMO is a ▱.	13

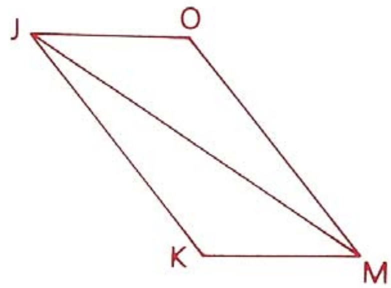
6 Given: $\overline{CD} \parallel \overline{AB}$,
$\angle EDA \cong \angle CBF$

Prove: ABCD is a parallelogram.



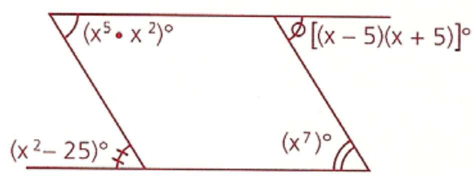
9 JKMO is a $\square$.
$\overleftrightarrow{JM}$ bisects $\angle OJK$ and $\angle OMK$.
$OJ = x + 5$, $KM = y - 3$,
$JK = 2x - 4$

a Solve for x.
b Solve for y.
c Find the perimeter of OJKM.



11 Answer Always, Sometimes, or Never: A quadrilateral is a parallelogram if **a** Diagonals are congruent **b** One pair of opposite sides are congruent and one pair of opposite sides are parallel **c** Each pair of consecutive angles are supplementary **d** All angles are right angles	11a 11b 11c 11d

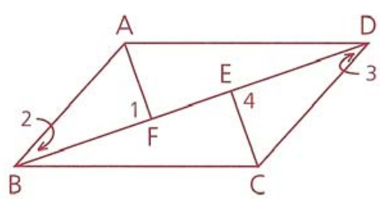
13 Prove that the quadrilateral is a parallelogram.

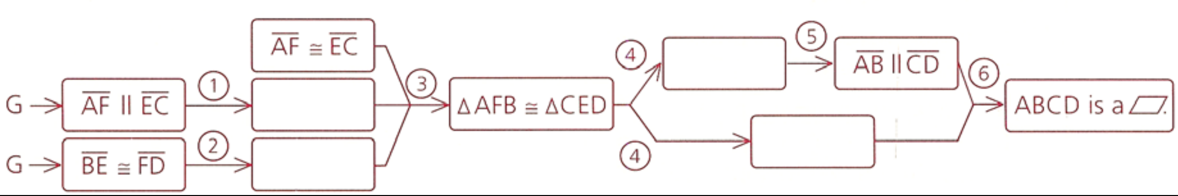


18 Given: $\overline{AF} \parallel \overline{EC}$,
$\overline{AF} \cong \overline{EC}$,
$\overline{BE} \cong \overline{FD}$

Prove ABCD is a $\square$.

Copy and complete the flow diagram for the proof. Be sure to list reasons 1–6.





1	3	5
2	4	6

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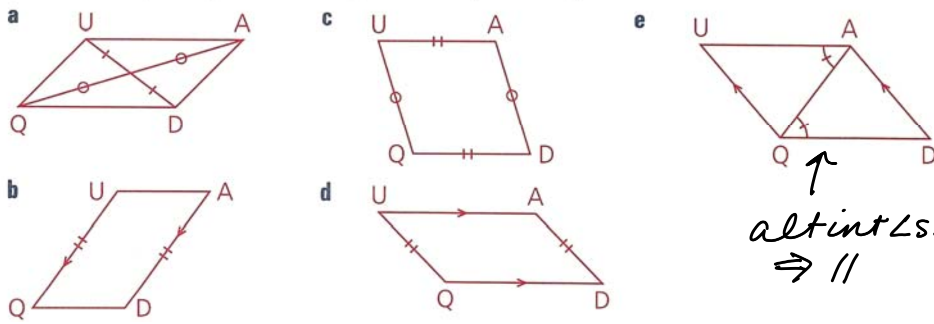
CLASS WORK

1 For each quadrilateral QUAD, state the property or definition (if there is one) that proves that QUAD is a parallelogram.



alt int ∠s ≅ ⇒ //

1A Diags bis ea. other ⇒ ▱

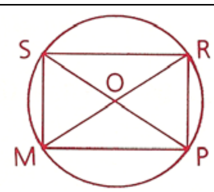
1B 1 pr sds ≅ & // ⇒ ▱

1C Both prs sds ≅ ⇒ ▱

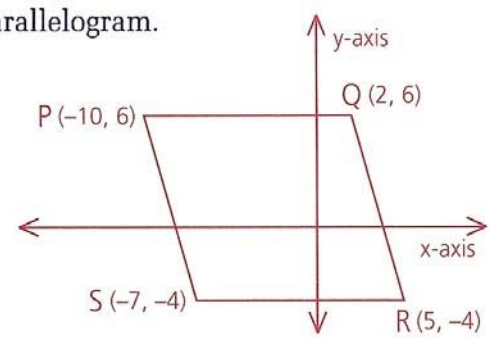
1D NONE

1E Both prs opp sds // ⇒ ▱

3 Given: ⊙O
Conclusion: SMPR is a ▱.



5 Show that PQRS is a parallelogram.



slope PQ $= \frac{\Delta y}{\Delta x} = 0$

slope SR $= 0$

} same slope ⇒ PQ ∥ SR

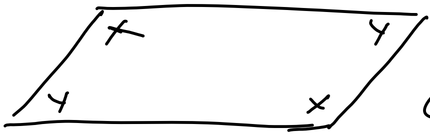
PQ $= |-10| + |2| = 12$

SR $= |-7| + |5| = 12$

} same meas ⇒ $\overline{PQ} \cong \overline{SR}$

If one pr of sides is both ≅ & ∥ then ▱

10 The measure of one angle of a parallelogram is 40 more than 3 times another. Find the measure of each angle.



$y = 3x + 40$

G: ▱

▱ ⇒ opp sides ∥

∥ ⇒ int ∠s sst supp

$4x + 40 = 180$

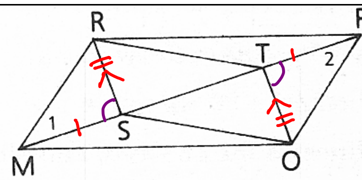
$4x = 140$

$x = 35$

$y = 145$

14 Given: RSOT is a ▱.
$\overline{MS} \cong \overline{TP}$

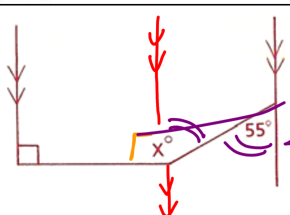
Prove: MOPR is a ▱.



1 RSOT is a ▱.	1 Given
2 $\overline{MS} \cong \overline{TP}$	2 Given
3 $\overline{RS} \parallel \overline{TO}$	3 ▱ ⇒ opp sds ∥
4 $\angle RSM \cong \angle PTO$	4 ∥ ⇒ alt ext ∠s ≅
5 $\overline{RS} \cong \overline{TO}$	5 ▱ ⇒ opp sds ≅
6 $\triangle RSM \cong \triangle OTP$	6 SAS (2, 4, 5)
7 $\overline{RM} \cong \overline{PO}$	7 CPCTC
8 $\angle 1 \cong \angle 2$	8 CPCTC
9 $\overline{RM} \parallel \overline{PO}$	9 alt int ∠s ≅ ⇒ ∥
10 MOPR is a ▱.	10 one pr of sides ≅ & ∥ ⇒ ∥ (7 & 10)

17 Find the value of x.

∥ ⇒ int ∠s SST supp



② ← ∥ ⇒ alt int ∠s ≅

① parallel postulate

∴ $x = 90 + 55$

$= 145°$