Polar and Parametric Review | Ad majorem Dei gloriam | Accelerated Algebra 2 / Precalculus
Mr. Reis

Name KEY

For questions one through eight, do the following:

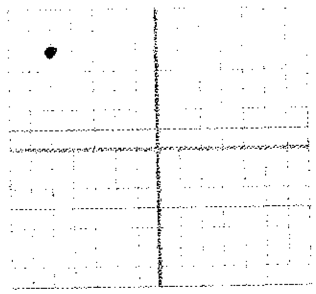
- Convert to the alternative form (polar → rectangular or rectangular → polar
- Graph on the rectangular plane
- Graph on the polar plane

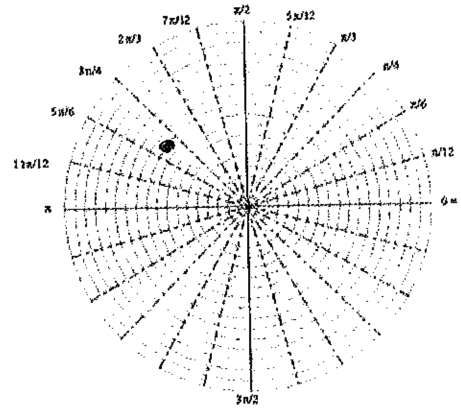
1. The rectangular coordinate point (-6, 5)

$r = \sqrt{(-6)^2 + (5)^2} = \sqrt{61} \approx 7.810$

$\theta = \tan^{-1}\left(\frac{5}{-6}\right) \approx -.695$

$\theta \approx -.695 + \pi \approx 2.447$

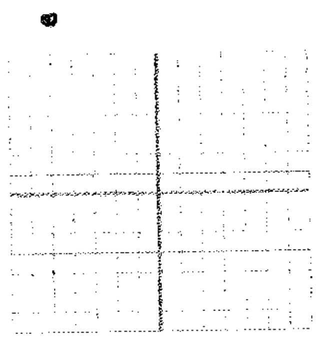


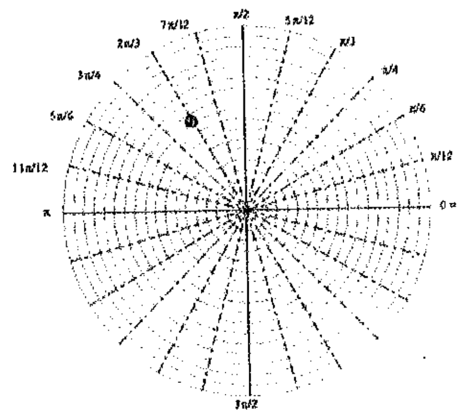


2. The polar coordinate point $\left(-10, -\frac{\pi}{3}\right)$

$x = -10 \cos \frac{-\pi}{3} = -5$

$y = -10 \sin \frac{-\pi}{3} = \frac{10\sqrt{3}}{2} = 5\sqrt{3} \approx 8.660$



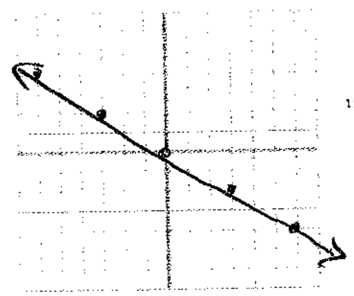


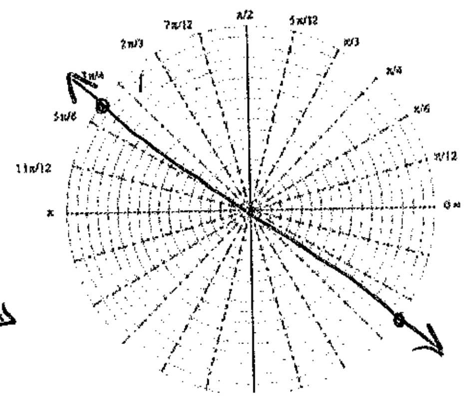
3. The rectangular equation $y = -\frac{2}{3}x$

$r \sin\theta = \frac{-2}{3} r \cos\theta$

$\tan\theta = \frac{-2}{3}$

$\theta = \tan^{-1}\left(\frac{-2}{3}\right) \approx -.588$

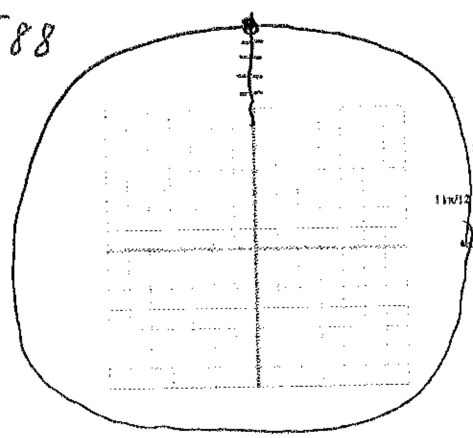


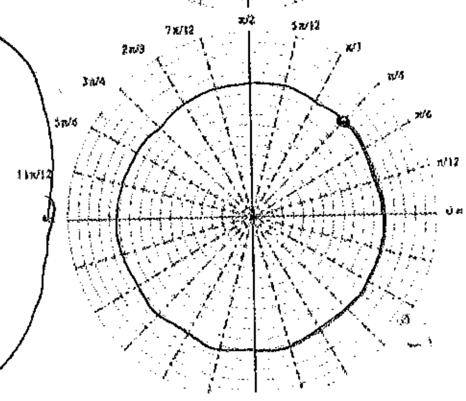


4. The polar equation $r = 12$

$\sqrt{x^2 + y^2} = 12$

$x^2 + y^2 = 144$

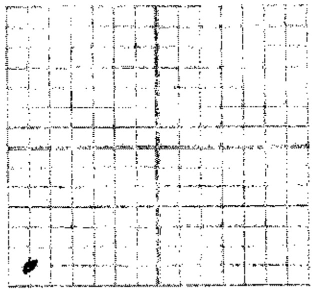


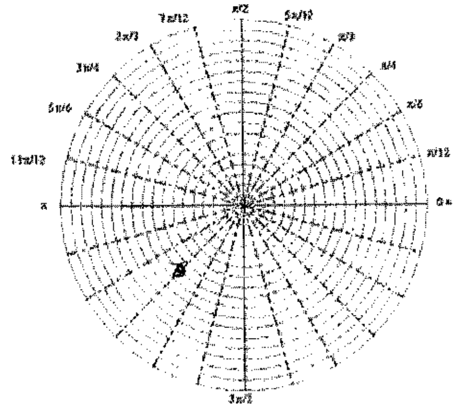


✓ 5. The rectangular coordinate point (-6, -6)

$$\left(6\sqrt{2},\ \frac{5\pi}{4}\right)$$

≈ 8.5



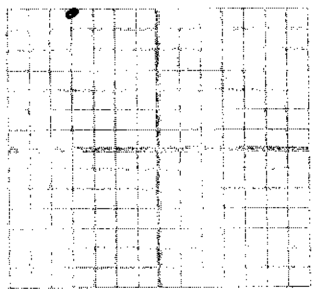


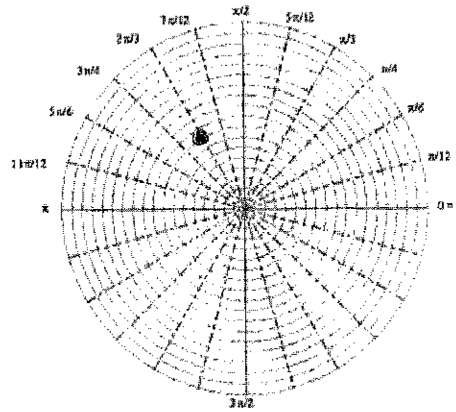
✓ 6. The polar coordinate point $\left(8, \frac{2\pi}{3}\right)$

$$\left(8\cos\frac{2\pi}{3},\ 8\sin\frac{2\pi}{3}\right)$$

$$\left(-4,\ \frac{8\sqrt{3}}{2}\right)$$

$$\left(-4,\ 4\sqrt{3}\right) \approx (-4, 6.9)$$





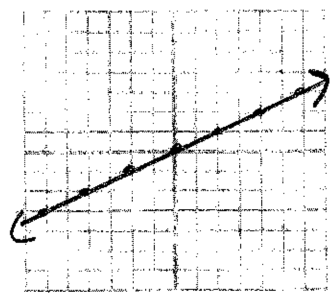
✓ 7. The rectangular equation $y=\frac{1}{2}x$

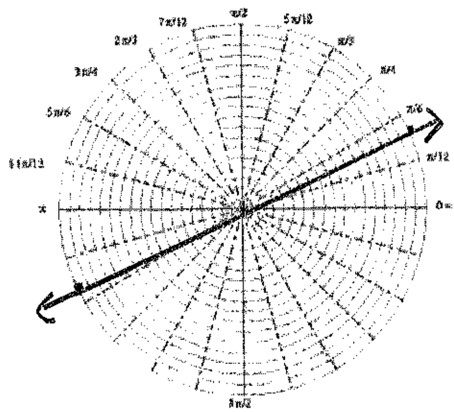
$$r\sin\theta = \frac{1}{2} r\cos\theta$$

$$\tan\theta = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) \approx 26.6°$$

$\approx$ little less than $\pi/6$

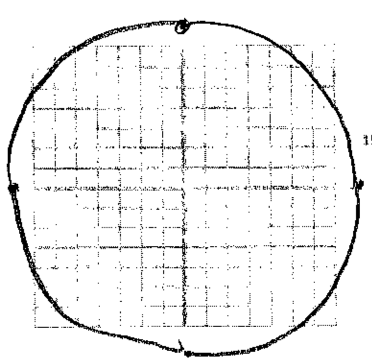


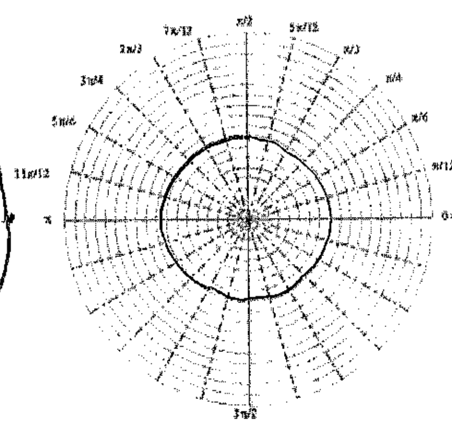


✓ 8. The polar equation $r=8$

$$\sqrt{x^2+y^2} = 8$$

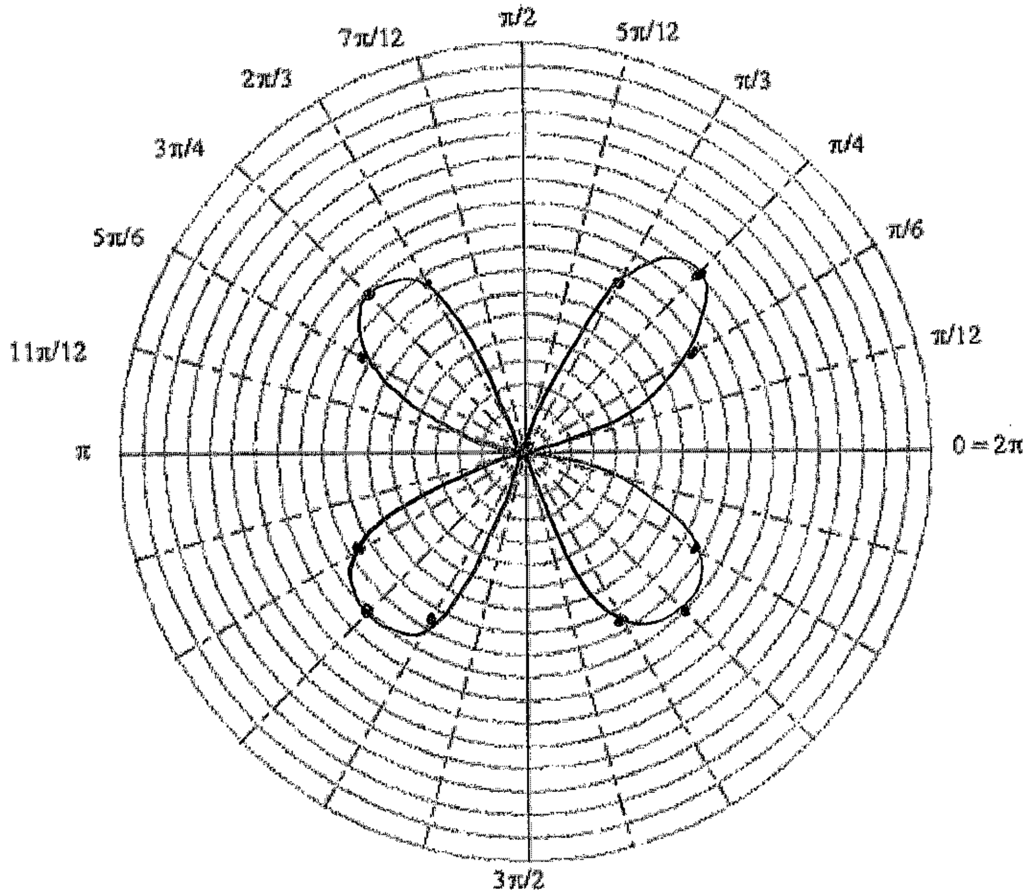
$$x^2+y^2 = 64$$





9. Complete the chart of values for the polar equation $r = 10\sin(2\theta)$. Round values to the nearest tenth. Plot the points on the polar coordinate plane and connect the points **in order** as you plot them. You may have to fill in more points than your chart contains to get a complete graph.

θ	r
0	0
$\frac{\pi}{6}$	8.7
$\frac{\pi}{4}$	10
$\frac{\pi}{3}$	8.7
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-8.7
$\frac{3\pi}{4}$	-10
$\frac{5\pi}{6}$	-8.7
π	0



10. What type of symmetry does the graph from question nine have? Write yes/no below:

(a) With respect to the pole yes
(b) With respect to the polar axis yes
(c) With respect to $\theta = \frac{\pi}{2}$ yes
(d) No symmetry no

11. Convert the equation $r = 2\sec\theta$ from polar to rectangular form.

$x = r\cos\theta$

$\frac{x}{\cos\theta} = r$

$\frac{x}{\cos\theta} = \frac{2}{\cos\theta}$

$\boxed{x = 2}$

12. Convert the equation $y = x$ from rectangular to polar form.

$r\sin\theta = r\cos\theta$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\boxed{\theta = \frac{\pi}{4}}$

13. The center field wall in a baseball stadium is 8 feet high and 415 feet from home plate. A baseball is hit at a point 3.2 feet above the ground. It leaves the bat at an angle of θ degrees with a speed of 95 mph. There are 5,280 feet in a mile.

Horizontal distance $x=(v_0 \cos\theta)t$ (distance in feet, time in seconds)

Vertical distance $y=-16t^2+(v_0 \sin\theta)t+s_0$ (distance in feet, time in seconds, speed in ft/s)

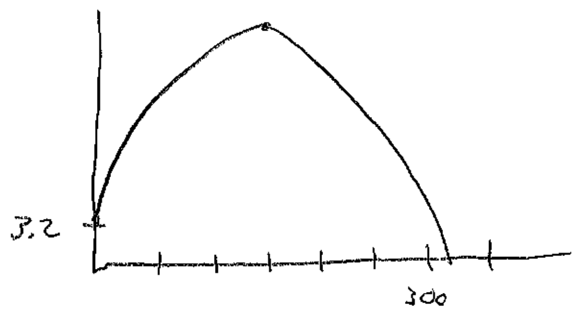
(a) Write a set of parametric equations to model the path of the baseball.

$$x = \left(\frac{418}{3} \cos \theta\right) t$$

$$y = -16t^2 + \left(\frac{418}{3} \sin \theta\right)t + 3.2$$

$$95 \frac{mi}{hr} \cdot \frac{1 \ hr}{3600 s} \cdot \frac{5280 \ ft}{1 \ mi} = \frac{418}{3} \ ft./s$$

(b) Use your calculator to graph the path of the baseball at $\theta=16^\circ$. Sketch the graph.



(c) Is the hit a home run if it is hit at 16 degrees? Justify your answer.

$$8 = -16t^2 + \left(\frac{418}{3} \sin 16^\circ\right)t + 3.2$$

$$= -16t^2 + 38.405t - 4.8$$

$$t \approx \frac{-38.405 \pm \sqrt{(38.405)^2 - 4(-16)(-4.8)}}{-32}$$

$$t \approx 0.132 s \text{ or } 2.268 s$$

$$x = \left(\frac{418}{3} \cos 16\right)(2.268)$$

$$x \approx 303.766 \ ft.$$

No, it's below 8 ft. after 303 ft, fence is at 415 ft.

(d) Is the hit a home run if it is hit at 22 degrees? Justify your answer.

$$8 = -16t^2 + \left(\frac{418}{3} \sin 22^\circ\right)t + 3.2$$

$$0 = -16t^2 + 52.195t - 4.8$$

$$t \approx \frac{-52.195 \pm \sqrt{(52.195)^2 - 4(-16)(-4.8)}}{-32}$$

$$t \approx 0.095 s \text{ or } t \approx 3.167 s$$

$$x = \frac{418}{3}(\cos 22^\circ)(3.167)$$

$$x \approx 409.137$$

No, it's below 8 ft. after 409 ft, fence is at 415 ft.

14. A baseball is thrown from the outfield with an initial velocity of 60 mph at an angle of 44 degrees. It is 6.5 feet above the ground when it leaves the outfielder's hand.

Horizontal distance $x=(v_0 \cos\theta)t$ (distance in feet, time in seconds, speed in ft/s)

Vertical distance $y=-16t^2+(v_0 \sin\theta)t+s_0$ (distance in feet, time in seconds, speed in ft/s)

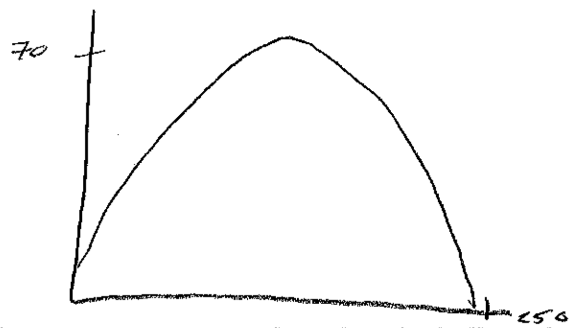
✓ (a) Express the parametric equations which model the flight of the ball.

$x = (88 \cos 44)t$

$y = -16t^2 + (88 \sin 44)t + 6.5$

$\frac{60 \text{ mi}}{\text{hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = 88 \text{ ft/s}$

✓ (b) Use your calculator to graph the path of the ball. Sketch your graph.



✓ (c) If the shortstop jumps up and catches the ball at a height of 8 feet above the ground, what is the horizontal distance the ball traveled?

$8 = -16t^2 + (88 \sin 44)t + 6.5$

$0 = -16t^2 + 61.130t - 1.5$

$t \approx \frac{-61.130 \pm \sqrt{61.130^2 - 4(-16)(-1.5)}}{2(-16)}$

$t \approx .025$ or $t \approx 3.796$

$x \approx (88 \cos 44)(3.796) \approx \boxed{240.294}$

✓ (d) How long was the ball in the air?

$\boxed{3.796 \text{ s}}$

✓ (e) What is the maximum height the ball reached and how many seconds did it take to reach this height?

$t_{max} = \frac{-61.130}{2(-16)} \approx \boxed{1.910 \text{ s}}$

$\boxed{y_{max} \approx 64.889 \text{ s}}$

✓ (f) If the shortstop had missed the ball and the pitcher had to dive to the ground to catch it, how far would the ball have traveled horizontally?

$0 = -16t^2 + 61.130t + 6.5$

$t \approx \frac{-61.130 \pm \sqrt{(61.130)^2 - 4(-16)(6.5)}}{-32}$

$t \approx -.104$ or $t \approx 3.924$

$\boxed{x \approx 248.397 \text{ ft}}$

15. A rocket is shot off the root of a builder 100 meters above the ground at an angle of 65 degrees. The initial velocity of the rocket is 30 meters per second (assume there is no wind).

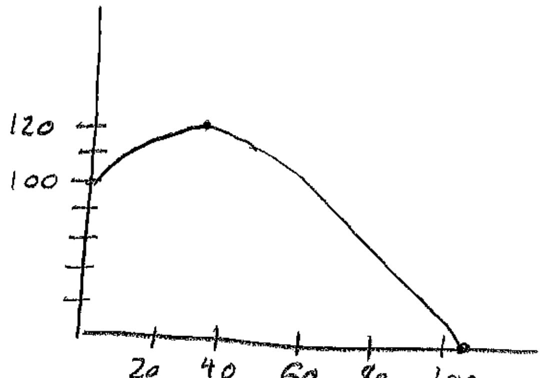
Horizontal distance $x=(v_0\cos\theta)t$ (distance in meters, time in seconds, speed in m/s)
Vertical distance $y=-4.9t^2+(v_0\sin\theta)t+s_0$ (distance in meters, time in seconds, speed in m/s)

(a) Express the parametric equations which model the flight of the rocket.

$x = (30\cos 65°)t$

$y = -4.9t^2 + (30\sin 65°)t + 100$

(b) Use your calculator to graph the path of the rocket. Sketch your graph.



(c) What is the maximum height of the rocket?

$t_{max} = \dfrac{-30\sin 65}{-9.8} \approx 2.774$ s

$y_{max} = -4.9(2.774)^2 + (30\sin 65)(2.774) + 100 \approx \boxed{137.717 \text{ ft}}$

(d) If the rocket is launched at exactly 9:30 am, at what time does the rocket hit the ground?

$0 = -4.9t^2 + (30\sin 65)t + 100$

$t \approx \dfrac{-30\sin 65 \pm \sqrt{(30\sin 65)^2 - 4(-4.9)(100)}}{2(-4.9)} \approx -2.257$ or 8.076

$\boxed{9:30:08.076 \text{ AM}}$

(e) What was the horizontal distance the rocket traveled?

$x = (30\cos 65)(8.076) \approx \boxed{102.392 \text{ ft}}$

16. A football is kicked from the ground at an angle of 53 degrees and an initial velocity of 68 ft/s.

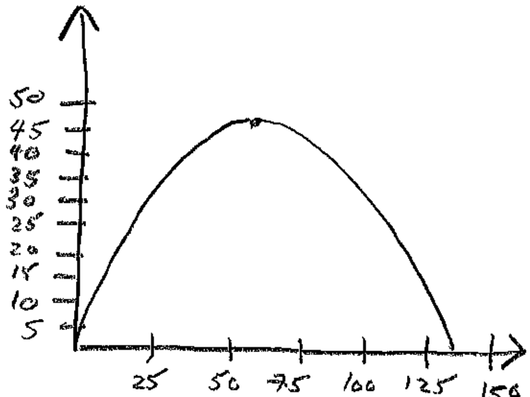
Horizontal distance $x=(v_0 \cos\theta)t$ (distance in feet, time in seconds, speed in ft/s)

Vertical distance $y=-16t^2+(v_0 \sin\theta)t+s_0$ (distance in feet, time in seconds, speed in ft/s)

✓ (a) Express the parametric equations which model the flight of the ball.

$$x = (68 \cos 53)t$$
$$y = -16t^2 + (68 \sin 53)t + 0$$

✓ (b) Use your calculator to graph the path of the football. Sketch your graph.



✓ (c) If the ball is caught 5 feet above the ground, what was the horizontal length of the kick?

$$5 = -16t^2 + (68 \sin 53)t$$
$$0 = -16t^2 + (68 \sin 53)t - 5$$
$$t = \frac{-68 \sin 53 \pm \sqrt{(68 \sin 53)^2 - 4(-16)(-5)}}{2(-16)}$$
$$t \approx 0.095 \quad \text{or} \quad \boxed{t \approx 3.299}$$

$$x = (68 \cos 53^\circ)(3.299)$$
$$x \approx \boxed{135.006 \text{ ft.}}$$

✓ (d) How long was the ball in the air, and what was its maximum height?

$$\boxed{t \approx 3.299}$$

$$t_{max} = \frac{-68 \sin 53^\circ}{-32} \approx 1.697 \text{ s}$$

$$y_{max} = -16(1.697)^2 + (68 \sin 53)(1.697) \approx \boxed{46.082 \text{ ft.}}$$

17. A baseball player hits a line drive off the end of his bat 4 feet above the ground. The ball's initial velocity is 84 mph and it is hit at an angle of 27 degrees toward the alley in left field.

Horizontal distance $x=(v_0 \cos\theta)t$ (distance in feet, time in seconds, speed in ft/s)

Vertical distance $y=-16t^2+(v_0 \sin\theta)t+s_0$ (distance in feet, time in seconds, speed in ft/s)

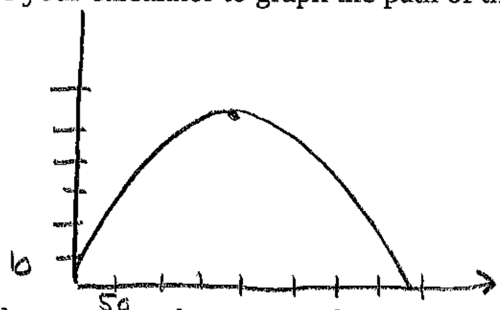
✓ (a) Write the parametric equations which model the flight of the ball.

$x = (123.2 \cos 27°)t$

$y = -16t^2 + (123.2 \sin 27°)t + 4$

$84 \text{ mph} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} = 123.2 \text{ ft/s}$

✓ (b) Use your calculator to graph the path of the ball. Sketch your graph.



✓ (c) If the opposing shortstop is playing in the direct path of the ball, 120 feet from home plate, does he have a chance to catch the ball? Justify your answer.

$120 = (123.2 \cos 27°)t$

$\boxed{t \approx 1.093 \text{ s}}$

$\boxed{y \approx 46.019 \text{ ft}}$ (no way, it's too high)

✓ (d) If the Brewers are playing in Wrigley Field and the fence at the left field alley is 11.5 feet high and 368 feet from home plate, does the ball clear the fence?

$368 = (123.2 \cos 27°)t$

$t \approx 3.352 \text{ s}$

$y \approx 11.708 \text{ ft}$ (yes)

✓ (e) Repeat part (d) if they are at Miller Park where the left field alley is 371 feet away from home plate and the fence is 8 feet high.

$371 = (123.2 \cos 27°)t$

$t \approx 3.386 \text{ s}$

$y \approx 10.259 \text{ ft}$ (yes)

✓ (f) What is the maximum height of the ball? How long is it in the air if it is not caught and it does not hit anything before it hits the ground?

$t_{max} = \frac{-123.2 \sin 27°}{-32} \approx 1.748 \text{ s}$

$y_{max} \approx \boxed{52.880 \text{ ft.}}$

$t = \frac{-123.2 \sin 27 \pm \sqrt{(123.2 \sin 27)^2 - 4(-16)(4)}}{-32}$

$t \approx -0.070$ or $\boxed{t \approx 3.566 \text{ s}}$