Mathematical Induction Practice   Ad majorem Dei gloriam   Accelerated Algebra 2 / Precalculus
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Prove each statement true by mathematical induction.

1. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

Show true for $k=1$

$\frac{1}{1\cdot 2}=\frac{1}{1+1}$

$\frac{1}{2}=\frac{1}{2}$ ✓

Assume true for k, show true for $k+1$

$$\underbrace{\frac{1}{1\cdot 2}+\ldots\frac{1}{k(k+1)}}_{}+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$$

$$\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}\rightarrow\frac{k(k+2)}{(k+1)(k+2)}+\frac{1}{(k+1)(k+2)}\rightarrow\frac{k^2+2k+1}{(k+1)(k+2)}$$

$$\uparrow \frac{(k+1)^2}{(k+1)(k+2)} \uparrow \boxed{\frac{k+1}{k+2}}$$

2. $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+..+\frac{1}{2^n}=1-\frac{1}{2^n}$

Show true for $k=1$

$\frac{1}{2^1}=1-\frac{1}{2^1}$

$\frac{1}{2}=1-\frac{1}{2}$

$\frac{1}{2}=\frac{1}{2}$ ✓

Assume true for k, show true for $k+1$

$$\underbrace{\frac{1}{2}+\ldots+\frac{1}{2^k}}_{}+\frac{1}{2^{k+1}}=1-\frac{1}{2^{k+1}}$$

$$1-\frac{1}{2^k}+\frac{1}{2^{k+1}}\rightarrow 1-\frac{2}{2^{k+1}}+\frac{1}{2^{k+1}}\rightarrow 1-\frac{1}{2^{k+1}}$$ ✓

3. The sum of the first n odd integers is the square of n:
$1+3+5+\ldots+(2n-1)=n^2$

Show true for $k=1$

$1 = 1^2$

$1 = 1$ ✓

Assume true for k, show true for $k+1$

$\underbrace{1 + \ldots + (2k-1)} + (2(k+1)-1) = (k+1)^2$

$k^2 + 2k + 2 - 1 =$

$k^2 + 2k + 1 =$

✓ $(k+1)^2 =$

4. $\sum_{k=1}^{n}(3k-2)=\frac{n}{2}(3n-1)$

Show true for $j=1$

$3(1)-2 = \frac{1}{2}(3(1)-1)$

$1 = \frac{1}{2}\cdot 2$

$1 = 1$ ✓

Assume true for j, show true for $j+1$ $= \frac{j+1}{2}(3(j+1)-1)$

$\sum_{k=1}^{j+1}(3k-2) = \left[\sum_{k=1}^{j}(3k-2)\right] + \left[3(j+1)-2\right]$

$= \frac{j}{2}(3j-1) + 3(j+1) - 2$

$= \frac{3j^2 - j}{2} + \frac{6j+6}{2} - \frac{4}{2}$

$\frac{3j^2+5j+2}{2} = \frac{(j+1)(3j+2)}{2}$

$\frac{(j+1)(3(j+1)-1)}{2}$ ✓

✓ 5. $\sum_{j=1}^{n}(2j)=n^2+n$

Show true for $k=1$

$2\cdot 1 = 1^2+1$

$2 = 2$ ✓

Assume true for k, show true for $k+1$

$\sum_{j=1}^{k+1}(2j) = (k+1)^2 + k+1$

$= \sum_{j=1}^{k}(2j) + 2(k+1)$

$= k^2+k + 2k+2$

$= k^2+2k+1+k+1 = (k+1)^2+(k+1)$ ✓

✓ 6. $1+2+2^2+2^3+\ldots+2^{n-1}=2^n-1$

Show true for $k=1$

$2^0 = 2^1-1$

$1 = 2-1$

$1 = 1$ ✓

Assume true for k, show true for $k+1$

$\underbrace{1+\ldots+2^{k-1}}_{2^k-1} + 2^k = 2^{k+1}-1$

$2^k-1+2^k =$

$2\cdot 2^k-1 =$

✓ $2^{k+1}-1 =$

7. $1+3+3^2+3^3+\ldots+3^{n-1}=\dfrac{3^n-1}{2}$

Show true for $k=1$

$3^{1-1} = \dfrac{3^1-1}{2}$

$3^0 = \dfrac{2}{2}$

$1 = 1$ ✓

Assume true for k, show true for $k+1$

$$\underbrace{1+\ldots+3^{k-1}} + 3^k = \frac{3^{k+1}-1}{2}$$

$$\frac{3^k-1}{2} + \frac{2\cdot 3^k}{2} =$$

$$\frac{3\cdot 3^k-1}{2} =$$

$$\frac{3^{k+1}-1}{2} = \checkmark$$

8. The sum of the squares of the first n Fibonacci numbers is the product of the nth and (n+1)th Fibonacci numbers.

$1, 1, 2, 3, 5, \ldots$

Show true for $k=1$

$1^2 = 1(1)$

$1 = 1$ ✓

Assume true for k, show true for $k+1$

$$\underbrace{F_1^2+\ldots+F_k^2} + F_{k+1}^2 = (F_{k+1})(F_{k+2})$$

$$F_k(F_{k+1}) + (F_{k+1})^2$$

$$(F_{k+1})(F_k + F_{k+1}) \leftarrow$$ this equals F_{k+2} by def.

$$(F_{k+1})(F_{k+2})$$ ✓