Chapter 7 Review
No calculator (#1-#8)

Ad majorem Dei gloriam

Accelerated Algebra 2 / Precalculus
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Name KEY

✓ 1. Solve the system by using a matrix and rewriting in row-echelon form.

$2x+2y-z=2$
$x-3y+z=28$
$-x+y=14$

$$\left[\begin{array}{ccc|c} 2 & 2 & -1 & 2 \\ 1 & -3 & 1 & 28 \\ -1 & 1 & 0 & 14 \end{array}\right]$$

① Add 2×3 to 1: $-2 \quad 2 \quad 0 \quad 28$

② Add 2 to 3

③ Add 2×3 to 1: $0 \quad -4 \quad 2 \quad 84$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 114 \\ 1 & -3 & 1 & 28 \\ 0 & -2 & 1 & 42 \end{array}\right]$$

$z = 114$ $\qquad -2y + 114 = 42$

$-2y = -72$

$y = 36$

$x - 108 + 114 = 28$

$x + 6 = 28$

$x = 22$

final EF

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 28 \\ 0 & -2 & 1 & 42 \\ 0 & 0 & 1 & 114 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 0 & 4 & -1 & 30 \\ 1 & -3 & 1 & 28 \\ -1 & 1 & 0 & 14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 0 & 4 & -1 & 30 \\ 1 & -3 & 1 & 28 \\ 0 & -2 & 1 & 42 \end{array}\right]$$

(22, 36, 114)

Given matrices $A=\begin{bmatrix} 3 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}, B=\begin{bmatrix} 4 & 1 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}, C=\begin{bmatrix} 1 & 2 & 3 \\ 8 & 7 & 5 \end{bmatrix}$

✓ 2. Find A + B

undefined

✓ 3. Find 2A – 3C

$$2A = \begin{bmatrix} 6 & 2 & 0 \\ -2 & 0 & 4 \end{bmatrix} \qquad -3C = \begin{bmatrix} -3 & -6 & -9 \\ -24 & -21 & -15 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -4 & -9 \\ -26 & -21 & -11 \end{bmatrix}$$

✓ 4. Find AB

2×3 by $3\times2 = 2\times2$

$$\begin{bmatrix} (3)(4)+(1)(-1)+(0)(2) & (3)(1)+(1)(0)+(0)(-1) \\ (-1)(4)+(0)(-1)+(2)(2) & (-1)(1)+(0)(0)+(2)(-1) \end{bmatrix} = \begin{bmatrix} 11 & 3 \\ 0 & -3 \end{bmatrix}$$

✓ 5. Find BA

3×2 by $2\times3 = 3\times3$

$$\begin{bmatrix} (4)(3)+(1)(-1) & (4)(1)+(1)(0) & (4)(0)+(1)(2) \\ (-1)(3)+(0)(-1) & (-1)(1)+(0)(0) & (-1)(0)+(0)(2) \\ (2)(3)+(-1)(1) & (2)(1)+(-1)(0) & (2)(0)+(-1)(2) \end{bmatrix} = \begin{bmatrix} 11 & 4 & 2 \\ -3 & -1 & 0 \\ 7 & 2 & -2 \end{bmatrix}$$

✓ 6. Write the partial fraction decomposition for $\frac{2x-3}{(x-1)^2}$

$\frac{A}{x-1} + \frac{B}{(x-1)^2}$

$\frac{A(x-1)}{(x-1)^2} + \frac{B}{(x-1)^2}$

$Ax - A + B = 2x - 3$

$A = 2$

$-A + B = -3$

$-2 + B = -3$

$B = -1$

$\frac{2}{x-1} - \frac{1}{(x-1)^2}$

✓ 7. Show that A and B are inverse matrices $A=\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}, B=\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} (1)(3)+(-2)(1) & (1)(2)+(-2)(1) \\ (-1)(3)+(3)(1) & (-1)(2)+(3)(1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$\begin{bmatrix} (3)(1)+(2)(-1) & (3)(-2)+(2)(3) \\ (1)(1)+(1)(-1) & (1)(-2)+(1)(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

✓ 8. Find the inverse of the matrix $C=\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$ by augmenting it with the identity matrix.

$\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ -2 & 2 & 0 & 1 \end{array}\right]$

① Add 2×1 to 2

$\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 4 & 0 & 2 & 1 \end{array}\right]$

② Divide 2 by 4

$\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 1 & 0 & \frac{1}{2} & \frac{1}{4} \end{array}\right]$

③ Add -3×2 to 1

$\left[\begin{array}{cc|cc} 0 & -1 & -\frac{1}{2} & -\frac{3}{4} \\ 1 & 0 & \frac{1}{2} & \frac{1}{4} \end{array}\right]$

④ Divide 1 by -1

$\left[\begin{array}{cc|cc} 0 & 1 & \frac{1}{2} & \frac{3}{4} \\ 1 & 0 & \frac{1}{2} & \frac{1}{4} \end{array}\right]$

⑤ Swap 1 and 2

$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & \frac{1}{2} & \frac{3}{4} \end{array}\right]$

$C^{-1} = \begin{bmatrix} 1/2 & 1/4 \\ 1/2 & 3/4 \end{bmatrix}$

9. Solve the system using a matrix equation and your calculator. Show all work.

$x+y+z=2$
$2x-y-2z=1$
$-x+3y-2z=-2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -2 \\ -1 & 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$A \cdot X = B$

$AX = B$

$A^{-1}AX = A^{-1}B$

$X = A^{-1}B = \begin{bmatrix} 23/19 \\ 3/19 \\ 12/19 \end{bmatrix}$

$\left(\frac{23}{19}, \frac{3}{19}, \frac{12}{19}\right)$

10. Find AB if $A=\begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}$ and $B=[3 \;\; 2 \;\; -4]$

$$\begin{bmatrix} 15 & 10 & -20 \\ -6 & -4 & 8 \\ 9 & 6 & -12 \end{bmatrix}$$

11. Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 0 & -1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 4 & -1 & -1 \\ 0 & 0 & -1 \\ -3 & 1 & 2 \end{bmatrix}$$

12. Give the 3x3 identity matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$