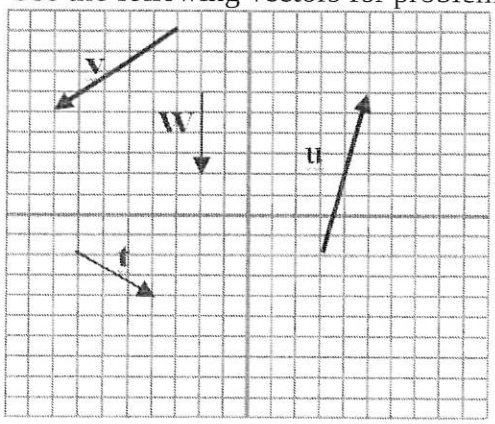
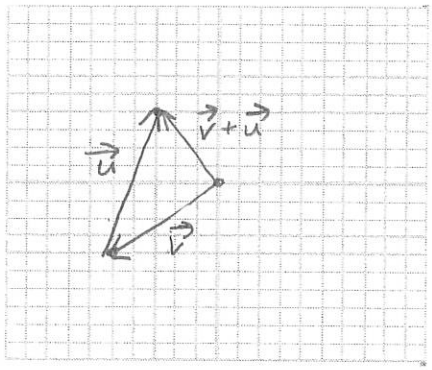
Vector Review Ad majorem Dei gloriam Accelerated Algebra 2 / Precalculus
Mr. Reis

Name KEY

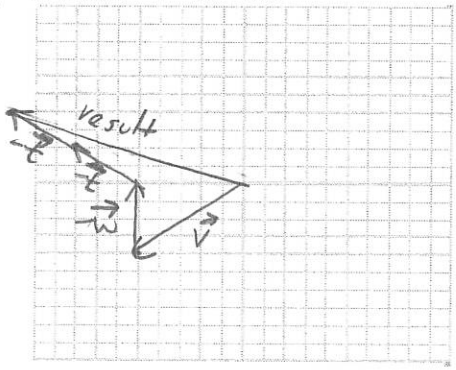
Use the following vectors for problems one through eight.



✓ 1. Draw $\vec{v}+\vec{u}$



✓ 2. Draw $\vec{v}-\vec{w}-2\vec{t}$



✓ 3. Write each vector in component form:

(a) $\vec{v}$ $\langle -5, -4 \rangle$

(b) $\vec{t}$ $\langle 3, -2 \rangle$

(c) $\vec{w}$ $\langle 0, -4 \rangle$

✓ 4. Find a unit vector in the same direction as $\vec{u}$ (in component form)

$\vec{u} = \langle 2, 8 \rangle$ $\|\vec{u}\| = \sqrt{2^2+8^2} = \sqrt{68} = 2\sqrt{17}$

unit vector $\left\langle \frac{2}{2\sqrt{17}}, \frac{8}{2\sqrt{17}} \right\rangle$

$\left\langle \frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17} \right\rangle$

5. Find the resultant vectors algebraically (in component form)

✓ (a) $\vec{v}+\vec{u}$

$\langle -3, 4 \rangle$

✓ (b) $3\vec{w}-\vec{u}+t$

$\langle 0, -12 \rangle$
$\langle -2, -8 \rangle$
$\langle 3, -2 \rangle$

$\langle 1, -22 \rangle$

✓ 6. Find the angle (in degrees) between the two vectors $\vec{u}=8\vec{i}+15\vec{j}$ and $\vec{v}=-4\vec{i}+5\vec{j}$

$\cos\theta = \frac{(8)\cdot(-4)+(15)(5)}{17\sqrt{41}}$ $\|\vec{u}\| = 17$ $\|\vec{v}\| = \sqrt{4^2+5^2} = \sqrt{41}$

$\cos\theta = \frac{43}{17\sqrt{41}} \approx 66.732°$

7. The Columbia River runs due west briefly between Oregon and Washington. A boat is crossing the river at a heading of 210°. The speed of the boat is 15 miles per hour. The current is moving at a speed of 4.7 miles per hour.

✓ (a) Express the boat velocity as a vector, b, in trigonometric form and then convert it to component form.

$\vec{b} = 15\langle \cos 240°, \sin 240° \rangle$

$= \langle 15 \cdot -\frac{1}{2}, 15 \cdot -\frac{\sqrt{3}}{2} \rangle$

$= \langle -\frac{15}{2}, -\frac{15\sqrt{3}}{2} \rangle$

✓ (b) Express the water current as a vector, c, in trigonometric form and then convert it to component form.

$\vec{c} = \langle 4.7 \langle \cos 180°, \sin 180° \rangle$

$= \langle 4.7 \cdot -1, 4.7 \cdot 0 \rangle$

$= \langle -4.7, 0 \rangle$

✓ (c) If you were standing on shore, what is the velocity vector, g, that you would observe? Express it in both component form and trigonometric form.

$\vec{g} = \langle -12.2, -12.990 \rangle$

$\|\vec{g}\| = \sqrt{(-12.2)^2 + (-12.990)^2}$

$= 17.821$ mph

$17.821 \langle \cos 226.796°, \sin 226.796° \rangle$

$\tan \theta = \frac{-12.990}{-12.2}$

$\theta \approx 46.796° + 180$ (opp. direction)

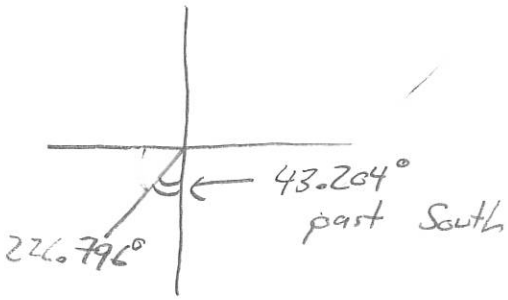
$\approx 226.796°$

see picture above

✓ (d) Find the "ground speed" of the boat.

17.821 mph

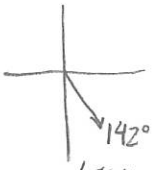
✓ (e) What is the actual compass heading of the boat? Draw a diagram.



223.203°

8. An airplane is flying at its cruising altitude at a compass heading of 142°. Its air speed is 680 mph. The wind is coming out of the SW at heading of 16° and a speed of 78 mph.

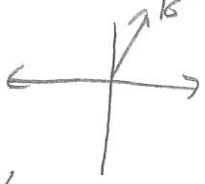
✓ (a) Express the velocity vector of the plane relative to the air in trig form and then convert to component form.



$680° \langle \cos 308°, \sin 308° \rangle$

$\langle 418.650, -535.847 \rangle$

✓ (b) Express the velocity vector of the wind in trigonometric form and then convert to component form.



$78 \langle \cos 74°, \sin 74° \rangle$

$\langle 21.500, 74.978 \rangle$

✓ (c) Express the resultant vector of the plane plus the wind in component form and then convert it to trigonometric form.

$\langle 440.15, -460.869 \rangle$

$\text{magnitude} = \sqrt{440.15^2 + (-460.869)^2}$

$\approx 637.285 \text{ mph}$

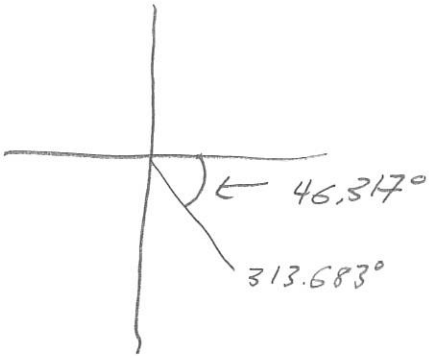
$\tan \theta = \frac{-460.869}{440.15}$ $\quad 637.285 \langle \cos 313.683, \sin 313.683 \rangle$

$\theta \approx -46.317 \approx 313.683°$

✓ (d) What is the ground speed of the plane?

637.285

✓ (e) What is the actual compass heading of the plane? Draw a diagram.

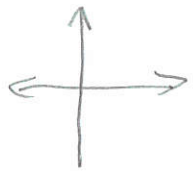


$90 + 46.317° = 136.317°$

9. Jacques is on a trip by canoe up the Wisconsin River at a point north of Merrimac where the river runs from the northeast to the southwest through Lake Wisconsin. Jacques can canoe at 5 mph and the current in the lake runs at 1 mph. Jacques starts across the lake from the south, canoeing straight north.

(a) Express the canoe velocity as a vector in trigonometric form and then convert it to component form.

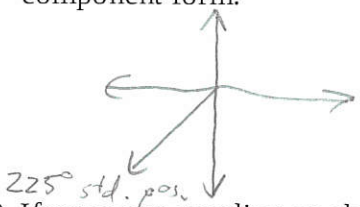
θ heading = 90° std. pos.



$5\langle \cos 90°, \sin 90° \rangle$

$\langle 0, 5 \rangle$

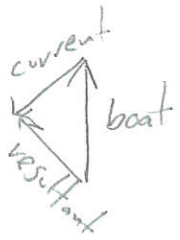
(b) Express the water current in the lake as a vector in trigonometric form and then convert it to component form.



$1\langle \cos 225°, \sin 225° \rangle$

$\left\langle \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2} \right\rangle$

(c) If you were standing on shore, what is the velocity vector that you would observe? Express it in both component form and trigonometric form.



$\left\langle \frac{-\sqrt{2}}{2}, 5 - \frac{\sqrt{2}}{2} \right\rangle$

magnitude $= \sqrt{\left(\frac{-\sqrt{2}}{2}\right)^2 + \left(5 - \frac{\sqrt{2}}{2}\right)^2}$

$= 4.351 \text{ mph}$

$\theta = \tan^{-1}\left(\frac{5 - \frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}}\right) \approx -80.646 + 180° \approx 99.354°$

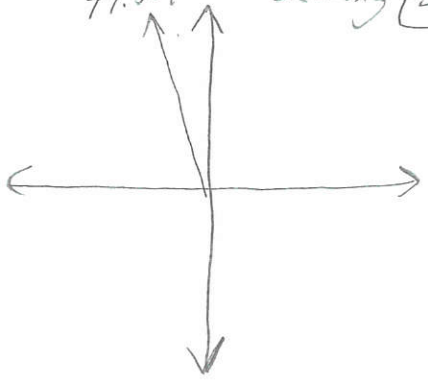
$4.351\langle \cos 99.354°, \sin 99.354° \rangle$

(d) Find the "ground speed" of the canoe.

4.351 mph

(e) What is the resultant compass heading of the canoe? Draw a diagram.

99.354° = bearing 350.646°

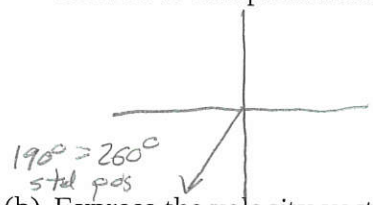


10. A plane flying at 650 mph is going to Mexico City from Milwaukee at a bearing of 190°. There is a wind blowing directly out of the northwest at 70 mph.

(a) Express the velocity vector of the plane relative to the air in trigonometric form and then convert to component form.

$650 \langle \cos 260°, \sin 260° \rangle$

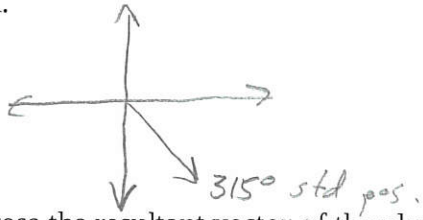
$\langle -112.871, -640.125 \rangle$



(b) Express the velocity vector of the wind in trigonometric form and then convert to component form.

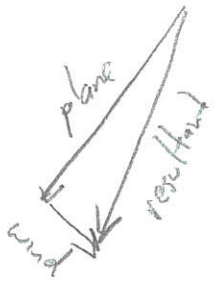
$70 \langle \cos 315°, \sin 315° \rangle$

$\langle 35\sqrt{2}, -35\sqrt{2} \rangle$



(c) Express the resultant vector of the plane plus the wind in component form and trigonometric form.

$\langle -63.374, -689.622 \rangle$



$$\text{magnitude} = \sqrt{(-63.374)^2 + (-689.622)^2}$$

$$= 692.528 \text{ mph}$$

$$\tan\theta = \tan^{-1}\left(\frac{-689.622}{-63.374}\right) \approx 84.749 + 180$$

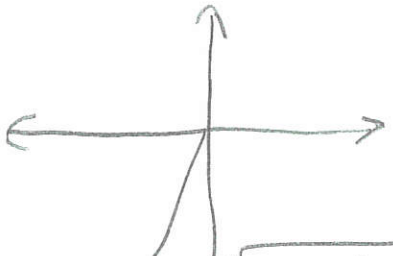
$$\approx 264.749°$$

$692.528 \langle \cos 264.749, \sin 264.749° \rangle$

(d) What is the ground speed of the plane?

692.528 mph

(e) What is the actual compass heading of the plane? Draw a diagram.



264.749 = bearing 185.251°