

Name: KEY1. Consider the function:  $h(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$ ✓ (a) Use synthetic division to verify that  $x = 1$  and  $x = 2$  are zeros of  $h(x)$ .

$$\begin{array}{r|rrrrr}
 1 & 1 & 2 & -7 & -8 & 12 \\
 & 0 & 1 & 3 & -4 & -12 \\
 \hline
 & 1 & 3 & -4 & -12 & 0
 \end{array}$$

$$\begin{array}{r|rrrr}
 2 & 1 & 3 & -4 & -12 \\
 & 0 & 2 & 10 & 12 \\
 \hline
 & 1 & 5 & 6 & 0
 \end{array}$$

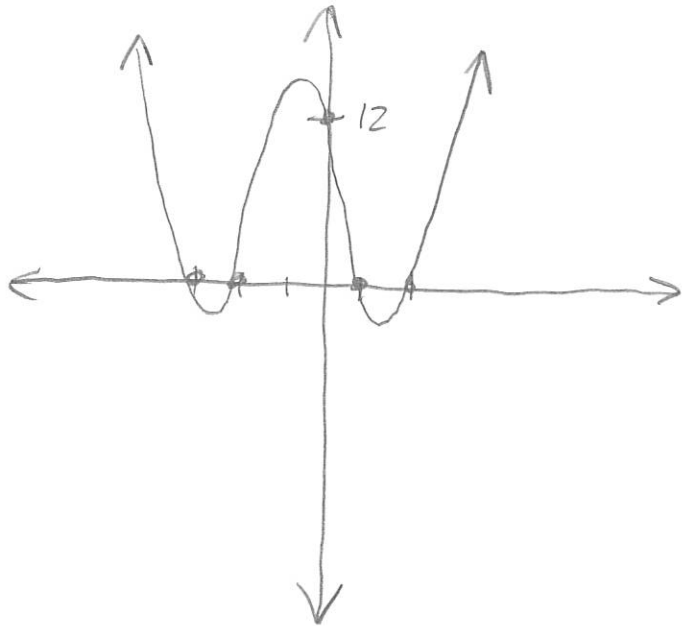
✓ (b) Find the other two zeros of  $h(x)$ , and then write the polynomial as a product of linear factors.

$$\begin{aligned}
 h(x) &= (x-1)(x-2)(x^2 + 5x + 6) \\
 &= (x-1)(x-2)(x+2)(x+3) \\
 \text{other zeros } &-2, -3
 \end{aligned}$$

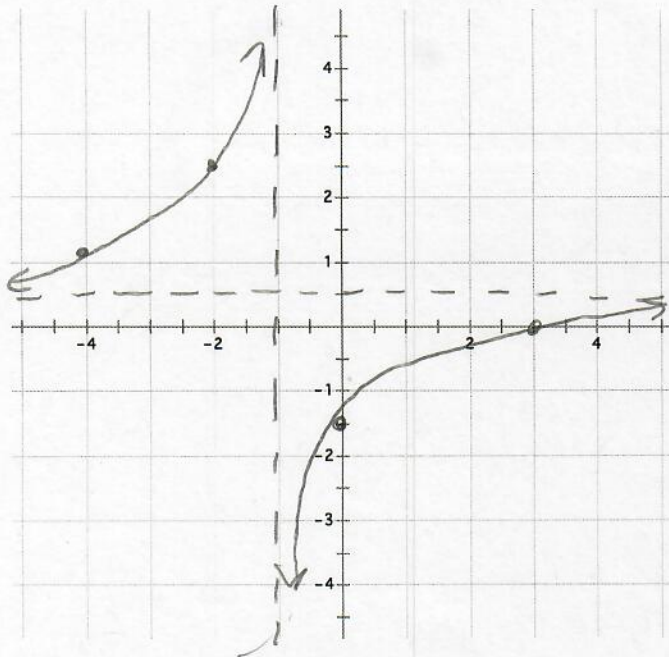
✓ (c) Graph  $h(x)$  in the coordinate plane at the right. Include the y-intercept and all x-intercepts.

y-int 12

no plane



2. Graph  $f(x) = \frac{x-3}{2x+2}$ , showing intercepts, asymptotes, and two additional points.



$$x\text{-int } 3$$

$$y\text{-int } \frac{-3}{2}$$

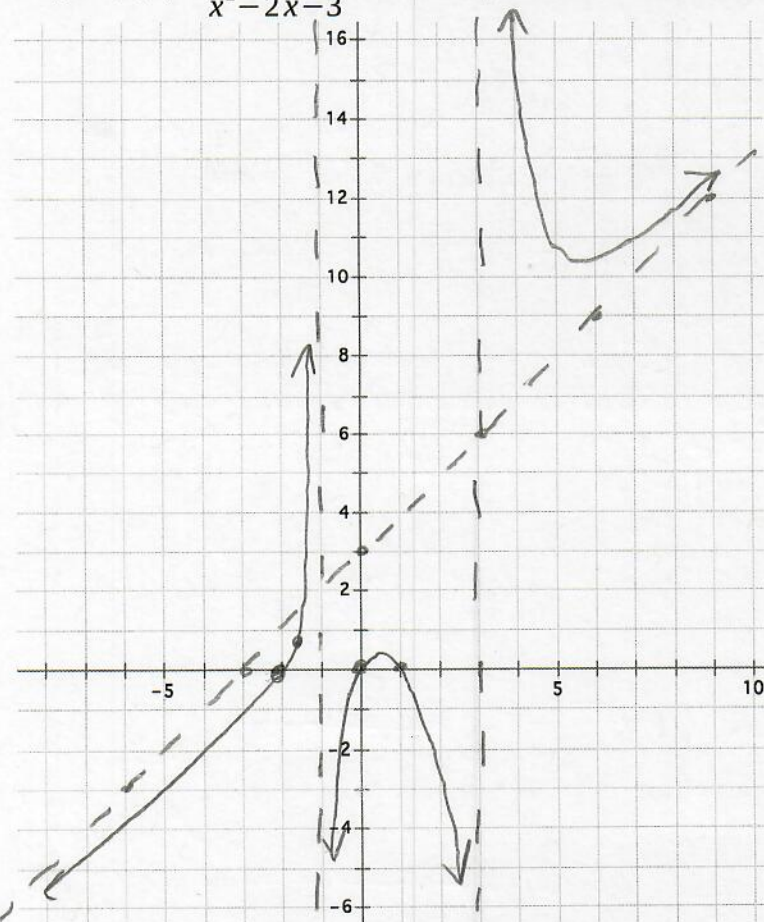
$$\text{vert } x = -1$$

$$\text{horiz } y = \frac{1}{2}$$

$$f(-2) = \frac{-5}{-2} = \frac{5}{2}$$

$$f(-4) = \frac{-7}{-6} = \frac{7}{6}$$

3. Graph  $f(x) = \frac{x^3+x^2-2x}{x^2-2x-3}$ , showing intercepts, asymptotes, and two additional points.



$$\frac{x(x^2+x-2)}{(x+1)(x-3)} = \frac{x(x-1)(x+2)}{(x-3)(x+1)}$$

$$x\text{-int } 0, 1, -2$$

$$y\text{-int } 0$$

$$\text{slant } y = x + 3$$

$$\text{vert } x = 3 \quad x = -1$$

$$f(4) = \frac{84}{5} = 16.8 \quad f(-1.5) = .8\bar{3}$$

$$\begin{array}{r}
 x^2 - 2x - 3 \overline{) x^3 + x^2 - 2x + 0} \\
 \underline{x^3 - 2x^2 - 3x} \phantom{+ 0} \\
 3x^2 + x + 0 \\
 \underline{3x^2 - 6x - 9} \\
 7x + 9
 \end{array}$$

4. Expand  $\log_3 \frac{xy^2}{\sqrt{z}}$  as far as possible.

$$\log_3 x + \log_3 y^2 - \log_3 \sqrt{z}$$

$$\log_3 x + 2 \log_3 y - \frac{1}{2} \log_3 z$$

5. Condense the expression to the logarithm of a single quantity:  $2 \ln x - 2 \ln(y+1) + 5 \ln z$

$$\ln x^2 - \ln (y+1)^2 + \ln z^5$$

$$\ln \left[ \frac{x^2 z^5}{(y+1)^2} \right]$$

Solve for x in problems six through nine.

6.  $3^{5x} + 2 = 973$

$$3^{5x} = 971$$

$$\log 3^{5x} = \log 971$$

$$5x \log 3 = \log 971$$

$$x = \frac{\log 971}{5 \log 3} \approx \boxed{1.253}$$

8.  $7^{x-2} = 3^x$

$$(x-2) \log 7 = x \log 3$$

$$x \log 7 - x \log 3 = 2 \log 7$$

$$x(\log 7 - \log 3) = 2 \log 7$$

$$x = \frac{2 \log 7}{\log 7 - \log 3}$$

10. Find all six trigonometric functions for  $495^\circ$

$$\sin 495^\circ = \frac{\sqrt{2}}{2}$$

$$\csc 495^\circ = \sqrt{2}$$

$$\cos 495^\circ = -\frac{\sqrt{2}}{2}$$

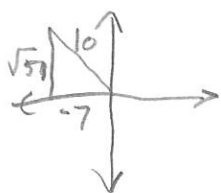
$$\sec 495^\circ = -\sqrt{2}$$

$$\tan 495^\circ = -1$$

$$\cot 495^\circ = -1$$

12. If  $\cos(x) = -0.7$  and x is in quadrant II, find:

(a)  $\sin(x) = \frac{\sqrt{51}}{10}$



(b)  $\sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos x} = \frac{1}{-0.7} = \frac{-10}{7}$

(c)  $\cos(x+\pi) = \cos x \cos \pi - \sin x \sin \pi = (-0.7) \cdot (-1) - \left(\frac{\sqrt{51}}{10}\right) \cdot 0 = 0.7$

7.  $\log_6(x+3) - \log_6(x+30) = -1$

$$\log_6 \frac{x+3}{x+30} = -1$$

$$6^{-1} = \frac{x+3}{x+30}$$

$$\frac{1}{6} = \frac{x+3}{x+30}$$

$$x+30 = 6x+18$$

$$12 = 5x$$

$$x = \frac{12}{5}$$

9.  $\log_5(2x+1) + \log_5(x-1) = 1$

$$\log_5 \frac{2x+1}{x-1} = 1$$

$$5^1 = \frac{2x+1}{x-1}$$

$$5x-5 = 2x+1$$

$$3x = 6$$

$$x = 2$$

11. Find all six trigonometric functions for  $-\frac{2\pi}{3}$

$$\sin \frac{-2\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\csc \frac{-2\pi}{3} = -\frac{2\sqrt{3}}{3}$$

$$\cos \frac{-2\pi}{3} = -\frac{1}{2}$$

$$\sec \frac{-2\pi}{3} = -2$$

$$\tan \frac{-2\pi}{3} = \sqrt{3}$$

$$\cot \frac{-2\pi}{3} = \frac{\sqrt{3}}{3}$$

13. Solve on the interval  $[0, 2\pi)$  :

✓(a)  $\sin \theta = -\frac{\sqrt{3}}{2}$

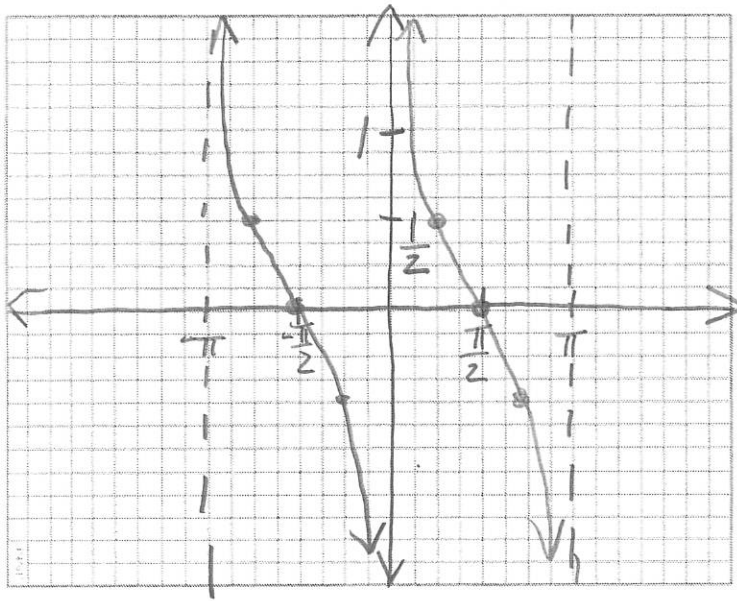
$\theta = \left\{ \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$

✓(b)  $\sec \theta = \sqrt{2}$

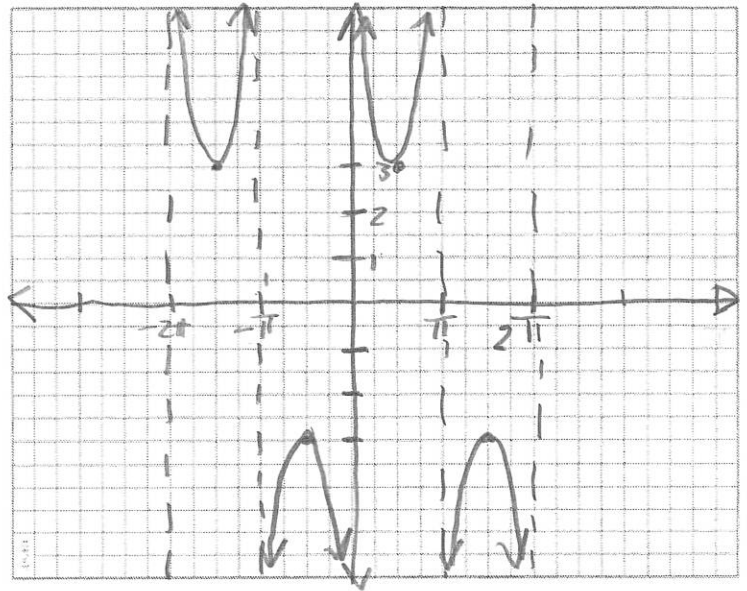
$\cos \theta = \frac{\sqrt{2}}{2}$

$\theta = \left\{ \frac{\pi}{4}, \frac{7\pi}{4} \right\}$

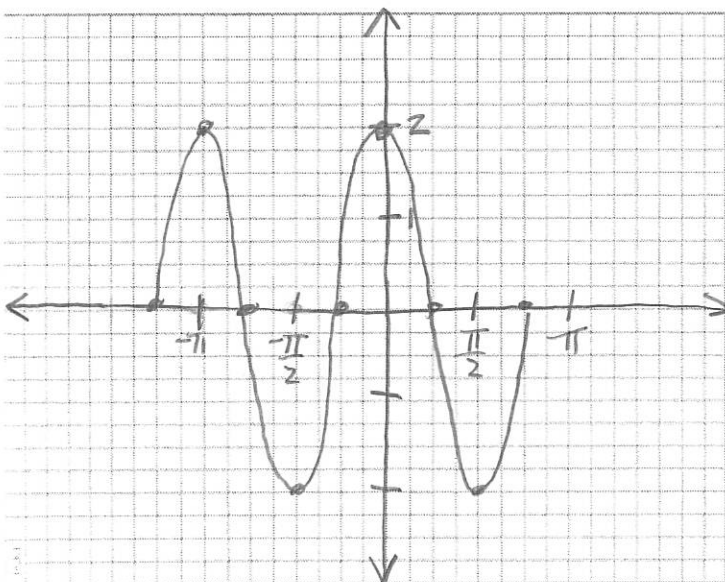
✓14. Graph  $y = \frac{1}{2} \cot x$



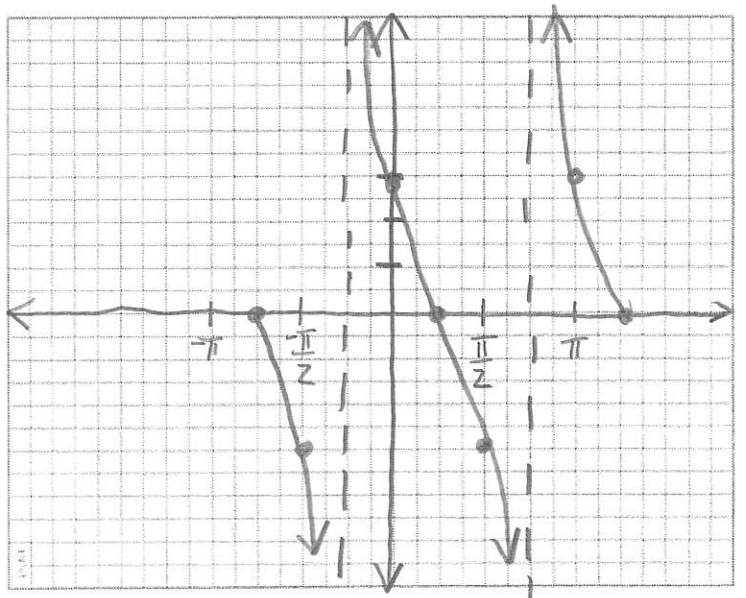
✓15. Graph  $y = 3 \csc x$



✓16. Graph  $y = 2 \sin\left(2x + \frac{\pi}{4}\right)$



✓17. Graph  $y = -3 \tan\left(x - \frac{\pi}{4}\right)$



18. Find the exact value:

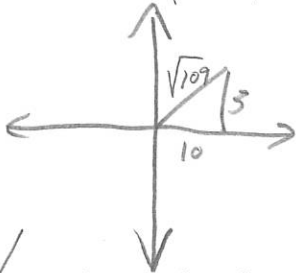
✓ (a)  $\arcsin\left(\frac{\sqrt{3}}{2}\right)$   
 $\frac{\pi}{3}$

✓ (b)  $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$   
 $\frac{3\pi}{4}$

✓ (c)  $\arccos(-1)$   
 $\pi$

✓ (d)  $\tan^{-1}(-\sqrt{3})$   
 $\frac{5\pi}{3}$

✓ (e)  $\cos\left(\tan^{-1}\left(\frac{3}{10}\right)\right) = \frac{10}{\sqrt{109}} = \frac{10\sqrt{109}}{109}$



✓ 19. When a plane leaves the runway its angle of climb is  $19^\circ$  and its speed is 250 feet per second. Find the plane's altitude after 1 minute.

$60 \cdot 250 = 15000$



$\frac{x}{15000} = \sin 19^\circ$

$x = 15000 \sin 19^\circ$

$x \approx 4883.522 \text{ ft}$

✓ 20. Precalculus books have many uses. One day Mr. Griesbach walked into the kitchen, holding his Precalculus book, of course. He spotted a huge millipede crawling on the floor. The millipede froze to avoid detection. Mr. Griesbach froze because he doesn't like huge millipedes. Those tiny, tiny legs send shivers up his spine. The millipede was exactly 10 feet from Mr. Griesbach's feet. The book was thrown from a height of 4 feet, smashing the millipede into many pieces. As the millipede looked up (its last line of sight on earth), what was the angle of elevation from the millipede to the approaching book?



$\tan \theta = \frac{4}{10}$

$\theta = \tan^{-1}\left(\frac{4}{10}\right) \approx 21.801^\circ$

21. Let  $f(x) = \sqrt{x}$  and  $g(x) = x - 2$ .

✓ (a) Find  $h(x) = f \circ g$

$$h(x) = \sqrt{x-2}$$

✓ (b) Find the domain of  $h(x)$

$$x \geq 2$$

✓ (c) Find  $h^{-1}(x)$

$$x = \sqrt{y-2}$$

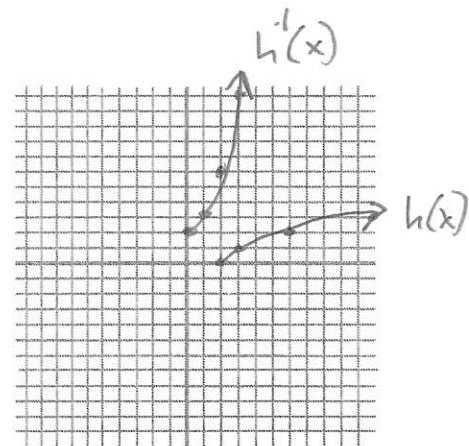
$$x^2 = y - 2$$

$$y = x^2 + 2$$

✓ (d) Find the domain of  $h^{-1}(x)$

$$x \geq 0$$

(e) Graph  $h(x)$  and  $h^{-1}(x)$  on the same plane.



✓ 22. Solve on  $[0, 2\pi)$ :  $2 \cos^2 x = 2 - \sin x$

$$2 - 2 \sin^2 x = 2 - \sin x$$

$$-2 \sin^2 x = -\sin x$$

$$0 = 2 \sin^2 x - \sin x$$

$$0 = \sin x (2 \sin x - 1)$$

$$\sin x = 0 \quad \sin x = \frac{1}{2}$$

$$x = 0, \pi \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \left\{ 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

✓ 23. Solve on  $(-\infty, \infty)$ :  $\cot^2(2x) = \frac{1}{3}$

$$\tan^2(2x) = 3 \quad 2x = \frac{2\pi}{3} \quad 2x = \frac{5\pi}{3}$$

$$\tan 2x = \pm\sqrt{3} \quad x = \frac{\pi}{3} \quad x = \frac{5\pi}{6}$$

$$2x = \frac{\pi}{3} \quad 2x = \frac{4\pi}{3} \quad 2x = \frac{7\pi}{3} \quad 2x = \frac{10\pi}{3}$$

$$x = \frac{\pi}{6} \quad x = \frac{2\pi}{3} \quad x = \frac{7\pi}{6} \quad x = \frac{5\pi}{3}$$

$$x = \left\{ \frac{\pi}{6} + \pi n, \frac{2\pi}{3} + \pi n, \frac{5\pi}{6} + \pi n, \frac{\pi}{3} + \pi n, n \in \mathbb{Z} \right\}$$