Advanced Identities | Ad majorem Dei gloriam | Accelerated Algebra 2/Precalculus
Sum/Difference/Half/Double | Mr. Reis

Name KEY

1. Find the exact values of:

(a) $\sin\left(-\frac{7\pi}{12}\right) = \sin -105^\circ$

$= \sin(45 - 150)$

$= \sin 45 \cos 150 - \cos 45 \sin 150$

$= \frac{\sqrt{2}}{2} \cdot \frac{-\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{-\sqrt{6} - \sqrt{2}}{4}$

(b) $\cos\left(-\frac{7\pi}{12}\right) =$

$\cos(45 - 150^\circ)$

$\cos 45 \cos 150 + \sin 45 \sin 150$

$\frac{\sqrt{2}}{2} \cdot \frac{-\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{-\sqrt{6} + \sqrt{2}}{4}$

(c) $\tan\left(-\frac{7\pi}{12}\right)$

$\tan(45 - 150)$

$\frac{\tan 45 - \tan 150}{1 + \tan 45 \tan 150}$

$\frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} \cdot \frac{3}{3} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$

$\frac{(3+\sqrt{3})^2}{9-3} = \frac{9 + 6\sqrt{3} + 3}{6}$

$= \frac{12 + 6\sqrt{3}}{6}$

$= 2 + \sqrt{3}$

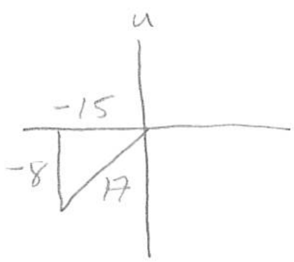
2. Write the exact value of $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16}$

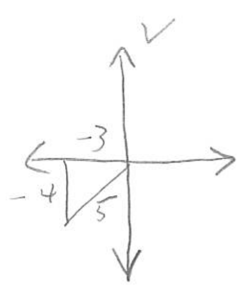
$= \cos\left(\frac{\pi}{16} + \frac{3\pi}{16}\right)$

$= \cos \frac{4\pi}{16}$

$= \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

3. Find the exact value of $\cos(u+v)$ given that $\sin u = -\frac{8}{17}$ and $\cos v = -\frac{3}{5}$ and both u, v are in quadrant III.





$\cos(u+v) = \cos u \cos v - \sin u \sin v$

$= \left(\frac{-15}{17}\right)\left(\frac{-3}{5}\right) - \left(\frac{-8}{17}\right)\left(\frac{-4}{5}\right)$

$= \frac{45}{85} - \frac{32}{85}$

$= \frac{13}{85}$

4. Solve the equation on the interval $[0, 2\pi)$:

(a) $2\sin\left(x+\frac{\pi}{2}\right)+3\tan(\pi-x)=0$

$2\left[\sin x\cos\frac{\pi}{2}+\cos x\sin\frac{\pi}{2}\right]+3\left[\frac{\tan\pi-\tan x}{1+\tan\pi\tan x}\right]=0$

$2\cos x-3\tan x=0$

$2\cos x-3\frac{\sin x}{\cos x}=0$

$2\cos^2 x-3\sin x=0$

$2-2\sin^2 x-3\sin x=0$

$2-2u^2-3u=0$

$2u^2+3u-2=0$

$(2u-1)(u+2)$

$u=\frac{1}{2}$ $u=-2$ invalid

$\sin x=\frac{1}{2}$

$x=\left\{\frac{\pi}{6},\frac{5\pi}{6}\right\}$

(b) $\sin(2x)=-\frac{1}{2}$

$2x=\frac{7\pi}{6}$ $2x=\frac{11\pi}{6}$ $2x=\frac{19\pi}{6}$ $2x=\frac{23\pi}{6}$

$x=\frac{7\pi}{12}$ $x=\frac{11\pi}{12}$ $x=\frac{19\pi}{12}$ $x=\frac{23\pi}{12}$

$x=\left\{\frac{7\pi}{12},\frac{11\pi}{12},\frac{19\pi}{12},\frac{23\pi}{12}\right\}$

(c) $\sin(2x)\sin x=\cos x$

$2\sin^2 x\cos x=\cos x$

$2\sin^2 x\cos x-\cos x=0$

$\cos x(2\sin^2 x-1)=0$

$\cos x=0$ $\sin^2 x=\frac{1}{2}$

$\sin x=\pm\frac{\sqrt{2}}{2}$

$x=\left\{\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

(d) $\cos 4x=1-8\sin^2 x+8\sin^4 x$

$1-2\sin^2(2x)=1-8\sin^2 x+8\sin^4 x$

$-2(2\sin x\cos x)^2=-8\sin^2 x+8\sin^4 x$

$-8\sin^2 x\cos^2 x+8\sin^2 x=8\sin^4 x$

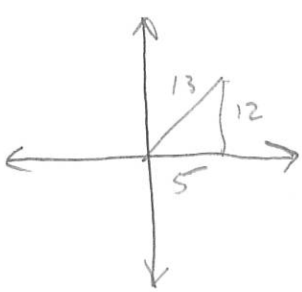
$8\sin^2 x(-\cos^2 x+1)=8\sin^4 x$

$8\sin^2 x(\sin^2 x)=8\sin^4 x$

$8\sin^4 x=8\sin^4 x$

$0=0$ all reals $\{x\in\mathbb{R}\}$

5. If $\sin\theta=\frac{12}{13}$ and θ is in quadrant I, find $\cos(2\theta)$, $\cot(2\theta)$, $\sin\frac{\theta}{2}$, $\cos\frac{\theta}{2}$



$\cos(2\theta)=\cos^2\theta-\sin^2\theta$

$=\left(\frac{5}{13}\right)^2-\left(\frac{12}{13}\right)^2$

$=\boxed{\frac{-119}{169}}$

$\sin(2\theta)=2\sin\theta\cos\theta=2\cdot\frac{5}{13}\cdot\frac{12}{13}=\frac{120}{169}$

$\cot(2\theta)=\frac{-119}{169}\div\frac{120}{169}=\boxed{\frac{-119}{120}}$

$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}$

$\sin\frac{\theta}{2}=\sqrt{\frac{1-\frac{5}{13}}{2}}=\sqrt{\frac{8/13}{2}}=\sqrt{4/13}=\frac{2}{\sqrt{13}}$

$=\frac{2\sqrt{13}}{13}$ positive b/c θ is positive

$\cos\frac{\theta}{2}=\pm\sqrt{\frac{1+\cos\theta}{2}}$ positive b/c θ positive

$=\pm\sqrt{\frac{18/13}{2}}$

$=\pm\sqrt{9/13}=\frac{3}{\sqrt{13}}=\frac{3\sqrt{13}}{13}$

6. Use the half angle formulas to find the exact values of the sine, cosine, and tangent of $\frac{\pi}{12}$

$$\sin\frac{\pi}{12} = \pm\sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}$$

$$= \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$

$$= \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

$$= \pm\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \frac{\pm\sqrt{2-\sqrt{3}}}{2}$$

$$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

+ b/c $\pi/12$ in QI

$$\cos\frac{\pi}{12} = \pm\sqrt{\frac{1+\cos\frac{\pi}{6}}{2}}$$

$$= \pm\frac{\sqrt{2+\sqrt{3}}}{2}$$

(same algebra as sine)

$$= \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$$

+ b/c $\frac{\pi}{12}$ in QI

$$\tan\frac{\pi}{12} = \frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}} = \frac{\sqrt{2-\sqrt{3}}}{2} \div \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$= \frac{\sqrt{2-\sqrt{3}}}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{2+\sqrt{3}}}$$

$$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}}$$