Trigonometric Equations | Ad majorem Dei Gloriam | Accelerated Algebra 2 / Precalculus
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Name KEY

Solve each equation on the interval $[0, 2\pi)$

1. $\sqrt{2}\csc x = 2$

$\csc x = \frac{2}{\sqrt{2}}$

$\sin x = \frac{\sqrt{2}}{2}$

$x = \left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$

2. $16\sin x + 20 = 15\sin x + 19$

$\sin x = -1$

$x = \frac{3\pi}{2}$

3. $3\csc^2 x - 4 = 0$

$\csc^2 x = \frac{4}{3}$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$x = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

4. $2\sin^2 x - 3\sin x = -1$

$2u^2 - 3u + 1 = 0$

$(2u - 1)(u - 1) = 0$ $\quad x = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}\right\}$

$u = \frac{1}{2}$ $\quad u = 1$

$\sin x = \frac{1}{2}$ $\quad \sin x = 1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad x = \frac{\pi}{2}$

5. $\cos(3x) = 1$

$3x = 0$ $\quad 3x = 2\pi$ $\quad 3x = 4\pi$

$x = 0$ $\quad x = \frac{2\pi}{3}$ $\quad x = \frac{4\pi}{3}$

$x = \left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

6. $\csc(2x) = 2$

$\sin(2x) = \frac{1}{2}$

$2x = \frac{\pi}{6}$ $\quad 2x = \frac{13\pi}{6}$ $\quad 2x = \frac{5\pi}{6}$ $\quad 2x = \frac{17\pi}{6}$

$x = \left\{\frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}\right\}$

7. $\sin^2 x = 3\cos^2 x$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

8. $2\sin^2 x + 3\cos x - 3 = 0$

$2(1 - \cos^2 x) + 3\cos x - 3 = 0$

$2 - 2u^2 + 3u - 3 = 0$

$-2u^2 + 3u - 1 = 0$

$2u^2 - 3u + 1 = 0$ $\quad x = \left\{0, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$

$(2u - 1)(u - 1) = 0$

$u = \frac{1}{2}$ $\quad u = 1$

$\cos x = \frac{1}{2}$ $\quad \cos x = 1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad x = 0$

9. $\sec^2 x\cos x - 2\cos x = 0$

$\cos x(\sec^2 x - 2) = 0$

$\cos x = 0$ $\quad \sec^2 x - 2 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \sec^2 x = 2$

invalid b/c $\sec(\frac{\pi}{2})$ undefined, $\sec(\frac{3\pi}{2})$ undefined $\quad \sec x = \pm\sqrt{2}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = \{\frac{\pi}{4} + \frac{\pi}{2}n,\ n \in \mathbb{Z}\}$

10. $\sqrt{3}\tan x\sin x + \sin x = 0$

$\sin x(\sqrt{3}\tan x + 1) = 0$

$\sin x = 0$ $\quad \sqrt{3}\tan x + 1 = 0$

$x = 0, \pi$ $\quad \tan x = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

$x = \{0, \pi, \frac{5\pi}{6}, \frac{11\pi}{6}\}$

Solve each equation on the interval $(-\infty, \infty)$

11. $\sin^2 x + \cos x = 0$

$1 - \cos^2 x + \cos x = 0$

$1 - u^2 + u = 0$

$u^2 - u - 1 = 0$

$u = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2}$

$u = \frac{1 \pm \sqrt{5}}{2}$

note $\frac{1+\sqrt{5}}{2}$ outside domain of cos

$x = \cos^{-1}\left(\frac{1 \pm \sqrt{5}}{2}\right)$

$x \approx \{2.237 + 2\pi n,\ 4.046 + 2\pi n,\ n \in \mathbb{Z}\}$

12. $\cos x = \sec x$

$\cos x = \frac{1}{\cos x}$

$\cos^2 x = 1$

$\cos x = \pm 1$

$x = \{0, \pi\}$

$x = \pi n \quad n \in \mathbb{Z}$

13. $2\sin^2 x - 3\sin x = -1$

identical to #4

$x = \{\frac{\pi}{6} + 2\pi n,\ \frac{5\pi}{6} + 2\pi n,\ \frac{\pi}{2} + 2\pi n,\ n \in \mathbb{Z}\}$

14. $\cos^2 x + \cos x = 2$

$u^2 + u - 2 = 0$

$(u-1)(u+2) = 0$

$u = 1$ $\quad u = 2$

$\cos x = 1$ $\quad \cos x = 2$

$x = 0$ $\quad$ no solution

$x = 0 + 2\pi n = \{2\pi n,\ n \in \mathbb{Z}\}$

15. $2\sin^2 x + \sin x = 0$

$2u^2 + u = 0$

$u(2u+1) = 0$

$u = 0$ $\quad u = -\frac{1}{2}$

$\sin x = 0$ $\quad \sin x = -\frac{1}{2}$

$x = 0, \pi$ $\quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = \{\pi n, \frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n,\ n \in \mathbb{Z}\}$

16. $12\cos^2 x + \cos x - 1 = 0$

$12u^2 + u - 1 = 0$

$(3u+1)(4u-1) = 0$

$u = -\frac{1}{3}$ $\quad u = \frac{1}{4}$

$x = \cos^{-1}(-\frac{1}{3})$ $\quad x = \cos^{-1}(\frac{1}{4})$

$x \approx 1.911, 4.373$ $\quad x \approx 1.318, 4.965$

$x = \{1.318 + 2\pi n, 1.911 + 2\pi n, 4.373 + 2\pi n, 4.965 + 2\pi n,\ n \in \mathbb{Z}\}$