Trigonometric Identities | Ad majorem Dei gloriam | Accelerated Algebra 2 / Precalculus
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Name KEY

Verify the identities:

✓ 1. $\frac{\sin x}{1-\cos x}=\frac{1+\cos x}{\sin x}$

$\frac{1+\cos x}{1+\cos x} \cdot \frac{\sin x}{1-\cos x} =$

$\frac{(1+\cos x)\sin x}{1-\cos^2 x} =$

$\frac{(1+\cos x)(\sin x)}{\sin^2 x} =$

$\frac{1+\cos x}{\sin x} =$

✓ 2. $\csc 2x=\frac{\csc x}{2\cos x}$

$= \frac{1}{\sin 2x}$

$= \frac{1}{2\sin x \cos x}$

$= \frac{\csc x}{2\cos x}$

✓ 3. $\csc^4 x-2\csc^2 x+1=\cot^4 x$

$(\csc^2 x - 1)^2 =$

$(\cot^2 x)^2 =$

$\cot^4 x =$

✓ 4. $\sin 4x=4\sin x\cos x(1-2\sin^2 x)$

$2 \sin 2x \cos 2x =$

$2 \cdot 2 \sin x \cos x (1-2\sin^2 x) =$

$4 \sin x \cos x (1-2\sin^2 x) =$

✓ 5. $\cos^2 2x-\sin^2 2x=\cos(4x)$

$= 2\cos^2(2x) - 1$

$= \cos^2(2x) + \cos^2(2x) - 1$

$= \cos^2(2x) + 1 - \sin^2(2x) - 1$

$= \cos^2(2x) - \sin^2(2x)$

Simplify:

6. $\sec x \cos x$

$\frac{1}{\cos x} \cdot \cos x$

1

7. $\tan x \csc x$

$\frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$

$\frac{1}{\cos x}$

$\sec x$

8. $\cot^2 x - \csc^2 x$

$-(\csc^2 x - \cot^2 x)$

-1

9. $(1-\cos^2 x)(\csc x)$

$\sin^2 x \cdot \frac{1}{\sin x}$

$\sin x$

10. $\sec^2 x \tan^2 x + \sec^2 x$

$\sec^2 x (\tan^2 x + 1)$

$\sec^2 x (\sec^2 x)$

$\sec^4 x$

11. $\sec^4 x - \tan^4 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$1 \cdot (\sec^2 x + \tan^2 x)$

$\sec^2 x + \tan^2 x$

12. $\frac{\sin(-x)}{\cos(-x)}$

$= \frac{-\sin x}{\cos x}$

$= -\tan x$

13. $\frac{\sec x}{\csc x}$

$= \frac{1}{\cos x} \div \frac{1}{\sin x}$

$= \frac{1}{\cos x} \cdot \frac{\sin x}{1}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

14. $\frac{1}{\sec x + 1} - \frac{1}{\sec x - 1}$

$\frac{\sec x - 1}{\sec x - 1} \cdot \frac{1}{\sec x + 1} - \frac{1}{\sec x - 1} \cdot \frac{\sec x + 1}{\sec x + 1}$

$\frac{\sec x - 1}{\sec^2 x - 1} - \frac{\sec x + 1}{\sec^2 x - 1}$

$\frac{-2}{\sec^2 x - 1}$

15. $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}$

$\dfrac{\tan x}{\tan x}\cdot\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}\cdot\dfrac{1+\sec x}{1+\sec x}$

$=\dfrac{\tan^2 x+1+2\sec x+\sec^2 x}{\tan x(1+\sec x)}$

$=\dfrac{\sec^2 x+2\sec x+\sec^2 x}{\tan x(1+\sec x)}$

$=\dfrac{2\sec^2 x+2\sec x}{\tan x(1+\sec x)}$

$=\dfrac{2\sec x(\sec x+1)}{\tan x(1+\sec x)}$

$=2\cdot\dfrac{1}{\cos x}\div\dfrac{\sin x}{\cos x}$

$2\cdot\dfrac{1}{\cos x}\cdot\dfrac{\cos x}{\sin x}$

$=\boxed{2\csc x}$

16. $\dfrac{\cos^2\left(\frac{\pi}{2}-x\right)}{\cos x}$

$=\dfrac{\sin^2 x}{\cos x}$

$=\dfrac{\sin x}{\cos x}\cdot\sin x$

$=\boxed{\tan x\cdot\sin x}$

17. $\dfrac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}$

$=\dfrac{\cos x}{\sin x}$

$=\boxed{\cot x}$