Graphing Rational Functions | Ad majorem Dei gloriam | Accelerated Algebra 2 and Precalculus
Mr. Reis

Name KEY

For each function, give the:

A. zeros B. y-intercept C. asymptotes (all) D. graph E. possible holes

Show all work. Use your calculator to check, but you must be able to find all zeros and asymptotes by hand

✓ 1. $f(x) = \dfrac{2x^2}{x^2-1}$

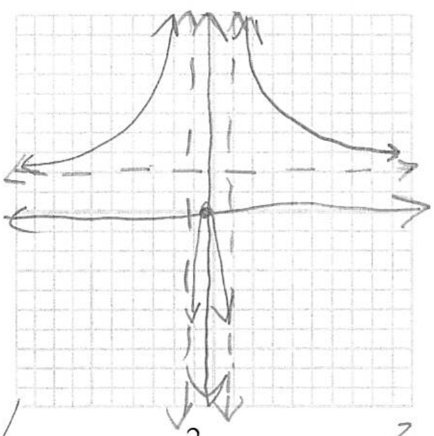
zeros 0

y-int 0

h. asymp $y = 2$

v. asymp $x = \pm 1$

holes none



✓ 2. $f(x) = \dfrac{x^2-9}{x^2-2x-3} = \dfrac{(x-3)(x+3)}{(x-3)(x+1)}$

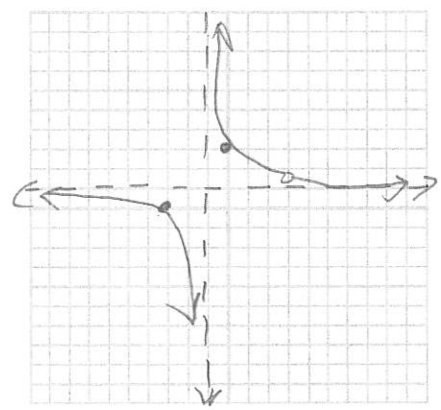
zeros -3

y-int 3

h. asymp $y = 1$

v. asymp $x = -1$

holes $\left(3, \frac{3}{2}\right)$



✓ 3. $f(x) = \dfrac{2}{x^3-3x^2} = \dfrac{2}{x^2(x-3)}$

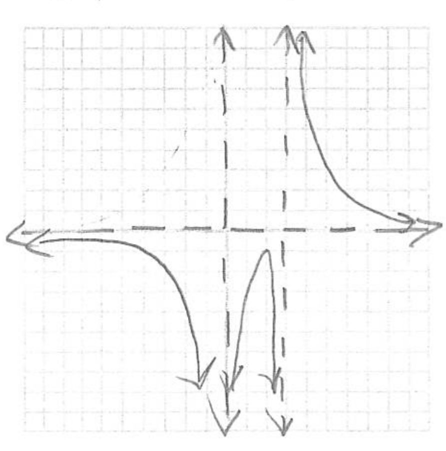
zeros none

y-int none

holes none

h. asymp. $y = 0$

v. asymp $x = 0,\ x = 3$



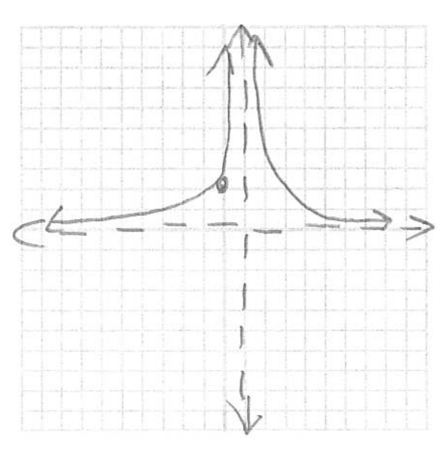
✓ 4. $f(x) = \dfrac{2}{x^2-2x+1} = \dfrac{2}{(x-1)^2}$

zeros none

y-int 2

h asymp. $y = 0$

v. asymp $x = 1$



5. $f(x) = -\dfrac{x}{x^2 - 4x + 4} = -\dfrac{x}{(x-2)^2}$

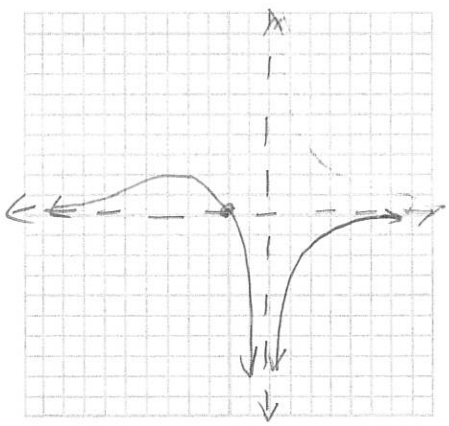
zeros 0

y-int 0

h. asymp 0

v. asymp. $x = 2$

holes none



6. $f(x) = \dfrac{4}{x^2 + 1}$

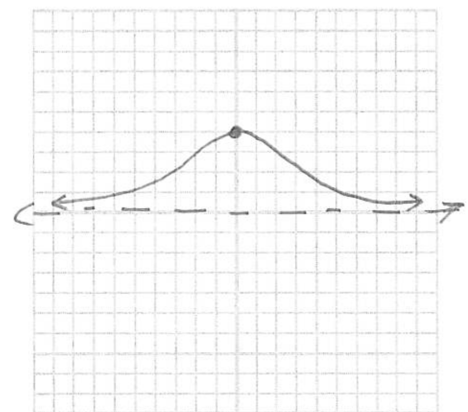
zeros none

y-int 4

horiz $y = 0$

vert. none

holes none



7. $f(x) = \dfrac{x^2 - 1}{x^2 + 4} = \dfrac{(x-1)(x+1)}{x^2+4}$

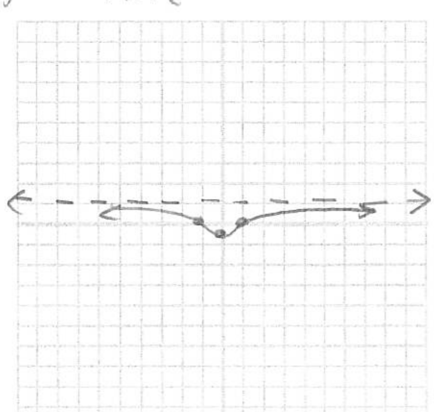
zeros 1, -1

y-int $-1/4$

horiz $y = 1$

vert. none

holes none



8. $f(x) = \dfrac{2x}{3x^2 + 1}$

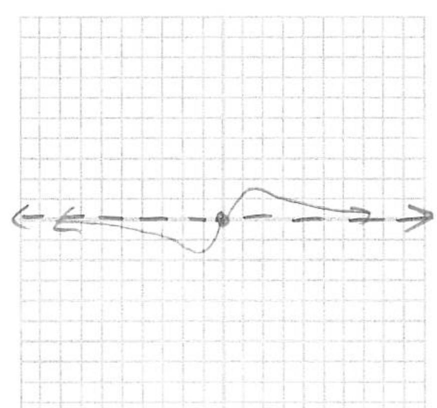
zeros 0

y-int 0

horiz $y = 0$

vert. none

holes none



9. $f(x)=\dfrac{x^2-x}{x+1} = \dfrac{x(x-1)}{x+1}$

$$\begin{array}{r} x-2 \\ x+1\,\big)\,\overline{x^2-x+0} \\ \underline{x^2+x} \\ -2x+0 \\ \underline{-2x-2} \\ 2 \end{array}$$

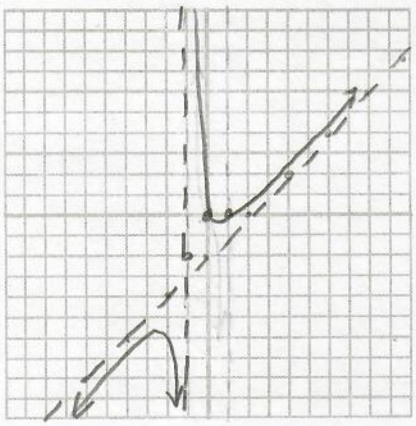
zeros $0, 1$

y-int 0

oblique $y = x-2$

vert $x = -1$

holes none



10. $f(x)=\dfrac{1-x^2}{x} = \dfrac{(1-x)(1+x)}{x}$

$$\begin{array}{r} -x+0 \\ x+0\,\big)\,\overline{-x^2+0x+1} \\ \underline{-x^2+0x} \\ 0x+1 \\ \underline{0x+0} \\ 1 \end{array}$$

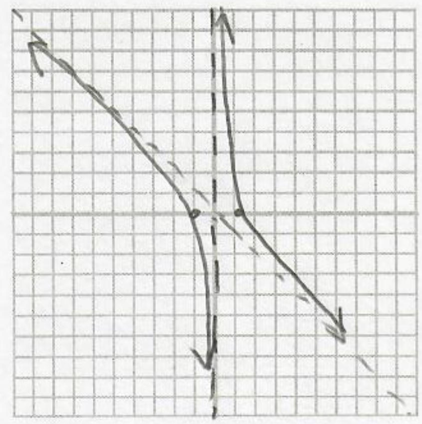
zeros ± 1

y-int none

vert. asymp. $x=0$

oblique $y=-x$

holes none



11. $f(x)=\dfrac{x^3}{x^2-1}$ $\dfrac{x^3}{(x-1)(x+1)}$

$$\begin{array}{r} x+0 \\ x^2+0x-1\,\big)\,\overline{x^3+0x^2+0x+0} \\ \underline{x^3+0x^2-x} \\ 0x^2-x+0 \\ \underline{0x^2+0x+0} \\ -x+0 \end{array}$$

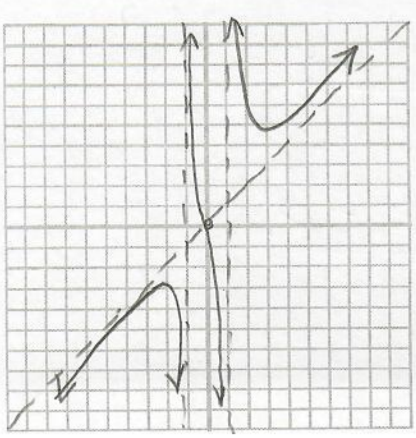
zeros 0

y-int 0

oblique $y=x$

vert $x=\pm 1$

holes none



12. $f(x)=\dfrac{2x^2-5x+5}{x-2}$ ← does not factor solutions complex

zeros none

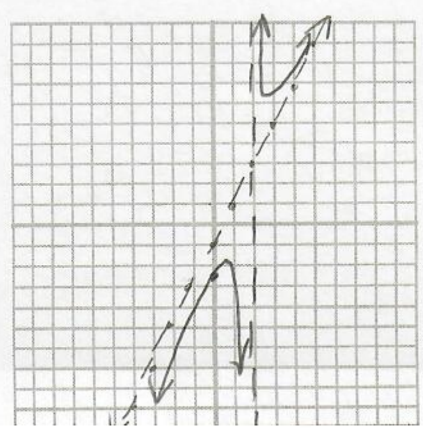
y-int $-5/2$

oblique $y=2x-1$

vert $x=2$

holes none

$$\begin{array}{r} 2x-1 \\ x-2\,\big)\,\overline{2x^2-5x+5} \\ \underline{2x^2-4x} \\ -x+5 \\ \underline{-x+2} \\ 3 \end{array}$$



13. $f(x)=\dfrac{x^2-16}{x+4}$ $=\dfrac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}}$

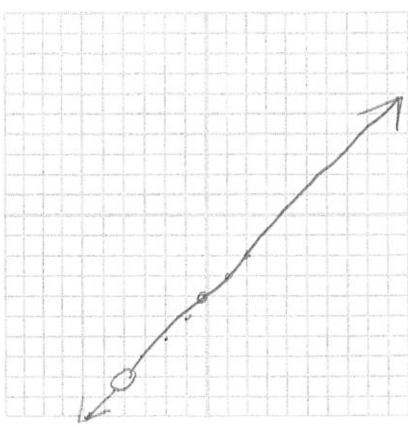
zeros 4

y-int -4

oblique none

vert. none

hole $(-4,-8)$



14. $f(x)=\dfrac{5x+20}{x^2+x-12}$ $=\dfrac{5(x+4)}{(x+4)(x-3)}$

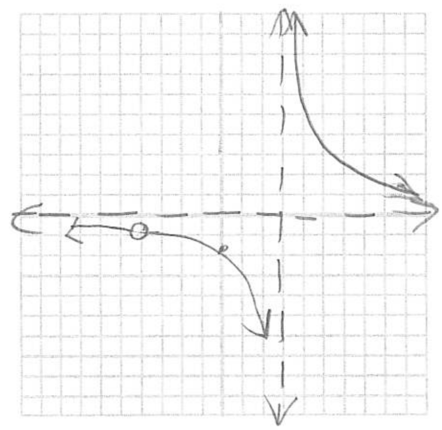
horiz $y=0$

vert $x=3$

zeros none

y-int $-5/3$

hole $(-4,-5/7)$



15. $f(x)=\dfrac{2x^3}{x^3-x}$ $=\dfrac{2x^3}{x(x^2-1)}=\dfrac{2x^3}{x(x-1)(x+1)}$

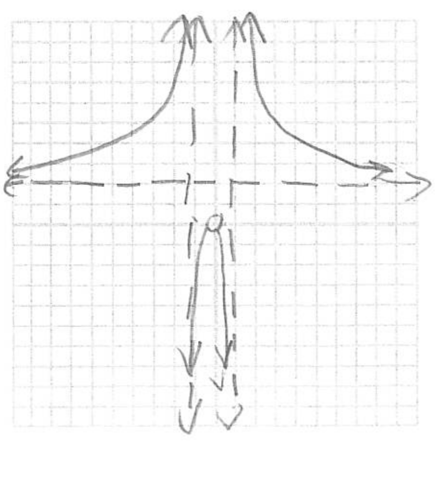
zeros none

y-int none

horiz $y=2$

vert. $x=\pm 1$

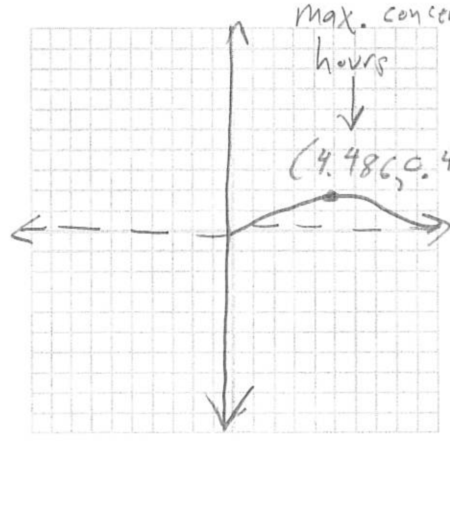
hole $(0,0)$



16. The concentration of a medicine, M, in the blood stream t hours after being injected into muscle tissue is given by: $M(t)=\dfrac{3t^2+t}{t^3+50}, t\ge 0$. Find the horizontal asymptote and interpret its meaning. Graph using your calculator and trace to find the time of the highest concentration and when the concentration is less than 0.3.

horiz $y=0$

Ultimately the medicine concentration should go to zero



$\boxed{\text{0.3 after about 9.83 hours}}$