AYRFC

(1) a) $\frac{x(x^2-9)}{(x-3)(x-4)} = \frac{x(x-3)(x+3)}{(x-3)(x-4)} = \frac{x(x+3)}{x-4}$

b) $\frac{(x+2)(x-4)}{x(x^2+x-2)} = \frac{(x+2)(x-4)}{x(x-1)(x+2)} = \frac{x-4}{x(x-1)}$

c) $\frac{\frac{25x^2}{x} - \frac{25x^2}{5}}{\frac{25x^2}{x^2} - \frac{25x^2}{25}} = \frac{25x-5x^2}{25-x^2} = \frac{5x(5-x)}{(5-x)(5+x)} = \frac{5x}{5+x}$

d) $\frac{9x^2-x^0}{3x^2+x} = \frac{9x^2-1}{x(3x+1)} = \frac{(3x+1)(3x-1)}{x(3x+1)} = \frac{3x-1}{x}$

(2) a) $\frac{2(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$ $\frac{2(\sqrt{3}-\sqrt{2})}{3-2} = \frac{2(\sqrt{3}\cdot\sqrt{2})}{1} = 2(\sqrt{3}-\sqrt{2})$

b) $\frac{4(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$ $\frac{4(1+\sqrt{5})}{1-5} = \frac{4(1+\sqrt{5})}{-4} = -1-\sqrt{5}$

(3) a) $8a^6b^{-1}$

b) $3a^{1/2}b^{3/2}$

c) $\frac{2a}{b} \cdot \frac{a}{3} = \frac{2}{3}a^2b^{-1}$

d) $\frac{a(b+1)}{b(b+1)}$ ab^{-1}

e) $a^{-3/2}b$

f) $\frac{a^{4/3}}{b} \cdot \frac{b^{2/3}}{a^{1/2}} = a^{5/6}b^{-1/3}$

(4) a) $5^{x+1} = 5^2$

$x+1=2$

$x=1$

b) $3^{-1} = 3^{2x+2}$

$-1 = 2x+2$

$-3 = 2x$

$\frac{-3}{2} = x$

c) $2^3 = x$

$x = 8$

d) $\log_3 x^2 = \log_3 \frac{4^2}{5^4}$

$x^2 = \frac{16}{625}$

$x = \pm\frac{4}{25}$

(5) a) $\log_2 \frac{5(x^2-1)}{x-1} = \log_2 \frac{5(x-1)(x+1)}{x-1} = \log_2 [5(x-1)]$

b) $3^{\log_3 5^2} = 5^2 = 25$

c) $1/2$

d) $\log_{10} 10^{-x} = -x$

e) $\log_{10} x(x) = \log_{10} x^2 \quad (x \geq 0)$

(6) a) $bcx + acy + abz = abc$

$bcx = abc - acy - abz$

$bcx = a(bc - cy - bz)$

$a = \frac{bcx}{bc - cy - bz}$

b) $V = 2ab + 2bc + 2ca$

$V - 2bc = 2ab + 2ca$

$V - 2bc = a(2b + 2c)$

$\frac{V - 2bc}{2b + 2c} = a$

c) $A = P(1 + nr)$

$\frac{A}{1+nr} = P$

d) $2x - y = xd + 2yd$

$2x - y = d(x + 2y)$

$\frac{2x - y}{x + 2y} = d$

e) $2x + \frac{4\pi(1-x)}{2} = 0$

$2x + 2\pi - 2\pi x = 0$

$2x - 2\pi x = -2\pi$

$x - \pi x = -\pi$

$x(1 - \pi) = -\pi$

$x = \frac{-\pi}{1-\pi} = \frac{\pi}{\pi - 1}$

(7) a) $y = x^2 + 4x + 4 - 1$

$y = (x+2)^2 - 1$

b) $-2y = 3x^2 + 3x$

$-2y = 3(x^2 + x)$

$-2y + \frac{3}{4} = 3(x^2 + x + \frac{1}{4})$

$-2y = 3(x + \frac{1}{2})^2 - \frac{3}{4}$

$y = \frac{-3}{2}(x + \frac{1}{2})^2 + \frac{3}{8}$

c) $9y^2 - 6y = x + 9$

$9(y^2 - \frac{2}{3}y + \frac{1}{9}) = x + 9 + 1$

$9(y - \frac{1}{3})^2 = x + 10$

$9(y - 1/3)^2 - 10 = x$

(8) a) $x^4(x^2-16)=0$

$x^4(x-4)(x+4)=0$

$x=\{0,4,-4\}$

b)

	$4x^3$	$-8x^2$
$4x^2$		
-25	$-25x$	50

$x=-2$

$(4x^2-25)(x-2)=0$

$(2x-5)(2x+5)(x-2)=0$

$x=\left\{\frac{5}{2},\frac{-5}{2},2\right\}$

c) $(2x+3)(4x^2-6x+9)=0$

$x=\frac{-3}{2}$ $\quad x=\frac{6\pm\sqrt{36-4(4)(9)}}{8}$

$x=\frac{6\pm\sqrt{-108}}{8}$

$x=\frac{6\pm 6i\sqrt{3}}{8}$

$x=\left\{\frac{-3}{2},\frac{3}{4}\pm\frac{3}{4}i\sqrt{3}\right\}$ $\quad x=\frac{3}{4}\pm\frac{3}{4}i\sqrt{3}$

d) $(x^2-1)(x^2+1)=0$

$x^2=-1$

$x=\pm 1$ $\quad x=\pm i$

$x=\{\pm 1,\pm i\}$

(9) a) $\frac{\sin^2 x}{\cos^2 x}=\frac{1}{3}$

$\tan^2 x=1/3$

$\tan x=\pm\sqrt{3}/3$

$x=\left\{\pi/6,\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}\text{ all }+2\pi n\right\}$ $\quad n\in\mathbb{Z}$

b) $1-\sin^2 x-\sin^2 x=\sin x$

$1-2\sin^2 x=\sin x$

$1-2u^2=u$

$0=2u^2+u-1$

$0=(2u-1)(u+1)$

$u=1/2$ $\quad u=-1$

$\sin x=\frac{1}{2}$ $\quad \sin x=-1$

$x=\left\{\frac{\pi}{6}+2\pi n,\frac{5\pi}{6}+2\pi n,\frac{3\pi}{2}+2\pi n,\ n\in\mathbb{Z}\right\}$

c) $\frac{\sin x}{\cos x}+\frac{1}{\cos x}=2\cos x$

$\sin x+1=2\cos^2 x$

$\sin x+1=2-2\sin^2 x$

$u+1=2-2u^2$

$2u^2+u-1=0$

$(2u-1)(u+1)=0$

$u=1/2$ $\quad u=-1$

$\sin x=1/2$ $\quad \sin x=-1$

invalid $x=\frac{3\pi}{2}$

$x=\frac{\pi}{6}+2\pi n,\frac{5\pi}{6}+2\pi n,$

* invalid b/c $\sec\frac{3\pi}{2}$ undef

$\tan\frac{3\pi}{2}$ undef

(10) a) $-\sqrt{3}/2$

b) $-\sqrt{2}/2$

c) $7\pi/4$

d) $3\pi/2$

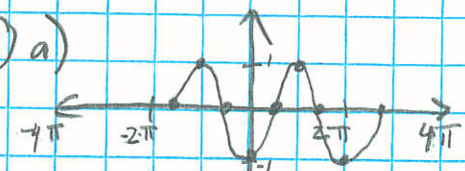
e) $\sqrt{2}/2$ $\quad \frac{9\pi}{4}=\pi/4$

f) $\pi/3$

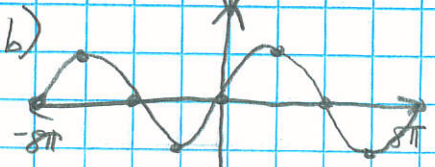
g) $\sqrt{3}/3$

h) π

(11) a)



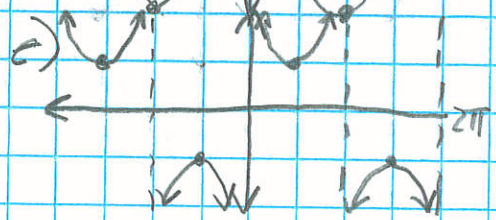
b)



c)

d)

e)



(12) a) $x = \frac{-12 \pm \sqrt{144-48}}{8}$

$x = \frac{-12 \pm \sqrt{96}}{8}$

$x = \frac{-12 \pm 4\sqrt{6}}{8}$

$x = \frac{-3 \pm \sqrt{6}}{2}$

b) $(2x+1)(x+2) = 5$

$2x^2 + 5x + 2 = 5$

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$x = \{1/2, -3\}$

c) $\frac{x+1}{x} = \frac{x}{x+1}$

$x^2 = x^2 + 2x + 1$

$0 = 2x + 1$

$x = -1/2$

(13) a)

-2	1	-4	1	0	-7	1
	0	-2	12	-26	52	-90
	1	-6	13	-26	45	-89

$x^4 - 6x^3 + 13x^2 - 26x + 45 - \frac{89}{x+2}$

b) $x^3 + 0x^2 + 0x + 1 \;\overline{)\; x^5 - x^4 + x^3 + 2x^2 - x + 4}$

$x^2 - x + 1$

$x^5 + 0x^4 + 0x + x^2$

$-x^4 + x^3 + x^2 - x$

$-x^4 + 0x^3 + 0x^2 - x$

$x^3 + x^2 + 0x + 4$

$x^3 + 0x + 0x + 1$

$x^2 + 3$

$x^2 - x + 1 + \frac{x^2+3}{x^3+1}$

(14)

2	12	-23	-3	2
	0	24	2	-2
	12	1	-1	0

$(x-2)(12x^2 + x - 1) = 0$

$(x-2)(4x-1)(3x+1) = 0$

$x = \{2, \frac{1}{4}, -\frac{1}{3}\}$

(15) a) $(x+1)(x-3) \leq 0$

$-1 \leq x \leq 3 \quad [-1, 3]$

b) $x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2}$

$x = \frac{-1 \pm i\sqrt{3}}{2}$

no x-int

parabola above x-axis since $a > 1$

$\therefore \quad x = \{R\} \quad (\infty, \infty)$

(15) c) $\frac{2x-1}{3x-1} - 1 \leq 0$

$\frac{2x-1-(3x-1)}{3x-1} \leq 0$

$\frac{-x}{3x-1} \leq 0$

− + −

0 $\frac{1}{3}$

$(-\infty, 0] \cup (\frac{1}{3}, \infty)$

$x \leq 0$ or $x > 1/3$

(16) a) $5x-2 = 8$ or $5x-2 = -8$

$x = 2$ or $x = -6/5$

$x = \{2, -6/5\}$

b) $2x+1 = x+3$ or $2x+1 = -x-3$

$x = 2$ $3x = -4$

$x = \{2, -4/3\}$ $x = -4/3$

(17) a) $m = -\frac{7}{3}$

$y - 3 = -\frac{7}{3}(x+1)$

$3y - 9 = -7x - 7$

$7x + 3y = 2$

b) $m = -3/2$

$y - 2 = -3/2(x+1)$

$2y - 4 = -3x - 3$

$3x + 2y = 1$

c) $(2,3)$ and $(1,3)$

$y = 3$

(18) $y = 3x - 7$

$x + 5(3x-7) + 3 = 0$

$x + 15x - 35 + 3 = 0$

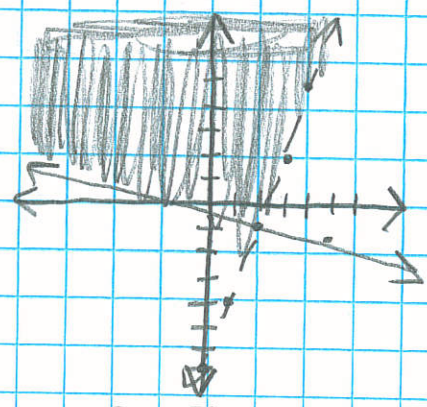
$16x - 32 = 0$

$x = 2$

$y = -1$

$(2, -1)$

(19)



$y > 3x - 7$ $5y \geq -x - 3$

$y \geq -\frac{1}{5}x - \frac{3}{5}$

(20) $r = \sqrt{(1+2)^2 + (2-1)^2}$

$r = \sqrt{10}$

$(x-1)^2 + (y-2)^2 = 10$

(21) a) $x^2 + 6x + 9 + y^2 - 4y + 4 = -3 + 9 + 4$

$(x+3)^2 + (y-2)^2 = 10$

center $(-3, 2)$

radius $\sqrt{10}$

b) $m_1 = \frac{5-2}{-2+3} = 3$

$m = -1/3$

$y - 5 = -\frac{1}{3}(x+2)$

$3y - 15 = -x - 2$

$x + 3y = 13$

(22) $\sqrt{(x+1)^2 + (y-1)^2} = 3\sqrt{(x-2)^2 + (y+1)^2}$

$(x+1)^2 + (y-1)^2 = 9[(x-2)^2 + (y+1)^2]$

$x^2 + 2x + 1 + y^2 - 2y + 1 = 9(x^2 - 4x + 4 + y^2 + 2y + 1)$

$x^2 + 2x + y^2 - 2y + 2 = 9x^2 - 36x + 36 + 9y^2 + 18y + 9$

$2 = 8x^2 - 38x + 8y^2 + 20y + 45$

$-43 = 8x^2 - 38x + 8y^2 + 20y$

(circle)

(23) $x^2 + x - 2 > 0$

$(x-1)(x+2) > 0$

$x > 1$ or $x < -2$

$(-\infty, -2) \cup (1, \infty)$

(24) a) D: $(-\infty, \infty)$
R: $[7, 7]$
b) D: $x \neq -1/2$
R: $y \neq 5/2$
(location of horiz and vert asymptotes)

(25) If $x = 0$, $f(x)$ is undefined, If $x < 0$
$|x| = -x$ and
$f(x) = -1$ If $x > 0$
$|x| = x$ and $f(x) = 1$
D: $(-\infty, 0) \cup (0, \infty)$
R: $\{-1, 1\}$

(26) a) $= \dfrac{[2(x+h)+3] - [2x+3]}{h}$

$= \dfrac{2x + 2h + 3 - 2x - 3}{h}$

$= 2$

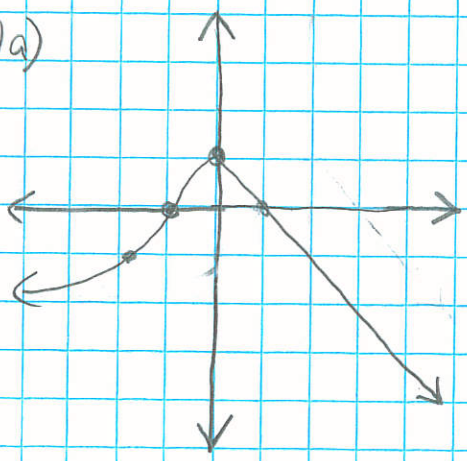
b) $\dfrac{(x+h)^2 - x^2}{h}$

$= \dfrac{x^2 + 2xh + h^2 - x^2}{h}$

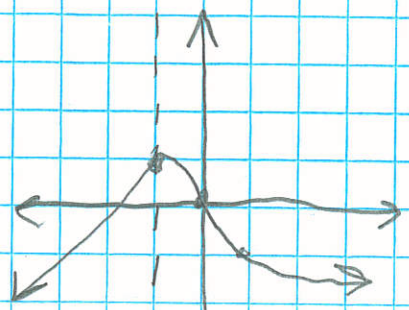
$= \dfrac{2xh + h^2}{h}$

$= 2x + h$

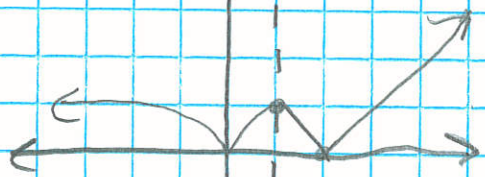
(27) a)



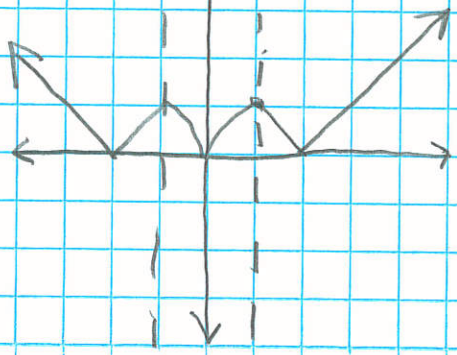
(27) b)



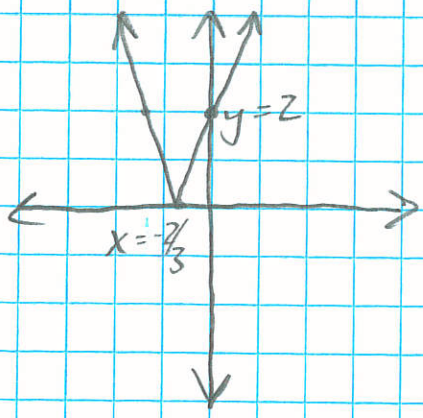
c)



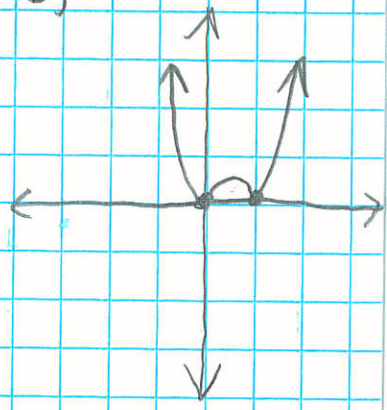
d)



(28) a)



b)



(29) a) $t = x-1$

$y = (x-1)^2 - (x-1)$

$y = x^2 - 2x + 1 - x + 1$

$y = x^2 - 3x + 2$

b) $t = (x+1)^3$

$y = (x+1)^6 - (x+1)^3$

(expansion not necessary)

c) $x^2 + y^2 = \sin^2 t + \cos^2 t$

$x^2 + y^2 = 1$

(30) a) $y = 2x + 3$

$x = 2y + 3$

$\frac{x-3}{2} = y$

$f^{-1}(x) = \frac{(x-3)}{2}$

b) If $x > 0$, $y > -1$

$x = y^2 + 2y - 1$

$x = y^2 + 2y + 1 - 2$

$x = (y+1)^2 - 2$

$\pm\sqrt{x+2} = y+1$

$y = \pm\sqrt{x+2} - 1$

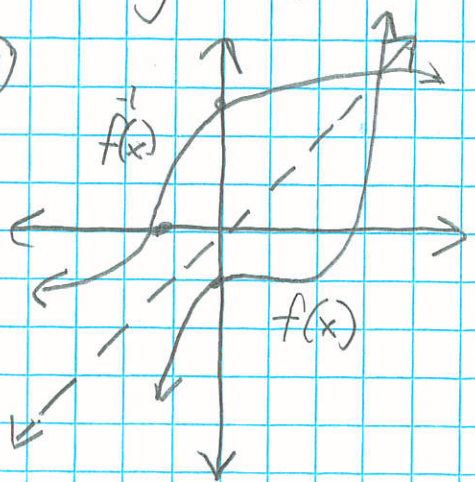
$x > -1$

$y > 0$

so $\pm$ can be discarded

$y = \sqrt{x+2} - 1$

(31)



(32) $\frac{t}{h} = \frac{t+x}{r}$

$tr = ht + xh$

$tr - ht = xh$

$\frac{tr - ht}{h} = x$

(33) $= \frac{(2r)^2 - \pi r^2}{(2r)^2}$

$= \frac{4r^2 - \pi r^2}{4r^2}$

$= \frac{4 - \pi}{4}$

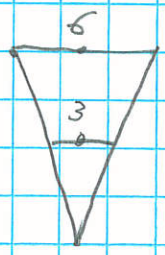
(34) $A = 2r(r) + \frac{1}{2}\pi r^2$

$= 2r^2 + \frac{1}{2}\pi r^2$

$P = \frac{1}{2} \cdot 2\pi r + 2r + 2r$

$= \pi r + 4r$

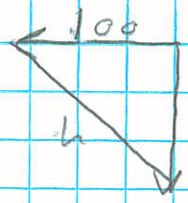
(35)



$\pi\left(\frac{3}{2}\right)^2$

$\frac{9\pi}{4}\ m^2$

(36)



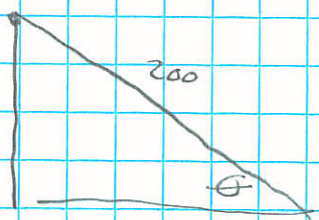
$h^2 = 100^2 + 200^2$

$h^2 = 50000$

$h = \sqrt{50000}$

$= 100\sqrt{5}$ km

(37)



$\theta = \sin^{-1}\left(\frac{100}{200}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \boxed{30}$

(38) a) $\sin(2x)$

$= \sin(x+x)$

$= \sin x \cos x + \cos x \sin x$

$= 2 \sin x \cos x$

b) $\cos(2x)$

$= \cos(x+x)$

$= \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

c) $\cos(2x)$

$= \cos^2 x - \sin^2 x$ (from (b))

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x$

$= \cos^2 x - (1 - \cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x$

$= 2\cos^2 x - 1$

d) $\cos(2x)$

$= \cos^2 x - \sin^2 x$ (from (b))

$\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$= 1 - \sin^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

e) $= \frac{1}{2} - \frac{1}{2}\cos 2x$

$= \frac{1}{2} - \frac{1}{2}(1 - 2\sin^2 x)$

$= \frac{1}{2} - \frac{1}{2} + \sin^2 x$

$= \sin^2 x$

f) $= \frac{1}{2} + \frac{1}{2}\cos(2x)$

$= \frac{1}{2} + \frac{1}{2}(2\cos^2 x - 1)$

$= \frac{1}{2} + \cos^2 x - \frac{1}{2}$

$= \cos^2 x$

(39) a) $\frac{40}{60} = \frac{2}{3}$

b) $\frac{15}{30} + \frac{25}{30} + \frac{48}{30} = \frac{88}{30} = \frac{44}{15}$

c) $\frac{1}{2} \cdot \frac{\cancel{6}^{3}}{\cancel{5}} \cdot \frac{\cancel{8}}{\cancel{8}} = \frac{3}{8}$

40) $= (2^6)^{1/2} (2^6)^{-2/3}$

$= 2^3 \cdot 2^{-4}$

$= 2^{-1} = 1/2$

41) $x_v = \frac{-b}{2a} = \frac{2}{2} = 1$

$y_v = 1^2 - 2 + 3 = 2$

$(1, 2)$

42) $\ln e^{-1} = -1$

43) $\lim_{x \to 2} \frac{-1(\cancel{2-x})(2+x)}{(x-1)(\cancel{x-2})}$

$\boxed{-4}$

(44) a) $f(-x) = \frac{-x}{(-x)^2-1} = -\frac{x}{x^2-1} = -f(x)$ odd

b) $f(-x) = (-x)^3 + 2(-x) = -x^3 - 2x = -(x^3+2x) = -f(x)$
odd

c) $f(-x) = (-x)^3 + 2(-x) + 1 = -x^3 - 2x + 1$ neither

d) $f(-x) = (-x)^3 + 2(-x)^2 + (-x) - 1$
$= -x^3 + 2x^2 - x - 1$ neither

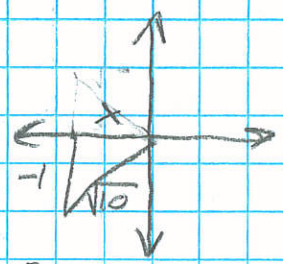
e) $f(-x) = (-x)^4 - (-x)^2 - 1 = x^4 - x^2 - 1 = f(x)$ even

f) $f(-x) = \frac{(-x)^2}{(-x)^4 - 2(-x)^2 + 1} = \frac{x^2}{x^4 - 2x^2 + 1} = f(x)$ even

g) $f(-x) = \frac{3(-x)}{(-x)^3 - 2(-x)} = \frac{-3x}{-x^3 + 2x} = \frac{-3x}{-(x^3-2x)} = \frac{3x}{x^3-2x}$
$= f(x)$ even

(45) 180°

(46) QIII



$x^2 + 1 = 10$
$x^2 = 9$
$x = -3$ $\tan\theta = 1/3$

(47) $\frac{1}{2} + \frac{-1}{2} = 0$

(48) $3 \cdot 3^{-1}\left[\frac{\frac{1}{3}}{5 - 1/3}\right] = \frac{1}{15-1} = \frac{1}{14}$

(49) $\frac{3}{20}$

(50) a) $50 \cdot 1.1 = 55$
b) $55 \cdot 0.9 = 49.5$

$$\begin{array}{r} 55 \\ \times\ .9 \\ \hline 45 \\ 495 \end{array}$$

(51) $3\sqrt{3} - 2 \cdot 3\sqrt{3} + 4 \cdot 2\sqrt{3}$
$3\sqrt{3} - 6\sqrt{3} + 8\sqrt{3}$
$5\sqrt{3}$

(52) a) $7x - 11 = 0$
$x = 11/7$ ✓

b) $7x - 11 = 36$
$7x = 47$
$x = 47/7$ ✓

c) $7x - 11 = 121$
$7x = 132$
$x = 132/7$
invalid no solution

(53) $\sqrt{12}\left(\sqrt{8} + \sqrt{18}\right)$
$2\sqrt{3}\left(2\sqrt{2} + 3\sqrt{2}\right)$
$2\sqrt{3}\,(5\sqrt{2})$
$10\sqrt{6}$

(54) $y = 1$
$x + \pi = \cos^{-1}(1)$
$x + \pi = 0$
$x = -\pi$ (not valid, add 2π)
$(\pi, 1)$

(55) a) $(3, 12]$
b) $(0, \infty)$
c) $(-\infty, 0]$
d) $[-5, 5]$
e) $\emptyset$
f) $(-\infty, 2) \cup (3, \infty)$

(56) a) D: $(-\infty, 0) \cup (0, \infty)$
R: $(-\infty, 0) \cup (0, \infty)$
b) D: $[-2, \infty)$
R: $[0, \infty)$
c) D: $(-\infty, \infty)$
R: $[-1, 1]$
d) D: $(0, \infty)$
R: $(-\infty, \infty)$

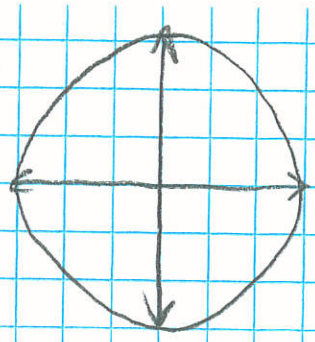
(57) a) $f(g(x)) = (5x+3) - 2$
$= 5x + 1$
$f(g(2)) = 11$
b) $\frac{g}{f}(-4) = \frac{-17}{-6} = \frac{17}{6}$
c) $f(1) = -1$
$f(-1) = -3$
d) $5x + 1$

(58) a) $\frac{0^2}{2} + \frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2} + \frac{4^2}{2}$
$0 + \frac{1}{2} + \frac{4}{2} + \frac{9}{2} + \frac{16}{2}$
$\frac{30}{2} = 15$

b) $\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} = \frac{1}{1} + \frac{1}{8} + \frac{1}{27} = \frac{216}{216} + \frac{27}{216} + \frac{8}{216}$
$= \frac{251}{216}$

(59) a) $-6\vec{i} + \frac{5}{2}\vec{j}$
b) $5\vec{i} - \vec{j}$
c) 5
d) $\|v\| = \sqrt{29}$
$\frac{-2}{\sqrt{29}}\vec{i} + \frac{5}{\sqrt{29}}\vec{j}$

60) a)



b)

θ	r
0	undef
$\pi/6$	6
$\pi/3$	$3\sqrt{3}/2 \approx 5.2$
$\pi/4$	$3\sqrt{2}/2 \approx 4.2$
$\pi/2$	3
$2\pi/3$	$3\sqrt{2}/2 \approx 4.2$
$3\pi/4$	$3\sqrt{3}/2 \approx 5.2$
$5\pi/6$	6
2π	undef
$7\pi/6$	-6
$5\pi/4$	-5.2
$4\pi/3$	-4.2
$3\pi/2$	-3
$5\pi/3$	-4.2
$7\pi/4$	-5.2
$11\pi/6$	-6
2π	undef

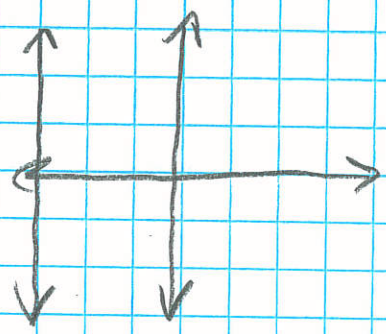
alternatively

$r = 3 \sec\theta$

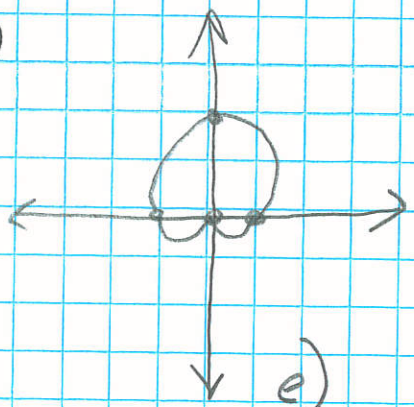
$r = \frac{3}{\cos\theta}$

$r\cos\theta = 3$

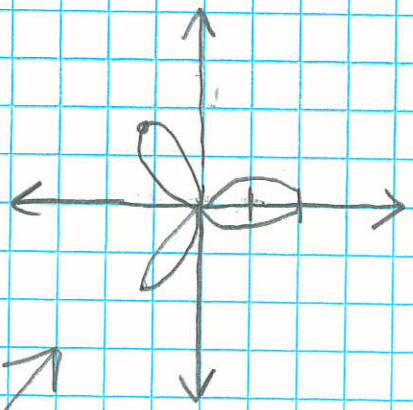
$x = 3$



c)



d)



e)