AMDG

Name — Ms. Kresovic

Adv Geo - 3.2: Three Ways to Prove Triangles Congruent — Date:

For tonight's homework, you are also asked to watch a couple of videos:

Objectives

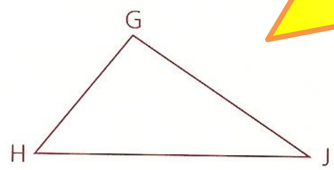
After studying this section, you will be able to

- Identify included angles and included sides
- Apply the SSS postulate
- Apply the SAS postulate
- Apply the ASA postulate

Part One: Introduction

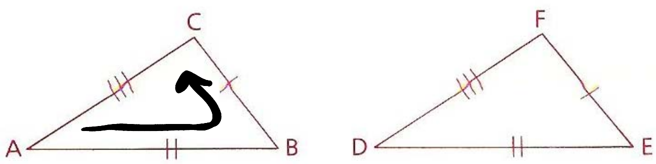
Included Angles and Included Sides

In the figure at the right, $\angle H$ is ***included*** by the sides $\overline{GH}$ and $\overline{HJ}$. Side $\overline{GH}$ is included by $\angle H$ and $\angle G$. Can you name the sides that include $\angle G$? Can you name the angles that include side $\overline{HJ}$?



The SSS Postulate

...



	S	R
S	1. $\overline{AC} \cong \overline{DF}$	1. given
S	2. $\overline{CB} \cong \overline{FE}$	2. given
S	3. $\overline{BA} \cong \overline{ED}$	3. given
	4. $\triangle ABC \cong \triangle DEF$	4. SSS (1,2,3)

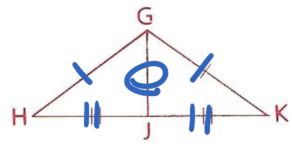
The tick marks on $\triangle ABC$ and $\triangle DEF$ show sufficient conditions for us to know that $\triangle ABC \cong \triangle DEF$. This special property of triangles can be expressed as a postulate, which we will refer to as the SSS postulate. Each S stands for a pair of congruent corresponding sides, such as $\overline{AC}$ and $\overline{DF}$.

Postulate ***If there exists a correspondence between the vertices of two triangles such that three sides of one triangle are congruent to the corresponding sides of the other triangle, the two triangles are congruent. (SSS)***

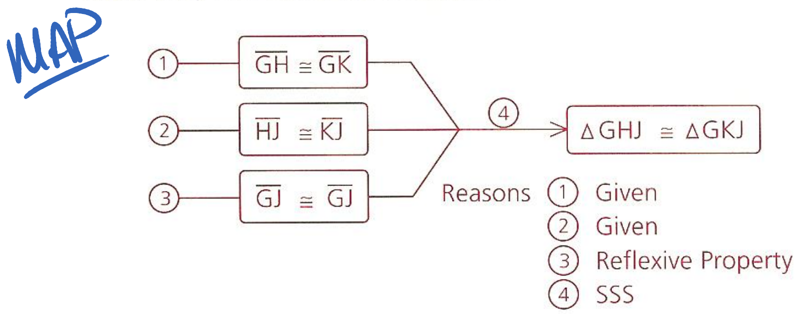
The video "Proving Triangles are Congruent - MathHelp.com - Math Help" (3:34) is hosted on YouTube: http://www.youtube.com/watch?v=NAhcmPS5k9g

The SSS relationship can be proved by methods that are not part of this course; we shall assume it and use the abbreviation SSS in proofs.

In the figure, is △GHJ congruent to △GKJ by SSS? The tick marks give us *two* pairs of congruent sides, but that is not enough. However, since $\overline{GJ}$ is a common side of both triangles, $\overline{GJ} \cong \overline{GJ}$ by the Reflexive Property. So we actually do have SSS!

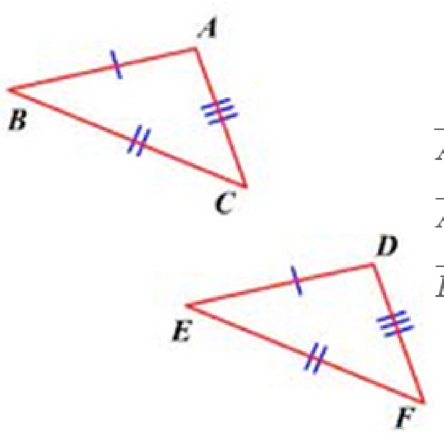


The following diagram illustrates the flow of logic that proves that △GHJ and △GKJ are congruent.



1. Side-Side-Side (SSS)

If we know that the three sides of a triangle are congruent to the three sides of another triangle, then the angles MUST be the same (or it wouldn't form a triangle).



$\overline{AB} \cong \overline{DE}$ (*side*)

$\overline{AC} \cong \overline{DF}$ (*side*)

$\overline{BC} \cong \overline{EF}$ (*side*)

$\therefore \Delta ABC \cong \Delta DEF$

The symbol ∴ means "therefore." If we are able to show that the three corresponding sides are congruent, then we have enough information to prove that the two triangles are congruent because of the SSS Postulate!

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The SAS Postulate

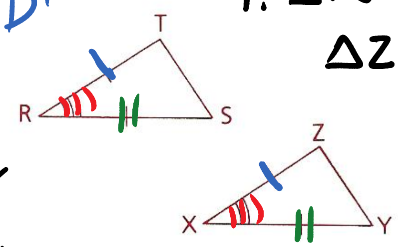
It can also be shown that only two pairs of congruent corresponding sides are needed to establish the congruence of two triangles if the angles included by the sides are known to be congruent.

Postulate ***If there exists a correspondence between the vertices of two triangles such that two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle, the two triangles are congruent.*** (*SAS*)

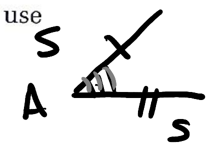
ORDER MATTERS!

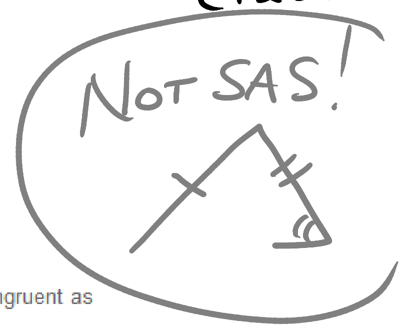
MARK DIAG.

	S	R
S	1. $\overline{TR} \cong \overline{ZX}$	1. Given
A	2. $\angle TRS \cong \angle ZXY$	2. Given
S	3. $\overline{SR} \cong \overline{XY}$	3. Given
	4. $\triangle TRS \cong \triangle ZXY$	4. SAS (123)



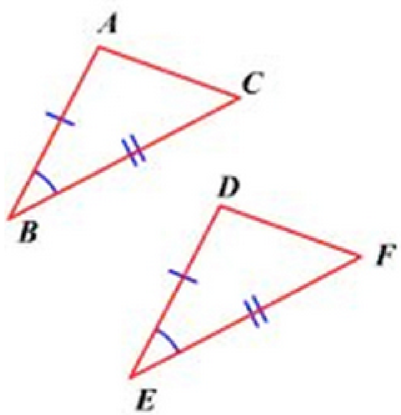
The fact that the *A* is between the *S*'s in SAS should help you remember that the congruent angles in the triangles must be the angles *included by* the pairs of congruent sides. Although this relationship, like SSS, can be proved, we shall assume it and use the abbreviation SAS in proofs.





2. Side-Angle-Side (SAS)

If we can show that two sides and the angle IN BETWEEN them are congruent, then the whole triangle must be congruent as well. It looks like this:



$\overline{AB} \cong \overline{DE}$ (side)

$\angle B \cong \angle E$ (angle)

$\overline{BC} \cong \overline{EF}$ (side)

$\therefore \Delta ABC \cong \Delta DEF$

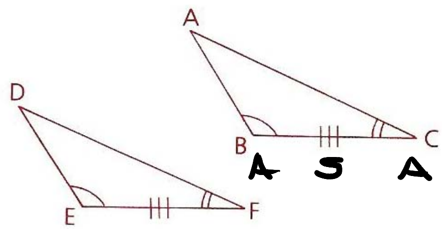
The angle HAS to be in between the two sides for the SAS Postulate to be used.

The ASA Postulate

The following postulate will give us a third way of proving triangles congruent.

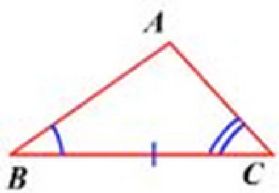
Postulate ***If there exists a correspondence between the vertices of two triangles such that two angles and the included side of one triangle are congruent to the corresponding parts of the other triangle, the two triangles are congruent. (ASA)***

Again, ASA can be proved, although we shall assume it. The arrangement of the letters in *ASA* matches the arrangement of marked parts in the triangles; the congruent sides must be the ones included by the pairs of congruent angles.



3. Angle-Side-Angle (ASA)

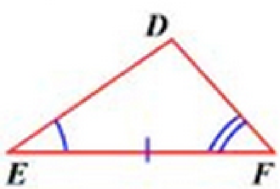
If we can show that two angles and the side IN BETWEEN them are congruent, then the whole triangle must be congruent as well.



$\angle B \cong \angle E$ (angle)

$\overline{BC} \cong \overline{EF}$ (side)

$\angle C \cong \angle F$ (angle)



$\therefore \Delta ABC \cong \Delta DEF$

The side HAS to be in between the two angles for the ASA Postulate to be used.

If you are curious, you may be wondering whether SSS, SAS, and ASA are the only shortcuts for proving that triangles are congruent. Not quite. These three postulates, however, are enough to get us started on proofs that triangles are congruent.

Study the sample problems carefully before you attempt the problem sets. Notice that we call SSS, SAS, and ASA methods of proof. Any definition, postulate, or theorem can be called a method if it is a key reason in proofs.

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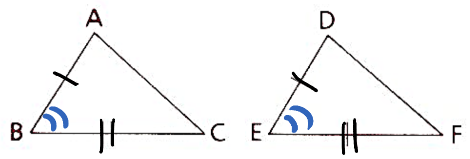
Part Two: Sample Problems

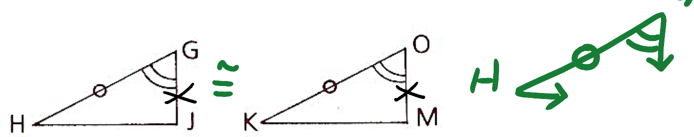
In problems 1–3 and 5, you are given the congruent angles and sides shown by the tick marks. Name the additional congruent sides or angles needed to prove that the triangles are congruent by each specified method.

Problem 1 a SSS b SAS



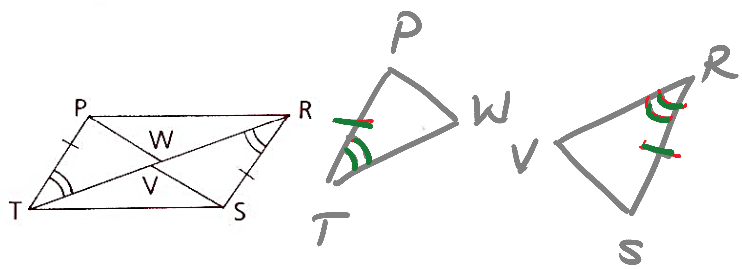
Answers a $\overline{AC} \cong \overline{DF}$
b $\angle ABC \cong \angle DEF$

Problem 2 a SAS b ASA



Answers a $\overline{GJ} \cong \overline{OM}$
b $\angle H \cong \angle K$

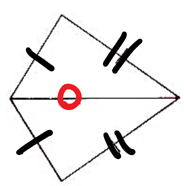
Problem 3 Prove: $\triangle PWT \cong \triangle SVR$
a SAS
b ASA

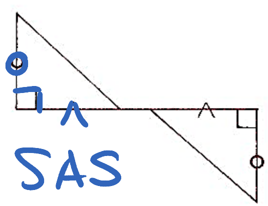


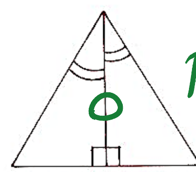
Answers a $\overline{WT} \cong \overline{VR}$
b $\angle P \cong \angle S$

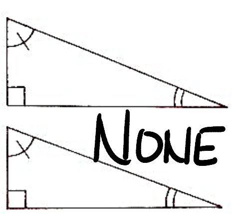
Problem 4 *Using the tick marks for each pair of triangles, name the method (SSS, SAS, or ASA), if any, that can be used to prove the triangles congruent.*

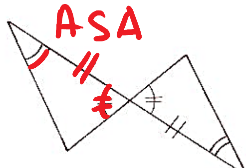
SSS
SAS
~~ASA~~

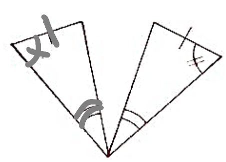
a  REF

c  SAS

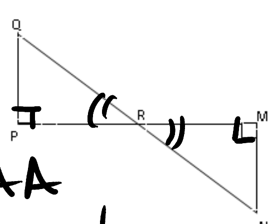
e  REF then ASA

b  NONE

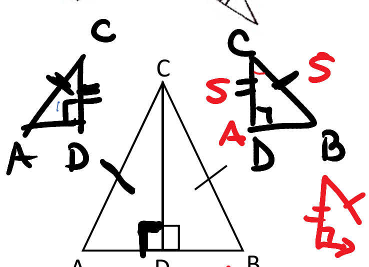
d  ASA

f  NONE

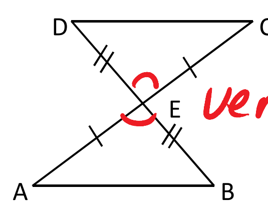
AA

g 

AA NONE

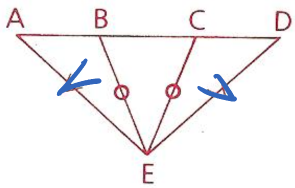
h 

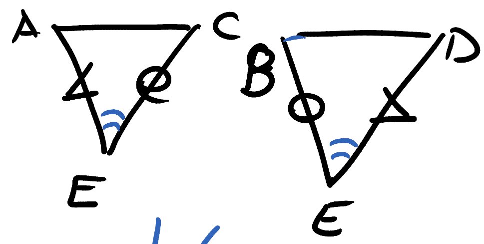
NONE

i 

vertks ⇒ ≅∠: SAS

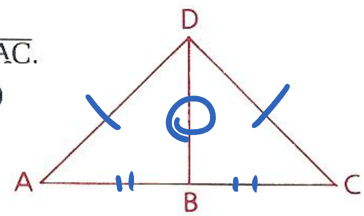
Problem 5 Prove: △AEC ≅ △DEB
a SSS
b SAS





Answers
a $\overline{AC} \cong \overline{BD}$
b ∠AEC ≅ ∠BED

Problem 6 Given: $\overline{AD} \cong \overline{CD}$;
B is the midpoint of $\overline{AC}$.
Conclusion: △ABD ≅ △CBD

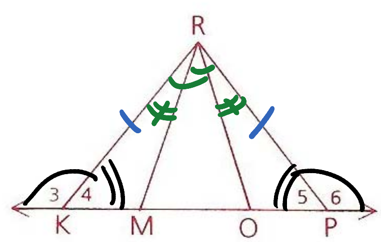


Proof

	Statements	Reasons
S	1 $\overline{AD} \cong \overline{CD}$	1 Given
	2 B is the midpt. of $\overline{AC}$.	2 Given
S	3 $\overline{AB} \cong \overline{CB}$	3 midpt ⇒ ≅ segs (2)
S	4 $\overline{BD} \cong \overline{BD}$	4 Ref
	5 △ABD ≅ △CBD	5 SSS (1 3 4)

Note After SSS, SAS, or ASA we shall identify the numbers of the statements in which the pairs of congruent parts were found.

Problem 7 Given: ∠3 ≅ ∠6,
$\overline{KR} \cong \overline{PR}$,
∠KRO ≅ ∠PRM
Prove: △KRM ≅ △PRO



Proof

	Statements	Reasons
	1 ∠3 ≅ ∠6	1 Given
	2 ∠3 is supp. to ∠4.	2 } St∠ ⇒ supp ∠s
	3 ∠5 is supp. to ∠6.	3 }
A	4 ∠4 ≅ ∠5	4 ∠s supp ≅ ∠s ⇒ ≅ ∠s
S	5 $\overline{KR} \cong \overline{PR}$	5 Given
	6 ∠KRO ≅ ∠PRM	6 Given
	7. ∠MRO ≅ ∠ORM	7. Ref
A	8 ∠KRM ≅ ∠PRO	8 Subtract
	9 △KRM ≅ △PRO	9 ASA (4, 5, 8)

* **Note** The assumption of straight angles and the fact that two angles that form a straight angle are supplementary may now be combined in one step (as in step 2 above).

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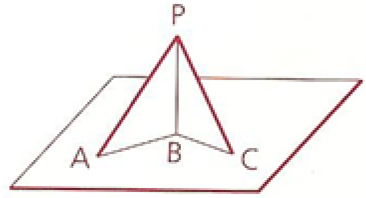
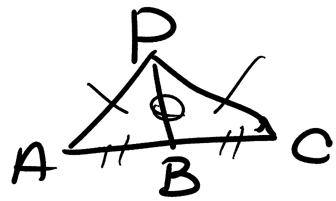
Problem 8

Two triangles are standing up on a table-top as shown. $\overline{PA} \cong \overline{PC}$ and $\overline{BA} \cong \overline{BC}$.

Prove: $\triangle PBA \cong \triangle PBC$



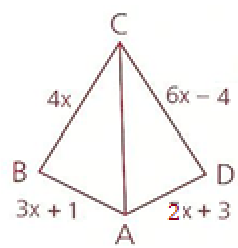


Maps are a good pre-proof-writing strategy! Use the map to organize the information ***before*** *you try writing the proof.*

Statements	Reasons
S 1. $\overline{PA} \cong \overline{PC}$	1. Given
S 2. $\overline{BA} \cong \overline{BC}$	2. Given
S 3. $\overline{PB} \cong \overline{PB}$	3. Reflexive
4. $\triangle PBA \cong \triangle PBC$	4. SSS(1,2,3)

Problem 9

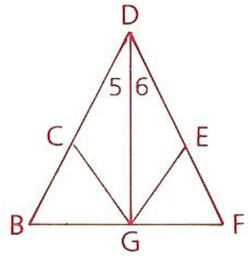
The perimeter of ABCD is 30. Find the value of x. Is $\triangle ABC$ congruent to $\triangle ADC$?



Problem 10

Given: $\overline{BC} \cong \overline{FE}$,
$\overline{DC} \cong \overline{DE}$,
$\angle 5 \cong \angle 6$

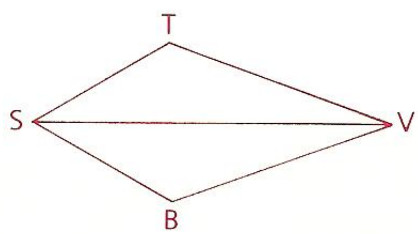
Prove: $\triangle BDG \cong \triangle FDG$



Problem 11

Given: $\overrightarrow{SV}$ bisects $\angle TSB$.
$\overrightarrow{VS}$ bisects $\angle TVB$.

Prove: $\triangle TSV \cong \triangle BSV$



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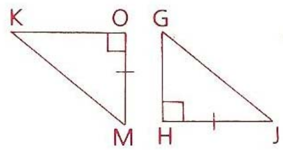
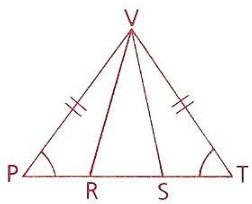
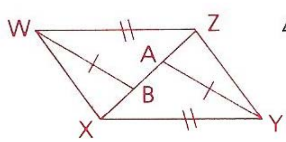
Homework

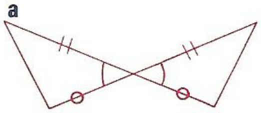
Problem Set A

1 Study the congruent sides and angles shown by the tick marks, then identify the additional information needed to support the specified method of proving that the indicated triangles are congruent.

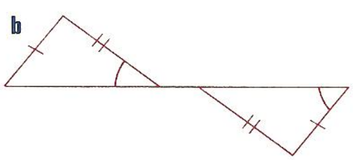
	Triangles	Method	Needed Information
a 	△HGJ and △OKM	SAS ASA	? ?
b 	△PSV and △TRV	SAS ASA	? ?
c 	△WBZ and △YAX	SSS SAS	? ?

2 Using the tick marks for each pair of △, name the method (SSS, SAS, or ASA), if any, that will prove the △ to be ≅.

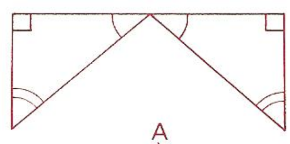
a



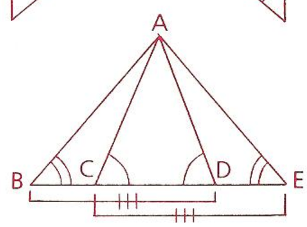
b



c

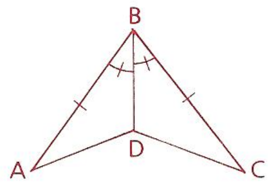


d

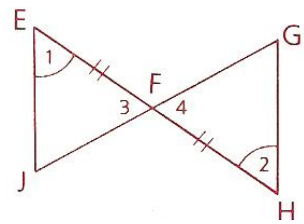


△ABD and △AEC

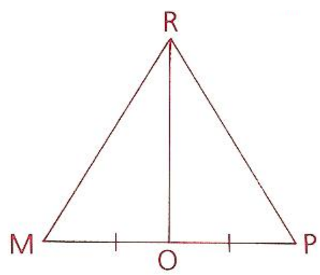
3 Given: $\overline{AB} \cong \overline{CB}$,
$\angle ABD \cong \angle CBD$
Prove: △ABD ≅ △CBD



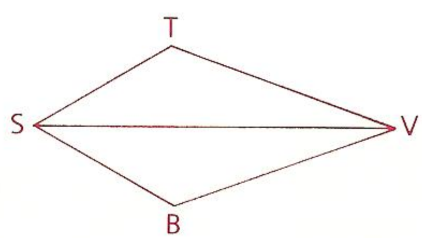
4 Given: $\angle 1 \cong \angle 2$,
$\overline{EF} \cong \overline{HF}$
Prove: $\triangle EFJ \cong \triangle HFG$



5 Given: $\overline{RO} \perp \overline{MP}$,
$\overline{MO} \cong \overline{OP}$
Prove: $\triangle MRO \cong \triangle PRO$



6 Given: $\overrightarrow{SV}$ bisects $\angle TSB$.
$\overrightarrow{VS}$ bisects $\angle TVB$.
Prove: $\triangle TSV \cong \triangle BSV$



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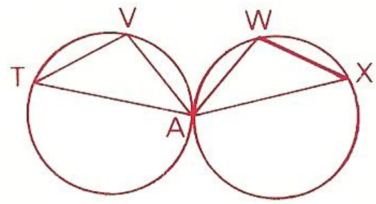
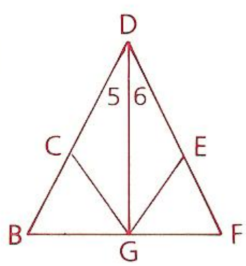
7 Given: $\overline{TV} \cong \overline{XW}$,
$\overline{VA} \cong \overline{WA}$,
$\overline{TA} \cong \overline{XA}$

Prove: $\triangle TVA \cong \triangle XWA$

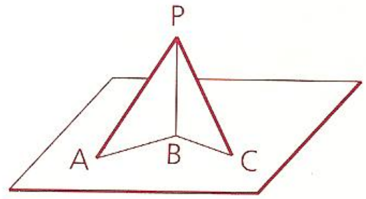


8 Given: $\overline{BC} \cong \overline{FE}$,
$\overline{DC} \cong \overline{DE}$,
$\angle 5 \cong \angle 6$

Prove: $\triangle BDG \cong \triangle FDG$



9 Two triangles are standing up on a table-top as shown. $\overline{PA} \cong \overline{PC}$ and $\overline{BA} \cong \overline{BC}$.
Prove: $\triangle PBA \cong \triangle PBC$



Hint: Make a 3D paper model of this.

10 The perimeter of ABCD is 85. Find the value of x. Is $\triangle ABC$ congruent to $\triangle ADC$?