Objectives

After studying this section, you will be able to

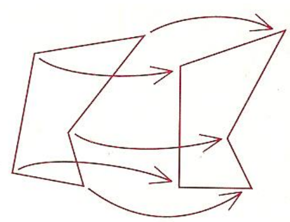
- Understand the concept of congruent figures
- Accurately identify the corresponding parts of figures

After each paragraph, write (underline, or highlight) a couple of words (no more than one sentence) that identifies its purpose or point.

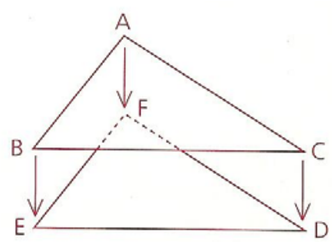
Congruent Figures

Although you learned a bit about the art of proof in Chapters 1 and 2, you may still be uneasy about proofs. You will, however, find your confidence growing as you work with triangles in this chapter. What you discover about congruent triangles will help you understand the characteristics of the other geometric figures you will meet in your studies.

In general, two geometric figures are congruent if one of them could be placed on top of the other and fit exactly, point for point, side for side, and angle for angle. *Congruent figures have the same size and shape.*



Every triangle has six parts—three angles and three sides. When we say that $\triangle ABC \cong \triangle FED$, we mean that $\angle A \cong \angle F$, $\angle B \cong \angle E$, and $\angle C \cong \angle D$ and that $\overline{AB} \cong \overline{FE}$, $\overline{BC} \cong \overline{ED}$, and $\overline{CA} \cong \overline{DF}$.



The order of the letters matter! Here's why: Even without a picture, we know – just by the statement – which segments and which angles are congruent.

$\triangle ABC \cong \triangle FED$ has congruent segments:

$\triangle ABC \cong \triangle FED$ has congruent angles:

Definition ***Congruent triangles*** $\Leftrightarrow$ all pairs of corresponding parts are congruent.

Remember, an arrow symbol ($\Rightarrow$) means "implies" ("If . . ., then . . ."). If the arrow is double ($\Leftrightarrow$), the statement is reversible.

Would the statement $\triangle ABC \cong \triangle DEF$ be correct? The answer is no! Corresponding letters must match in the correspondence.

Correct: $\triangle ABC \cong \triangle FED$ Incorrect: $\triangle ABC \cong \triangle DEF$

Congruent segments of $\triangle ABC \cong \triangle FED$

Congruent segments of $\triangle ABC \cong \triangle DEF$

Are these lists the same?

Does the order of the letters in the name of the triangle matter?

Definition ***Congruent polygons*** $\Leftrightarrow$ all pairs of corresponding parts are congruent.

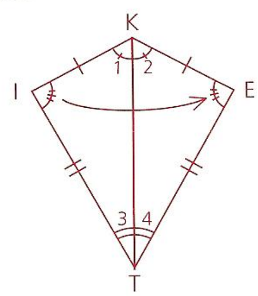
Writing proofs involving congruent triangles will be unnecessarily tedious unless we shorten some of the reasons. From now on, therefore, we will refer to many theorems and postulates in proofs only by the names or abbreviations we have assigned. You may wish to review the following properties, presented in Chapter 2:

Addition Property	Multiplication Property	Transitive Property
Division Property	Subtraction Property	Substitution Property

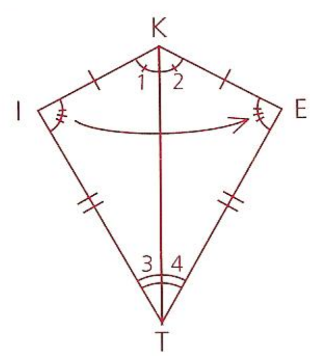
More About Correspondences

Notice that $\triangle KET$ is a ***reflection*** of $\triangle KIT$ over $\overline{KT}$.

$\angle I$ reflects onto $\angle E$.
$\angle 1$ reflects onto $\angle 2$.
$\angle 3$ reflects onto $\angle 4$.
$\overline{KI}$ reflects onto $\overline{KE}$.
$\overline{IT}$ reflects onto $\overline{ET}$.



Fold the paper at $\overline{KT}$ so that you can see that I, when reflected over $\overline{KT}$, is located at E.



Notice also that $\overline{KT}$ is the sixth corresponding part. $\overline{KT}$ reflects onto itself. In fact, it is actually a side shared by the two triangles. We often need to include a shared side in a proof. Whenever a side or an angle is shared by two figures, we can say that the side or angle is congruent to itself. This property is called the ***Reflexive Property.***

Postulate ***Any segment or angle is congruent to itself. (Reflexive Property)***

Do we prove postulates?

AMDG

Name | *3: Congruent Triangles* | Ms. Kresovic

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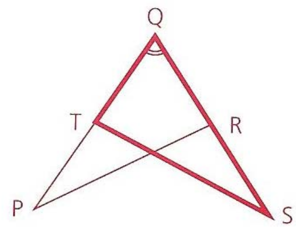
∠PQR, in △PQR, is congruent to ∠SQT, in △SQT, by the Reflexive Property.

Notice that ∠SQT and ∠PQR are actually different names for the same angle. We used different names so that you could see that the angle belonged to two different triangles.



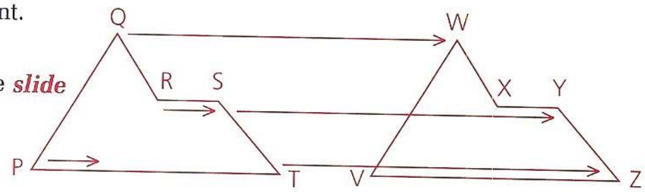
Triangles may overlap! A colored pencil (orange, yellow, pink, light blue) may help you in these diagrams.

Other transformations (slides and rotations) may be shown:

The two figures shown are congruent.

PQRST ≅ VWXYZ

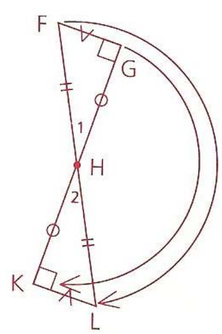
The correspondence is evident if we ***slide*** PQRST onto VWXYZ.



The triangles at the right are congruent. To determine the correspondence of the triangles, we can ***rotate*** △FGH onto △LKH about H.

Angle 1 at H rotates onto angle 2 at H.

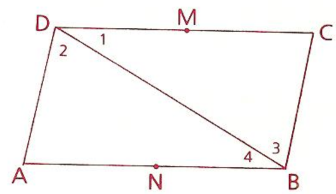
Thus, all six pairs of corresponding parts are congruent.



Part Two: Sample Problems

In the following two problems, try to justify each conclusion with one of the properties presented in Chapter 2 and in this section.

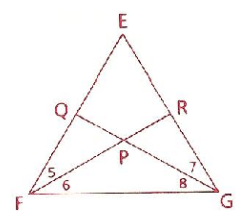
Problem 1 *Given:* M *and* N *are midpoints.*
$\overline{DC} \cong \overline{AB}$, $\overline{AB} \cong \overline{DB}$,
$\angle 1 \cong \angle 4$, $\angle 2 \cong \angle 3$



Conclusions: **a** $\angle ADC \cong \angle ABC$
b $\overline{CM} \cong \overline{AN}$
c $\overline{BD} \cong \overline{DB}$
d $\overline{DC} \cong \overline{DB}$

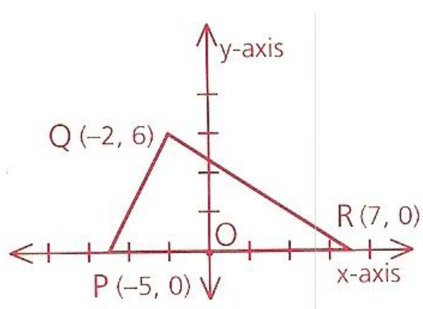
Axiom Bank:
Match these reasons to the conclusions at left.
Addition
Subtraction
Multiplication
Division
Reflexive
Symmetric
Transitive

Problem 2 *Given:* $\overrightarrow{FP}$ *and* $\overrightarrow{GP}$ *are angle bisectors.*
$\angle 5$ *is an acute angle.*
$\angle 5 \cong \angle 7$, $\overline{PF} \cong \overline{PG}$, $\overline{QG} \cong \overline{FR}$



Conclusions: **a** $\angle QFG \cong \angle RGF$
b $\overline{QP} \cong \overline{PR}$
c $\angle 7$ *is an acute angle.*
d $\angle FER \cong \angle GEQ$

Problem 3
Draw the rotation of ΔPQR 90° counter-clockwise about O. Label its vertices with their coordinates.

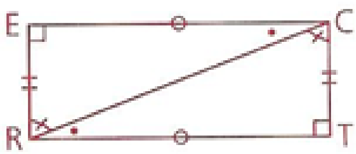


Homework

In problems 1–3, indicate which triangles are congruent. Be sure to have the correspondence of letters correct.

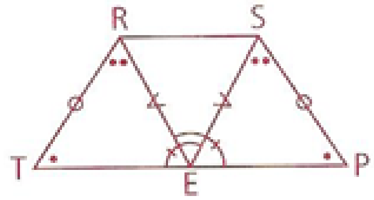
1 Why is $\overline{RC} \cong \overline{RC}$?

$\triangle ERC \cong$ ___?___



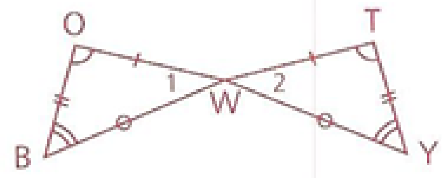
2 E is the midpt. of $\overline{TP}$.

$\triangle SPE \cong$ ___?___



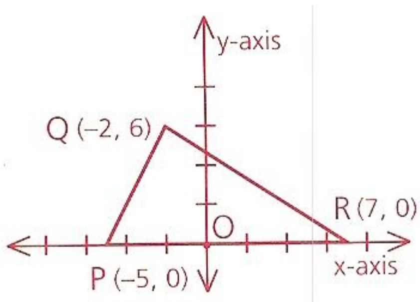
3 Why is $\angle 1 \cong \angle 2$?

$\triangle BOW \cong$ ___?___

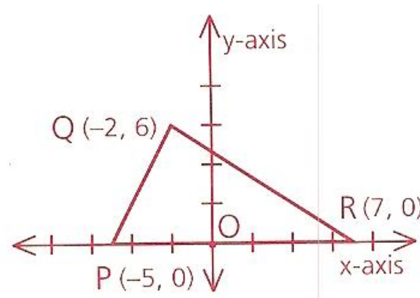


In problems 4 and 5, use the "prime" notation, that is P (x_1, y_1) once transformed is noted as P' (x_2, y_2).

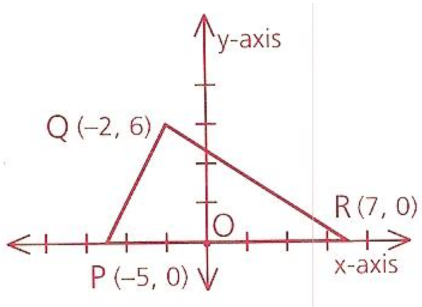
4 a Copy $\triangle PQR$. Draw its reflection over the x-axis and give the coordinates of the vertices.



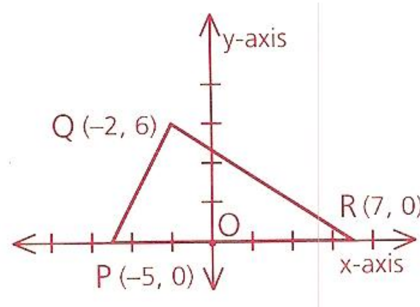
b Copy $\triangle PQR$. Draw its reflection over the y-axis and give the coordinates of the vertices.



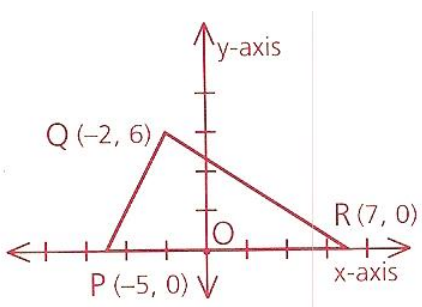
c Copy $\triangle PQR$. Slide it 3 units to the left and give the coordinates of the vertices.



5 a Draw the rotation of $\triangle PQR$ 180° clock-wise about O. Label its vertices with their coordinates.



b Draw the slide of $\triangle PQR$ along ray $\overrightarrow{PR}$ so that P is at O, and label its vertices with their coordinates.



c Draw the reflection of $\triangle PQR$ over the y-axis and label its vertices with their coordinates.