

Objective

After studying this section, you will be able to

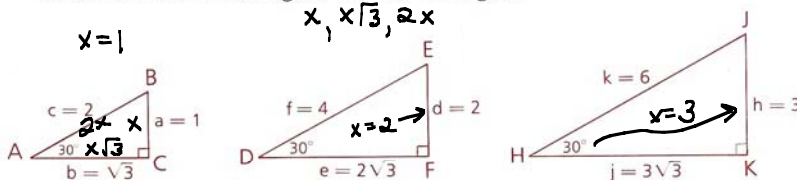
- Understand three basic trigonometric relationships

SOH CAH TOA

This section presents the three basic trigonometric ratios **sine**, **co-sine**, and **tangent**. The concept of similar triangles and the Pythagorean Theorem can be used to develop the **trigonometry of right triangles**.

$\sin \angle A$
 $\cos \angle A$
 $\tan \angle A$

Consider the following 30°-60°-90° triangles.



Compare the length of the leg opposite the 30° angle with the length of the hypotenuse in each triangle.

$$\frac{\text{Opp}}{\text{Hyp}} \Rightarrow \sin 30^\circ = \frac{1}{2}$$

In $\triangle ABC$, $\frac{a}{c} = \frac{1}{2} = 0.5$. In $\triangle DEF$, $\frac{d}{f} = \frac{2}{4} = 0.5$. In $\triangle HJK$, $\frac{h}{k} = \frac{3}{6} = 0.5$.

If you think about similar triangles, you will see that in every 30°-60°-90° triangle,

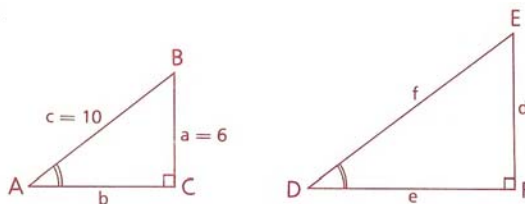
$$\sin 30^\circ = \frac{\text{leg opposite } 30^\circ \angle}{\text{hypotenuse}} = \frac{1}{2}$$

For each triangle shown, verify that $\frac{\text{leg adjacent to } 30^\circ \angle}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} = \cos 30^\circ$

For each triangle shown, find the ratio $\frac{\text{leg opposite } 30^\circ \angle}{\text{leg adjacent to } 30^\circ \angle} = \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

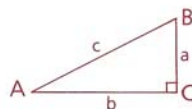
In $\triangle ABC$ and $\triangle DEF$,

$$\frac{a}{c} = \frac{d}{f} = \frac{6}{10} = \frac{3}{5}$$



Engineers and scientists have found it convenient to formalize these relationships by naming the ratios of sides. You should memorize these three basic ratios.

Definition Three Trigonometric Ratios



sine of $\angle A = \sin \angle A = \frac{\text{opposite leg}}{\text{hypotenuse}}$ **SOH**

cosine of $\angle A = \cos \angle A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$ **CAH**

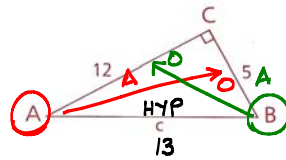
tangent of $\angle A = \tan \angle A = \frac{\text{opposite leg}}{\text{adjacent leg}}$ **TOA**

Class Examples

Problem 1 Find: a $\cos \angle A$
b $\tan \angle B$

$$\cos \angle A = \frac{12}{13}$$

CAH



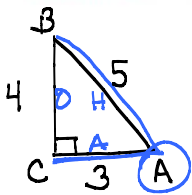
TOA

$$\tan \angle B = \frac{12}{5}$$



SOH CAH TOA

Problem 2 Find the three trigonometric ratios for $\angle A$ and $\angle B$.



$$\sin \angle A = \frac{4}{5}$$

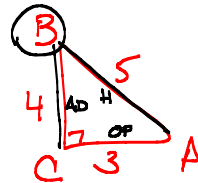
$$\cos \angle A = \frac{3}{5}$$

$$\tan \angle A = \frac{4}{3}$$

$$\sin \angle B = \frac{3}{5}$$

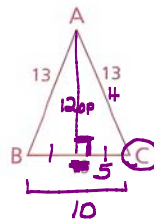
$$\cos \angle B = \frac{4}{5}$$

$$\tan \angle B = \frac{3}{4}$$



Problem 3 $\triangle ABC$ is an isosceles triangle as marked. Find $\sin \angle C$.

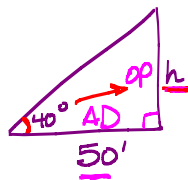
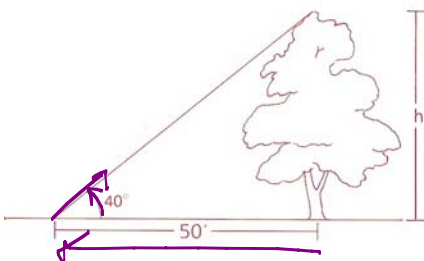
$$\sin \angle C = \frac{12}{13}$$



(5, 12, 13)

Problem 4 Use the fact that $\tan 40^\circ \approx 0.8391$ to find the height of the tree to the nearest foot.

SOH CAH TOA



$$\tan 40^\circ = \frac{h}{50}$$

$$0.8391 = \frac{h}{50}$$

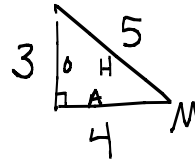
$$50(0.8391) = h$$

$$41.955 = h$$

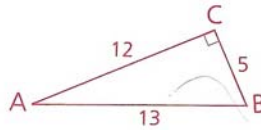
$$42 \text{ ft} \approx h$$

Homework

- 5 If $\tan \angle M = \frac{3}{4}$, find $\cos \angle M$. (Hint: Start by drawing the triangle.)



- 6 Using the figure as marked, name each missing angle.

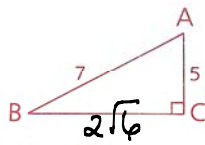


a $\frac{5}{12} = \tan \angle \underline{\hspace{1cm}}$

b $\frac{12}{13} = \cos \angle \underline{\hspace{1cm}}$

c $\frac{5}{13} = \sin \angle \underline{\hspace{1cm}}$

- 7 Find each quantity.



a BC

b $\sin \angle A$

c $\tan \angle B$

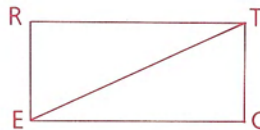
$$BC^2 + 5^2 = 7^2$$

$$BC = \sqrt{49 - 25} = \sqrt{24} = \sqrt{4} \cdot \sqrt{6} = 2\sqrt{6}$$

- 8 Given: RECT is a rectangle.
ET = 26, RT = 24

Find: a $\sin \angle RET$

b $\cos \angle RET$



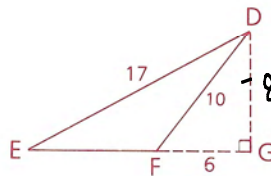
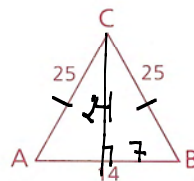
Problem Set B

- 9 Using the given figures, find

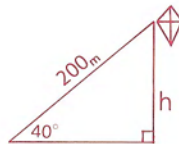
a $\cos \angle A$

b $\sin \angle E$

c $\sin \angle DFG$



- 10 Use the fact that $\sin 40^\circ \approx 0.6428$ to find the height of the kite to the nearest meter.

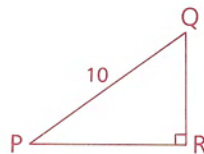


- 11 a If $\tan \angle A = 1$, find $m\angle A$.

b If $\sin \angle P = 0.5$, find $m\angle P$.

- 12 Given: $\sin \angle P = \frac{3}{5}$, PQ = 10

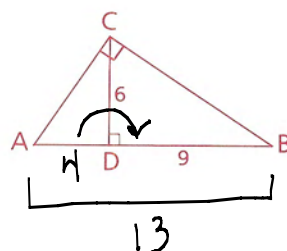
Find: $\cos \angle P$



- 13 Using the figure, find

a $\tan \angle ACD$

b $\sin \angle A$



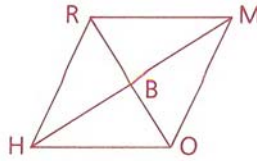
$$\frac{AD}{6} = \frac{6}{9} \rightarrow 9(AD) = 36 \rightarrow AD = 4$$

Problem Set B, continued

- 14 Given: RHOM is a rhombus.

RO = 18, HM = 24

Find: **a** $\cos \angle BRM$ **b** $\tan \angle BHO$



- 15 Given a trapezoid with sides 5, 10, 17, and 10, find the sine of one of the acute angles.

- 16 Given $\triangle ABC$ with $\angle C = 90^\circ$, indicate whether each statement is true Always (A), Sometimes (S), or Never (N).

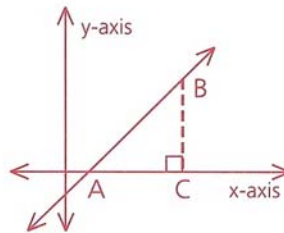
a $\sin \angle A = \cos \angle B$

b $\sin \angle A = \tan \angle A$

c $\sin \angle A = \cos \angle$

- 17 If $\triangle EQU$ is equilateral and $\triangle RAT$ is a right triangle with $RA = 2$, $RT = 1$, and $\angle T = 90^\circ$, show that $\sin \angle E = \cos \angle A$.

- 18 If the slope of $\overleftrightarrow{AB}$ is $\frac{5}{8}$, find the tangent of $\angle BAC$.



Problem Set C *C → Challenge*

- 19 Use the definitions of the trigonometric ratios to verify the following relationships, given $\triangle ABC$ in which $\angle C = 90^\circ$.

a $(\sin \angle A)^2 + (\cos \angle A)^2 = 1$

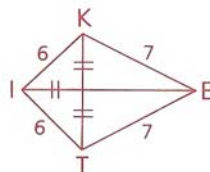
c $\frac{\sin \angle A}{\cos \angle A} = \tan \angle A$

b $\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$

d $\sin \angle A = \cos (90^\circ - \angle A)$

- 22 Given: KITE is a kite with sides as marked.

Find: $\tan \angle KEI$



Classwork

1 Find each ratio.

a $\sin \angle A$

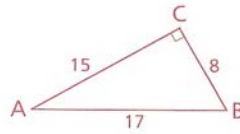
b $\cos \angle A$

c $\tan \angle A$

d $\sin \angle B$

e $\cos \angle B$

f $\tan \angle B$



2 Find each ratio.

a $\sin 30^\circ = \frac{x}{2} = \frac{1}{2}$

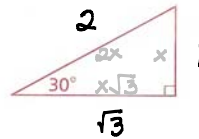
b $\cos 30^\circ$

c $\tan 30^\circ$

d $\sin 60^\circ$

e $\cos 60^\circ$

f $\tan 60^\circ$

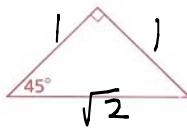


3 Find each ratio.

a $\sin 45^\circ$

b $\cos 45^\circ$

c $\tan 45^\circ$

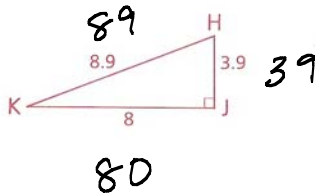


$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

4 Find each ratio.

a $\cos \angle H$

b $\tan \angle K$



1a	8/17
1b	15/17
1c	8/15
1d	15/17
1e	8/17
1f	15/8
2a	1/2
2b	sqrt(3)/2
2c	sqrt(3)/3
2d	sqrt(3)/2
2e	1/2
2f	sqrt(3)
3a	sqrt(2)/2
3b	sqrt(2)/2
3c	1
4a	39/89
4b	39/80

