

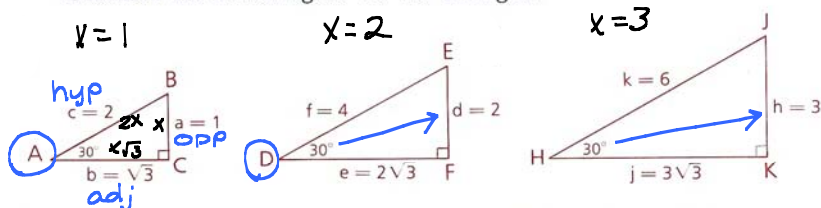
## Objective

After studying this section, you will be able to

- Understand three basic trigonometric relationships

This section presents the three basic trigonometric ratios **sine**, **co-sine**, and **tangent**. The concept of similar triangles and the Pythagorean Theorem can be used to develop the **trigonometry of right triangles**.

Consider the following 30°-60°-90° triangles.



$\sin A$   
 $\cos A$   
 $\tan A$

$SOH$   
 $CAH$   
 $TOA$

Compare the length of the leg opposite the 30° angle with the length of the hypotenuse in each triangle.

In  $\triangle ABC$ ,  $\frac{a}{c} = \frac{1}{2} = 0.5$ . In  $\triangle DEF$ ,  $\frac{d}{f} = \frac{2}{4} = 0.5$ . In  $\triangle HJK$ ,  $\frac{h}{k} = \frac{3}{6} = 0.5$ .

$\sin 30^\circ = \frac{1}{2}$

If you think about similar triangles, you will see that in every 30°-60°-90° triangle,

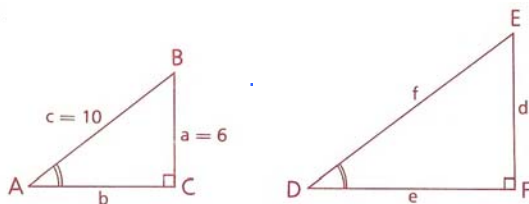
$$\frac{\text{leg opposite } 30^\circ \angle}{\text{hypotenuse}} = \frac{1}{2}$$

For each triangle shown, verify that  $\frac{\text{leg adjacent to } 30^\circ \angle}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$ .

For each triangle shown, find the ratio  $\frac{\text{leg opposite } 30^\circ \angle}{\text{leg adjacent to } 30^\circ \angle}$ .

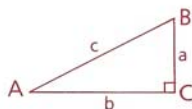
In  $\triangle ABC$  and  $\triangle DEF$ ,

$$\frac{a}{c} = \frac{d}{f} = \frac{6}{10} = \frac{3}{5}$$



Engineers and scientists have found it convenient to formalize these relationships by naming the ratios of sides. You should memorize these three basic ratios.

## Definition Three Trigonometric Ratios



**sine** of  $\angle A = \sin \angle A = \frac{\text{opposite leg}}{\text{hypotenuse}}$

**cosine** of  $\angle A = \cos \angle A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$

**tangent** of  $\angle A = \tan \angle A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

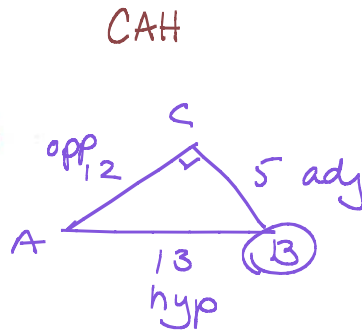
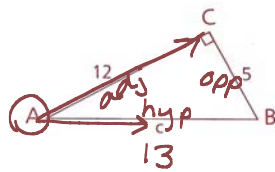
# Class Examples

**Problem 1** Find: **a**  $\cos \angle A$   
**b**  $\tan \angle B$

$$\cos \angle A = \frac{12}{13}$$

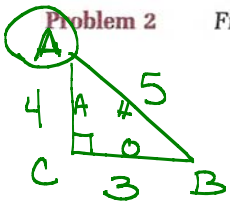
$$\tan \angle B = \frac{12}{5}$$

TOA



SOH CAH TOA

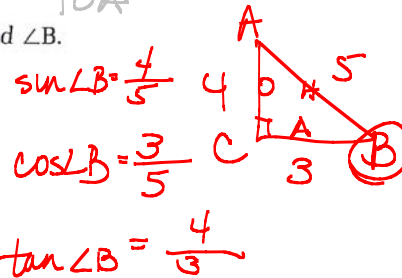
**Problem 2** Find the three trigonometric ratios for  $\angle A$  and  $\angle B$ .



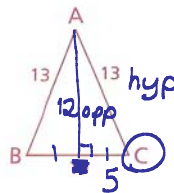
$$\sin \angle A = \frac{3}{5}$$

$$\cos \angle A = \frac{4}{5}$$

$$\tan \angle A = \frac{3}{4}$$



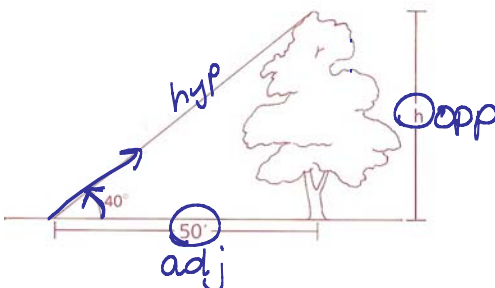
**Problem 3**  $\triangle ABC$  is an isosceles triangle as marked. Find  $\sin \angle C$ .



SOH

$$\sin \angle C = \frac{12}{13}$$

**Problem 4** Use the fact that  $\tan 40^\circ \approx 0.8391$  to find the height of the tree to the nearest foot.



TOA

$$\tan 40^\circ = \frac{h}{50}$$

$$50(0.8391) = h$$

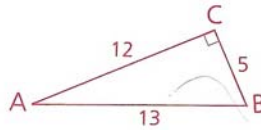
$$41.955 = h$$

$$42 \text{ ft} \approx h$$

## Homework

5 If  $\tan \angle M = \frac{3}{4}$ , find  $\cos \angle M$ . (Hint: Start by drawing the triangle.)

6 Using the figure as marked, name each missing angle.

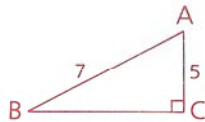


a  $\frac{5}{12} = \tan \angle \underline{\hspace{1cm}}$

b  $\frac{12}{13} = \cos \angle \underline{\hspace{1cm}}$

c  $\frac{5}{13} = \sin \angle \underline{\hspace{1cm}}$

7 Find each quantity.



a BC

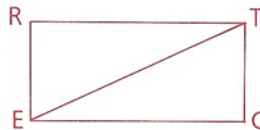
b  $\sin \angle A$

c  $\tan \angle B$

8 Given: RECT is a rectangle.  
ET = 26, RT = 24

Find: a  $\sin \angle RET$

b  $\cos \angle RET$



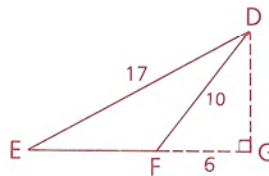
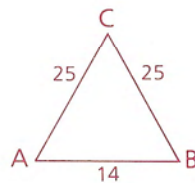
## Problem Set B

9 Using the given figures, find

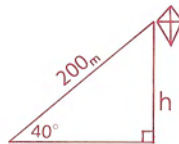
a  $\cos \angle A$

b  $\sin \angle E$

c  $\sin \angle DFG$



10 Use the fact that  $\sin 40^\circ \approx 0.6428$  to find the height of the kite to the nearest meter.

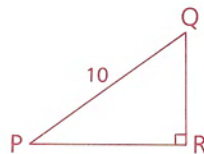


11 a If  $\tan \angle A = 1$ , find  $m\angle A$ .

b If  $\sin \angle P = 0.5$ , find  $m\angle P$ .

12 Given:  $\sin \angle P = \frac{3}{5}$ , PQ = 10

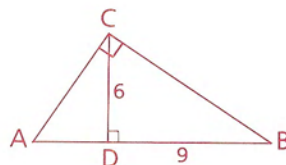
Find:  $\cos \angle P$



13 Using the figure, find

a  $\tan \angle ACD$

b  $\sin \angle A$

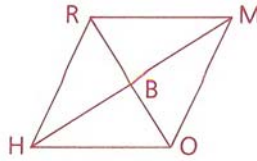


**Problem Set B, continued**

- 14 Given: RHOM is a rhombus.

$$RO = 18, HM = 24$$

Find: **a**  $\cos \angle BRM$       **b**  $\tan \angle BHO$



- 15 Given a trapezoid with sides 5, 10, 17, and 10, find the sine of one of the acute angles.

- 16 Given  $\triangle ABC$  with  $\angle C = 90^\circ$ , indicate whether each statement is true Always (A), Sometimes (S), or Never (N).

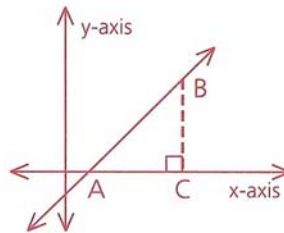
**a**  $\sin \angle A = \cos \angle B$

**b**  $\sin \angle A = \tan \angle A$

**c**  $\sin \angle A = \cos \angle$

- 17 If  $\triangle EQU$  is equilateral and  $\triangle RAT$  is a right triangle with  $RA = 2$ ,  $RT = 1$ , and  $\angle T = 90^\circ$ , show that  $\sin \angle E = \cos \angle A$ .

- 18 If the slope of  $\overleftrightarrow{AB}$  is  $\frac{5}{8}$ , find the tangent of  $\angle BAC$ .



**Problem Set C** *C → Challenge*

- 19 Use the definitions of the trigonometric ratios to verify the following relationships, given  $\triangle ABC$  in which  $\angle C = 90^\circ$ .

**a**  $(\sin \angle A)^2 + (\cos \angle A)^2 = 1$

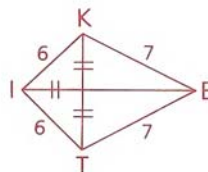
**c**  $\frac{\sin \angle A}{\cos \angle A} = \tan \angle A$

**b**  $\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$

**d**  $\sin \angle A = \cos (90^\circ - \angle A)$

- 22 Given: KITE is a kite with sides as marked.

Find:  $\tan \angle KEI$

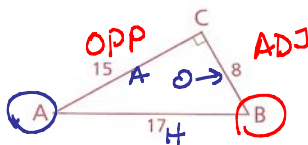


# Classwork

1 Find each ratio.

- a  $\sin \angle A$   
b  $\cos \angle A$   
c  $\tan \angle A$

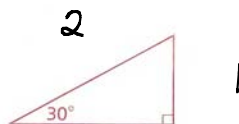
- d  $\sin \angle B$   
e  $\cos \angle B$   
f  $\tan \angle B$



2 Find each ratio.

- a  $\sin 30^\circ$   
b  $\cos 30^\circ$   
c  $\tan 30^\circ$

- d  $\sin 60^\circ$   
e  $\cos 60^\circ$   
f  $\tan 60^\circ$

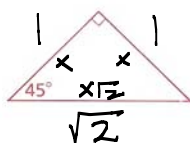


$\sqrt{3}$

$$\frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}\sqrt{3}} = \frac{1}{3}$$

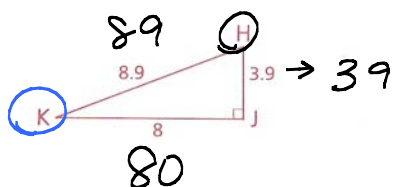
3 Find each ratio.

- a  $\sin 45^\circ$   
b  $\cos 45^\circ$   
c  $\tan 45^\circ$



4 Find each ratio.

- a  $\cos \angle H$   
b  $\tan \angle K$



<b>1a</b>	8/17
<b>1b</b>	15/17
<b>1c</b>	8/15
<b>1d</b>	15/17
<b>1e</b>	8/17
<b>1f</b>	15/8
<b>2a</b>	1/2
<b>2b</b>	$\sqrt{3}/2$
<b>2c</b>	$\sqrt{3}/3$
<b>2d</b>	$\sqrt{3}/2$
<b>2e</b>	1/2
<b>2f</b>	$\sqrt{3}$
<b>3a</b>	$\sqrt{2}/2$
<b>3b</b>	$\sqrt{2}/2$
<b>3c</b>	1
<b>4a</b>	39/89
<b>4b</b>	39/80

