Name StuDent

Adv Geo 7

9.9: Introduction to Trigonometry

Ms. Kresovic

Monday, March 24, 2014

Objective

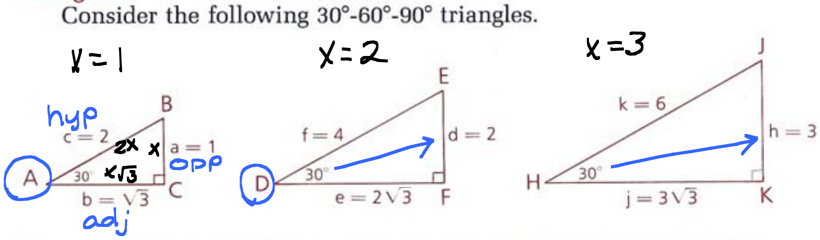
After studying this section, you will be able to

- Understand three basic trigonometric relationships

sin A SOH
cos A CAH
tan A TOA

This section presents the three basic trigonometric ratios ***sine***, ***cosine***, and ***tangent***. The concept of similar triangles and the Pythagorean Theorem can be used to develop the ***trigonometry of right triangles.***

Consider the following 30°-60°-90° triangles.



Compare the length of the leg opposite the 30° angle with the length of the hypotenuse in each triangle.

In $\triangle ABC$, $\frac{a}{c} = \frac{1}{2} = 0.5$. In $\triangle DEF$, $\frac{d}{f} = \boxed{\frac{2}{4}} = 0.5$. In $\triangle HJK$, $\frac{h}{k} = \frac{3}{6} = 0.5$.

$\sin 30^\circ = \frac{1}{2}$

If you think about similar triangles, you will see that in every 30°-60°-90° triangle,

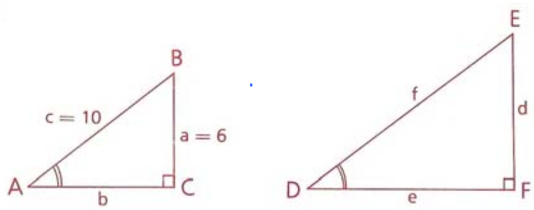
$$\frac{\text{leg opposite } 30^\circ \angle}{\text{hypotenuse}} = \frac{1}{2}$$

For each triangle shown, verify that $\frac{\text{leg adjacent to } 30^\circ \angle}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$.

For each triangle shown, find the ratio $\frac{\text{leg opposite } 30^\circ \angle}{\text{leg adjacent to } 30^\circ \angle}$.

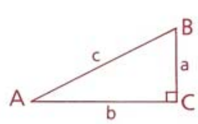
In $\triangle ABC$ and $\triangle DEF$,

$$\frac{a}{c} = \frac{d}{f} = \frac{6}{10} = \frac{3}{5}$$



Engineers and scientists have found it convenient to formalize these relationships by naming the ratios of sides. You should memorize these three basic ratios.

Definition **Three Trigonometric Ratios**



$$\textit{sine} \text{ of } \angle A = \sin \angle A = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\textit{cosine} \text{ of } \angle A = \cos \angle A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

$$\textit{tangent} \text{ of } \angle A = \tan \angle A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

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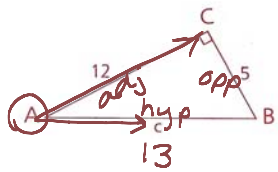
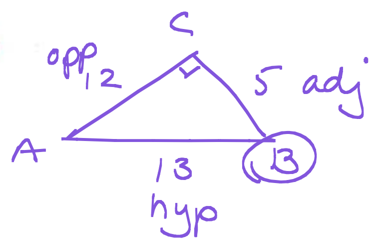
Class Examples

Problem 1 Find: **a** cos ∠A **b** tan ∠B



CAH



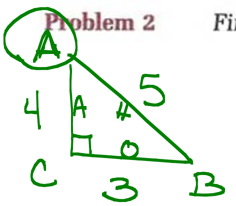
$\cos\angle A = \frac{12}{13}$

$\tan\angle B = \frac{12}{5}$

TOA

SOH CAH TOA

Problem 2 Find the three trigonometric ratios for ∠A and ∠B.



$\sin\angle A = \frac{3}{5}$

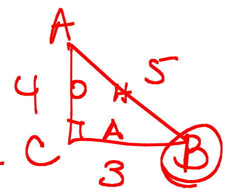
$\cos\angle A = \frac{4}{5}$

$\tan\angle A = \frac{3}{4}$

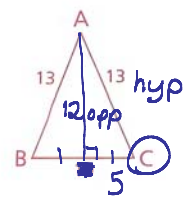
$\sin\angle B = \frac{4}{5}$

$\cos\angle B = \frac{3}{5}$

$\tan\angle B = \frac{4}{3}$



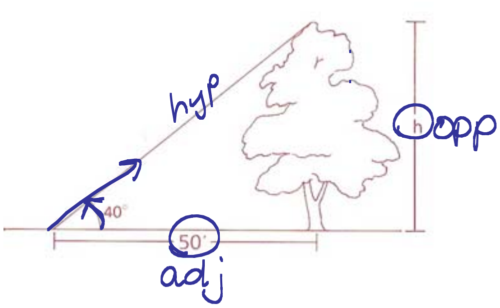
Problem 3 △ABC is an isosceles triangle as marked. Find sin ∠C.



SOH

$\sin\angle C = \frac{12}{13}$

Problem 4 Use the fact that tan 40° ≈ 0.8391 to find the height of the tree to the nearest foot.



TOA

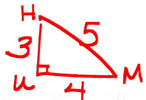
$\tan 40° = \frac{h}{50'}$

$50(.8391) = h$

$41.955 = h$

$42\text{ft} \approx h$

Homework

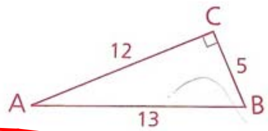


$\tan\angle = \frac{OPP}{ADJ}$ $\cos\angle M = \frac{ADJ}{HYP} = \frac{4}{5}$

SOH
CAH
TOA

5 If $\tan\angle M = \frac{3}{4}$, find $\cos\angle M$. (Hint: Start by drawing the triangle.)

6 Using the figure as marked, name each missing angle.

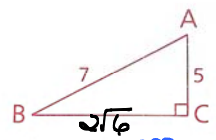


a $\frac{5}{12} = \tan\angle$ A $= \frac{OPP}{ADJ}$ **b** $\frac{12}{13} = \cos\angle$ A $\frac{ADJ}{HYP}$ **c** $\frac{5}{13} = \sin\angle$ A $= \frac{OPP}{HYP}$

7 Find each quantity.

$BC^2 = 7^2 - 5^2$

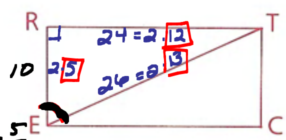
$BC = \sqrt{49-25} = 2\sqrt{6}$



a BC $= 2\sqrt{6}$ **b** $\sin\angle A = \frac{OPP}{HYP} = \frac{2\sqrt{6}}{7}$ **c** $\tan\angle B = \frac{OPP}{ADJ} = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12}$

8 Given: RECT is a rectangle.
ET = 26, RT = 24

Find: **a** $\sin\angle RET$ **b** $\cos\angle RET$



$\sin\angle RET = \frac{OP}{HY} = \frac{2\cdot 12}{2\cdot 13} = \frac{12}{13}$ $\cos\angle RET = \frac{ADJ}{HYP} = \frac{10}{26} = \frac{5}{13}$

Problem Set B

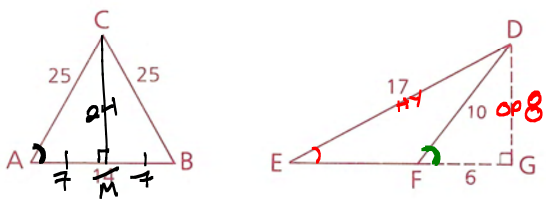
SOH CAH TOA

9 Using the given figures, find

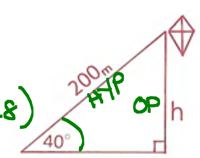
a $\cos\angle A = \frac{A}{H} = 7/25$

b $\sin\angle E = \frac{O}{H} = 8/17$

c $\sin\angle DFG = \frac{8}{10} = \frac{4}{5}$



10 Use the fact that $\sin 40° \approx 0.6428$ to find the height of the kite to the nearest meter.



$\sin 40 = \frac{h}{200} \rightarrow h = 200(.6428)$

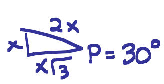
$h \approx 129$ m

Test quest

11 **a** If $\tan\angle A = 1$, find $m\angle A$. $\tan\angle = \frac{O}{A} \rightarrow$ $m\angle A = 45°$

b If $\sin\angle P = 0.5$, find $m\angle P$.

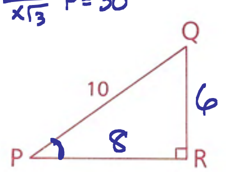
$\sin\angle P = \frac{1}{2}\cdot\frac{x}{x} = \frac{x}{2x}$



12 Given: $\sin\angle P = \frac{3}{5}$, PQ = 10

Find: $\cos\angle P$

$\cos\angle P = \frac{ADJ}{HYP} = \frac{8}{10} = \frac{4}{5}$



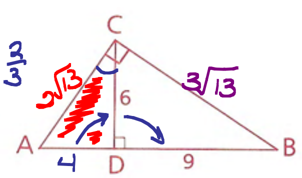
$\sin\angle P = \frac{OPP}{HYP} = \frac{3}{5} = \frac{QR}{10}$

$2(3, 4, 5)$

13 Using the figure, find

a $\tan\angle ACD$ $\frac{opp}{adj} = \frac{AD}{CD} = \frac{3}{3}$

b $\sin\angle A$



$AD^2 + CD^2 = AC^2$

$4^2 + 6^2 = AC^2$

$\sqrt{52} = AC$

$2\sqrt{13}$

$\frac{AD}{6} = \frac{6}{9} \rightarrow 36 = 9AD \rightarrow AD = 4$

$AC^2 + BC^2 = AB^2$

$BC^2 = AB^2 - AC^2$

$BC = \sqrt{AB^2 - AC^2}$

$= \sqrt{169 - 52}$

$= \sqrt{117} = 3\sqrt{13}$

then $\sin\angle A = \frac{OPP}{HYP} = \frac{3\sqrt{13}}{13}$

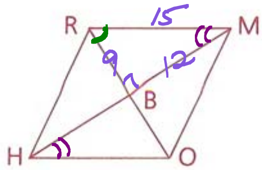
$\sin\angle A = \frac{6}{2\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{6\sqrt{13}}{26} = \frac{3\sqrt{13}}{13}$

Problem Set B, *continued*

14 Given: RHOM is a rhombus.
RO = 18, HM = 24

Find: **a** cos ∠BRM **b** tan ∠BHO

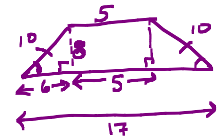
$\frac{ADJ}{HYP} = \frac{3}{5}$ $\frac{OPP}{ADJ} = \frac{9}{12} = \frac{3}{4}$

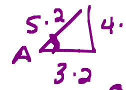


9, 12, RM
3(3, 4, 5)

15 Given a trapezoid with sides 5, 10, 17, and 10, find the sine of one of the acute angles.

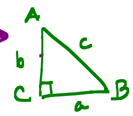
$\boxed{15}$





$\sin A = \frac{OPP}{HYP} = \frac{8}{10} = \frac{4}{5}$

16 Given △ABC with ∠C = 90°, indicate whether each statement is true Always (A), Sometimes (S), or Never (N).



a sin ∠A = cos ∠B $\frac{ADJ}{HYP}$ **b** sin ∠A = tan ∠A $\frac{OPP}{ADJ}$ **c** sin ∠A = cos ∠A $\frac{ADJ}{HYP}$

$\frac{OPP}{HYP}$ $\frac{a}{c} = \frac{a}{c}$ ALWAYS

$\frac{a}{c} = \frac{a}{b}$ → hyp ≠ side ∴ Never

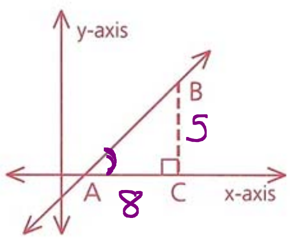
$\frac{a}{c} = \frac{b}{c}$
can a = b?
If 45, 45, 90
then yes → Sometimes

~~**17**~~ If △EQU is equilateral and △RAT is a right triangle with RA = 2, RT = 1, and ∠T = 90°, show that sin ∠E = cos ∠A.

Skip (time)

18 If the slope of $\overleftrightarrow{AB}$ is $\frac{5}{8}$, find the tangent of ∠BAC.

slope = $\frac{\Delta Y}{\Delta X}$

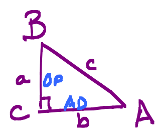


$\tan \angle A = \frac{5}{8}$

Problem Set C

C → Challenge

19 Use the definitions of the trigonometric ratios to verify the following relationships, given △ABC in which ∠C = 90°.



a $(\sin \angle A)^2 + (\cos \angle A)^2 = 1$ PYTH ID

b $\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$ law of sines

c $\frac{\sin \angle A}{\cos \angle A} = \tan \angle A$ quotient identity

d sin ∠A = cos (90° − ∠A) skip

$\boxed{a}$ $(\sin\angle A)^2 + (\cos\angle A)^2 = 1$

$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 =$

$\frac{a^2}{c^2} + \frac{b^2}{c^2}$

$\frac{a^2+b^2}{c^2}$

$\frac{c^2}{c^2}$

in any rt △ $a^2+b^2=c^2$

1 = 1

$\boxed{b}$ $\frac{a}{\left(\frac{a}{c}\right)} = \frac{b}{\left(\frac{b}{c}\right)}$

$a \cdot \frac{c}{a} = b \cdot \frac{c}{b}$

c = c

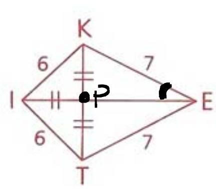
$\boxed{c}$ $\frac{\left(\frac{a}{c}\right)}{\left(\frac{b}{c}\right)}$

$\frac{a}{c} \cdot \frac{c}{b}$

$\frac{a}{b} = \tan\angle A$

22 Given: KITE is a kite with sides as marked.

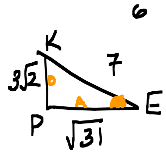
Find: tan ∠KEI = tan ∠KEP



△KIP → 45 45 90
x x $x\sqrt{2}$
6

$6 = x\sqrt{2}$

$3\sqrt{2} = \frac{6\sqrt{2}}{2} = x$ →



$KP^2 + PE^2 = KE^2$

$PE = \sqrt{KE^2 - KP^2} = \sqrt{49-18} = \sqrt{31}$

$\tan\angle KEP = \frac{3\sqrt{2}}{\sqrt{31}} \cdot \frac{\sqrt{31}}{\sqrt{31}} = \boxed{\frac{3\sqrt{62}}{31}}$

AMDG SOH CAH TOA

Name Ms. Kresovic

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Classwork

1 Find each ratio.

a sin ∠A

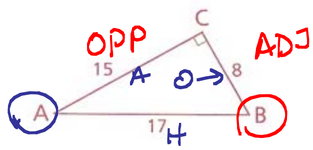
b cos ∠A

c tan ∠A

d sin ∠B

e cos ∠B

f tan ∠B



2 Find each ratio.

a sin 30°

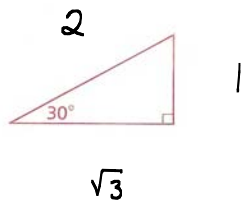
b cos 30°

c tan 30°

d sin 60°

e cos 60°

f tan 60°



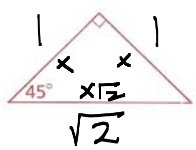
$\frac{x}{x\sqrt{3}} = \frac{1}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} =$

3 Find each ratio.

a sin 45°

b cos 45°

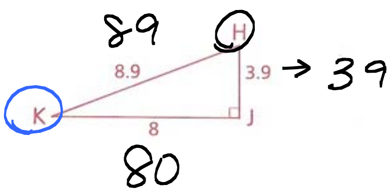
c tan 45°



4 Find each ratio.

a cos ∠H

b tan ∠K



1a	8/17
1b	15/17
1c	8/15
1d	15/17
1e	8/17
1f	15/8
2a	1/2
2b	√3/2
2c	√3/3
2d	√3/2
2e	1/2
2f	√3
3a	√2/2
3b	√2/2
3c	1
4a	39/89
4b	39/80