

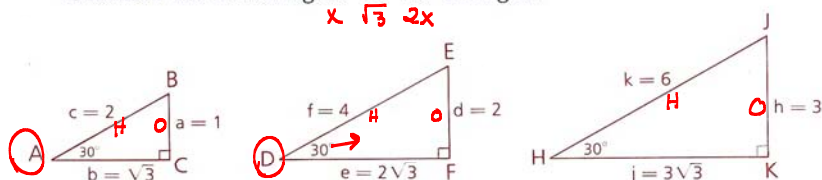
Objective

After studying this section, you will be able to

- Understand three basic trigonometric relationships

This section presents the three basic trigonometric ratios **sine**, **cosine**, and **tangent**. The concept of similar triangles and the Pythagorean Theorem can be used to develop the **trigonometry of right triangles**.

Consider the following 30°-60°-90° triangles.



Compare the length of the leg opposite the 30° angle with the length of the hypotenuse in each triangle.

In $\triangle ABC$, $\frac{a}{c} = \frac{1}{2} = 0.5$. In $\triangle DEF$, $\frac{d}{f} = \frac{2}{4} = 0.5$. In $\triangle HJK$, $\frac{h}{k} = \frac{3}{6} = 0.5$.

$\sin 30^\circ = \frac{1}{2}$

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If you think about similar triangles, you will see that in every 30°-60°-90° triangle,

$$\frac{\text{leg opposite } 30^\circ \angle}{\text{hypotenuse}} = \frac{1}{2}$$

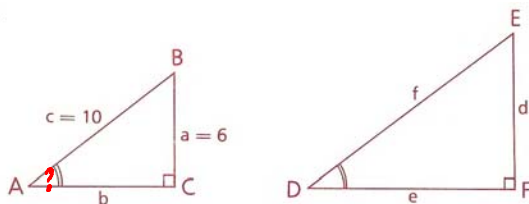
For each triangle shown, verify that $\frac{\text{leg adjacent to } 30^\circ \angle}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$.

For each triangle shown, find the ratio $\frac{\text{leg opposite } 30^\circ \angle}{\text{leg adjacent to } 30^\circ \angle}$.

In $\triangle ABC$ and $\triangle DEF$,

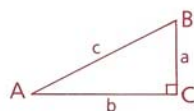
$$\frac{a}{c} = \frac{d}{f} = \frac{6}{10} = \frac{3}{5}$$

$\sin A = \frac{6}{10} = \frac{3}{5}$



Engineers and scientists have found it convenient to formalize these relationships by naming the ratios of sides. You should memorize these three basic ratios.

Definition Three Trigonometric Ratios



sine of $\angle A = \sin \angle A = \frac{\text{opposite leg}}{\text{hypotenuse}}$

$\sin L = \frac{O}{H}$

SOH

cosine of $\angle A = \cos \angle A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$

$\cos L = \frac{A}{H}$

CAH

tangent of $\angle A = \tan \angle A = \frac{\text{opposite leg}}{\text{adjacent leg}}$

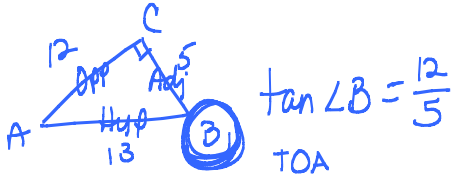
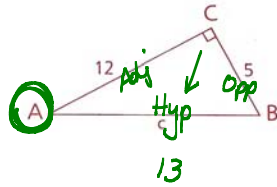
$\tan L = \frac{O}{A}$

TOA

Class Examples

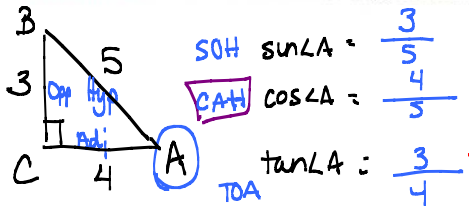
Problem 1 Find: **a** $\cos \angle A$
b $\tan \angle B$

CAH $\cos A = \frac{12}{13}$



$\tan \angle B = \frac{12}{5}$

Problem 2 Find the three trigonometric ratios for $\angle A$ and $\angle B$.



SOH $\sin A = \frac{3}{5}$

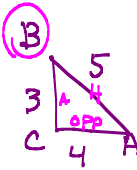
CAH $\cos A = \frac{4}{5}$

TOA $\tan A = \frac{3}{4}$

SOH $\sin B = \frac{4}{5}$

$\cos B = \frac{3}{5}$

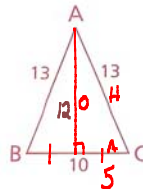
$\tan B = \frac{4}{3}$



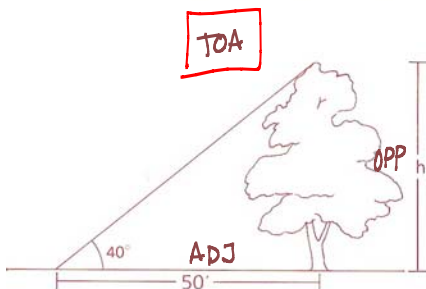
Problem 3 $\triangle ABC$ is an isosceles triangle as marked. Find $\sin \angle C$.

$\sin \angle C = \frac{12}{13}$

SOH



Problem 4 Use the fact that $\tan 40^\circ \approx 0.8391$ to find the height of the tree to the nearest foot.



$\tan 40^\circ = \frac{h}{50}$

$.8391 = \frac{h}{50}$

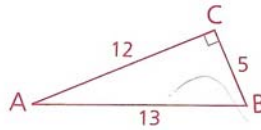
$41.955 = h$

or about 42 ft tall

Homework

5 If $\tan \angle M = \frac{3}{4}$, find $\cos \angle M$. (Hint: Start by drawing the triangle.)

6 Using the figure as marked, name each missing angle.

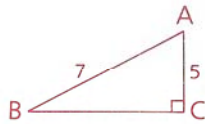


a $\frac{5}{12} = \tan \angle \underline{\hspace{1cm}}$

b $\frac{12}{13} = \cos \angle \underline{\hspace{1cm}}$

c $\frac{5}{13} = \sin \angle \underline{\hspace{1cm}}$

7 Find each quantity.



a BC

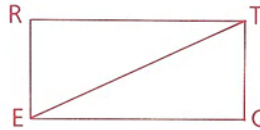
b $\sin \angle A$

c $\tan \angle B$

8 Given: RECT is a rectangle.
ET = 26, RT = 24

Find: a $\sin \angle RET$

b $\cos \angle RET$



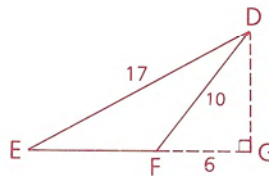
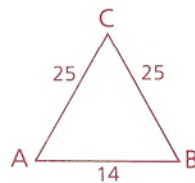
Problem Set B

9 Using the given figures, find

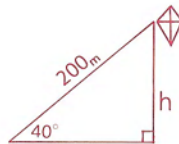
a $\cos \angle A$

b $\sin \angle E$

c $\sin \angle DFG$



10 Use the fact that $\sin 40^\circ \approx 0.6428$ to find the height of the kite to the nearest meter.

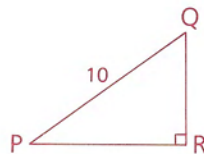


11 a If $\tan \angle A = 1$, find $m\angle A$.

b If $\sin \angle P = 0.5$, find $m\angle P$.

12 Given: $\sin \angle P = \frac{3}{5}$, PQ = 10

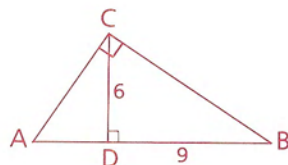
Find: $\cos \angle P$



13 Using the figure, find

a $\tan \angle ACD$

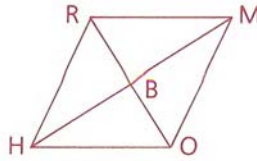
b $\sin \angle A$



Problem Set B, continued

- 14 Given: RHOM is a rhombus.
RO = 18, HM = 24

Find: **a** $\cos \angle BRM$ **b** $\tan \angle BHO$



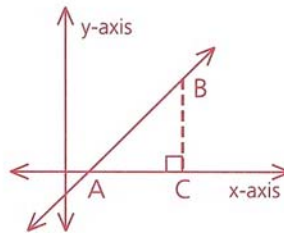
- 15 Given a trapezoid with sides 5, 10, 17, and 10, find the sine of one of the acute angles.

- 16 Given $\triangle ABC$ with $\angle C = 90^\circ$, indicate whether each statement is true Always (A), Sometimes (S), or Never (N).

a $\sin \angle A = \cos \angle B$ **b** $\sin \angle A = \tan \angle A$ **c** $\sin \angle A = \cos \angle$

- 17 If $\triangle EQU$ is equilateral and $\triangle RAT$ is a right triangle with $RA = 2$, $RT = 1$, and $\angle T = 90^\circ$, show that $\sin \angle E = \cos \angle A$.

- 18 If the slope of \overleftrightarrow{AB} is $\frac{5}{8}$, find the tangent of $\angle BAC$.



Problem Set C *Chis for Challenge* :)

- 19 Use the definitions of the trigonometric ratios to verify the following relationships, given $\triangle ABC$ in which $\angle C = 90^\circ$.

a $(\sin \angle A)^2 + (\cos \angle A)^2 = 1$

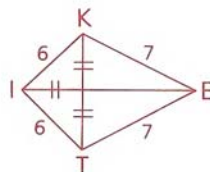
c $\frac{\sin \angle A}{\cos \angle A} = \tan \angle A$

b $\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$

d $\sin \angle A = \cos (90^\circ - \angle A)$

- 22 Given: KITE is a kite with sides as marked.

Find: $\tan \angle KEI$

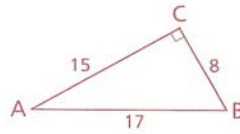


Classwork

1 Find each ratio.

- a $\sin \angle A$
b $\cos \angle A$
c $\tan \angle A$

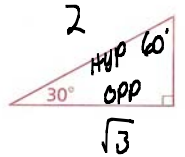
- d $\sin \angle B$
e $\cos \angle B$
f $\tan \angle B$



2 Find each ratio.

- a $\sin 30^\circ$
b $\cos 30^\circ$
c $\tan 30^\circ$

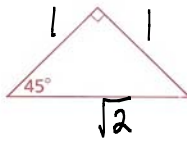
- d $\sin 60^\circ$
e $\cos 60^\circ$
f $\tan 60^\circ$



$$\sin 60 = \frac{\sqrt{3}}{2}$$

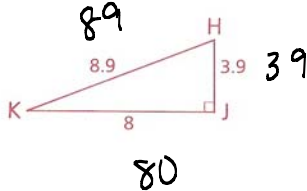
3 Find each ratio.

- a $\sin 45^\circ$
b $\cos 45^\circ$
c $\tan 45^\circ$



4 Find each ratio.

- a $\cos \angle H$
b $\tan \angle K$



| | |
|-----------|--------------|
| 1a | 8/17 |
| 1b | 15/17 |
| 1c | 8/15 |
| 1d | 15/17 |
| 1e | 8/17 |
| 1f | 15/8 |
| 2a | 1/2 |
| 2b | $\sqrt{3}/2$ |
| 2c | $\sqrt{3}/3$ |
| 2d | $\sqrt{3}/2$ |
| 2e | 1/2 |
| 2f | $\sqrt{3}$ |
| 3a | $\sqrt{2}/2$ |
| 3b | $\sqrt{2}/2$ |
| 3c | 1 |
| 4a | 39/89 |
| 4b | 39/80 |