Objective

After studying this section, you will be able to

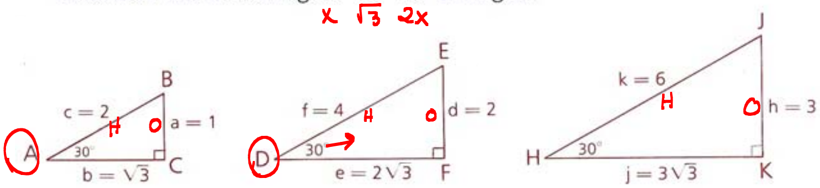
- Understand three basic trigonometric relationships

sin cos

This section presents the three basic trigonometric ratios ***sine***, ***cosine***, and ***tangent*** (tan). The concept of similar triangles and the Pythagorean Theorem can be used to develop the ***trigonometry of right triangles.***

Consider the following 30°-60°-90° triangles.

x √3 2x



Compare the length of the leg opposite the 30° angle with the length of the hypotenuse in each triangle.

In △ABC, $\frac{a}{c} = \frac{1}{2} = 0.5$. In △DEF, $\frac{d}{f} = \frac{2}{4} = 0.5$. In △HJK, $\frac{h}{k} = \frac{3}{6} = 0.5$.

$\sin 30° = \frac{1}{2}$ $\sin 30° = \frac{1}{2}$ $\sin 30° = \frac{1}{2}$

If you think about similar triangles, you will see that in every 30°-60°-90° triangle,

$$\frac{\text{leg opposite } 30° \angle}{\text{hypotenuse}} = \frac{1}{2}$$

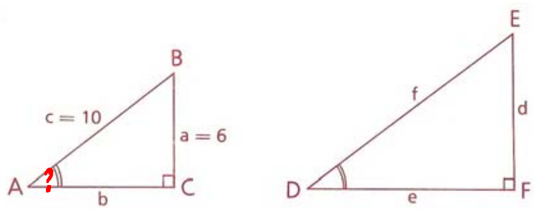
For each triangle shown, verify that $\frac{\text{leg adjacent to } 30° \angle}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$.

For each triangle shown, find the ratio $\frac{\text{leg opposite } 30° \angle}{\text{leg adjacent to } 30° \angle}$.

In △ABC and △DEF,

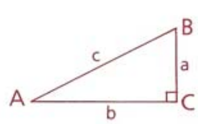
$\frac{a}{c} = \frac{d}{f} = \frac{6}{10} = \frac{3}{5}$

$\sin A = \frac{6}{10} = \frac{3}{5}$



Engineers and scientists have found it convenient to formalize these relationships by naming the ratios of sides. You should memorize these three basic ratios.

Definition **Three Trigonometric Ratios**



sine of $\angle A = \sin \angle A = \frac{\text{opposite leg}}{\text{hypotenuse}}$ $\sin \angle = \frac{O}{H}$ SOH

cosine of $\angle A = \cos \angle A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$ $\cos \angle = \frac{A}{H}$ CAH

tangent of $\angle A = \tan \angle A = \frac{\text{opposite leg}}{\text{adjacent leg}}$ $\tan \angle = \frac{O}{A}$ TOA

AMDG

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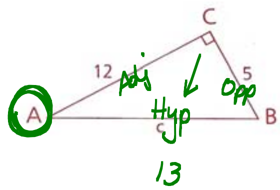
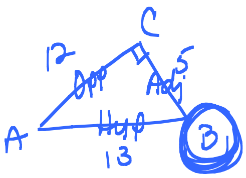
Class Examples

Problem 1 Find: **a** cos ∠A
b tan ∠B



CAH

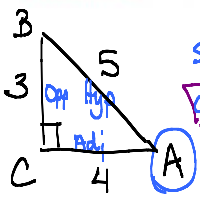
$\cos\angle A = \frac{12}{13}$



$\tan\angle B = \frac{12}{5}$

TOA

Problem 2 Find the three trigonometric ratios for ∠A and ∠B.



SOH $\sin\angle A = \frac{3}{5}$

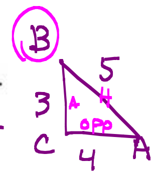
$\boxed{\text{CAH}}$ $\cos\angle A = \frac{4}{5}$

TOA $\tan\angle A = \frac{3}{4}$

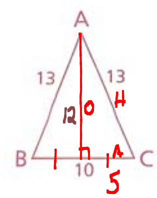
SOH $\sin\angle B = \frac{4}{5}$

$\cos\angle B = \frac{3}{5}$

$\tan\angle B = \frac{4}{3}$



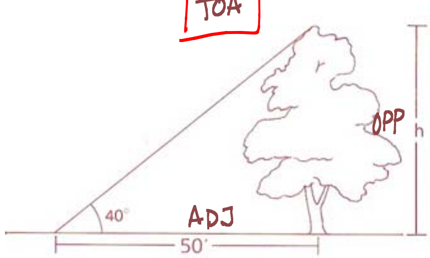
Problem 3 △ABC is an isosceles triangle as marked. Find sin ∠C.



$\sin\angle C = \frac{12}{13}$ SOH

Problem 4 *Use the fact that tan 40° ≈ 0.8391 to find the height of the tree to the nearest foot.*

$\boxed{\text{TOA}}$



$\tan 40° = \frac{h}{50}$

$.8391 = \frac{h}{50}$

$41.955 = h$

or about 42 ft tall

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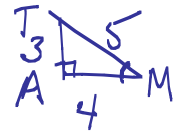
Homework

SOH CAH TOA

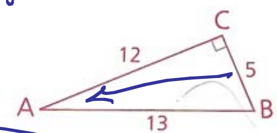
5 If $\tan \angle M = \frac{3}{4}$ (OPP / ADJ), find $\cos \angle M$. (Hint: Start by drawing the triangle.)

$\cos\angle M = \frac{ADJ}{HYP} = 4/5$

TAN $\angle = \frac{OPP}{ADJ}$

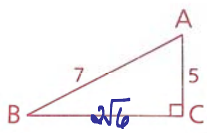


6 Using the figure as marked, name each missing angle.



a $\frac{5}{12} = \tan \angle$ A OPP=5, ADJ=12 **b** $\frac{12}{13} = \cos \angle$ A ADJ / HYP **c** $\frac{5}{13} = \sin \angle$ A OPP / HYP

7 Find each quantity.



$5^2 + BC^2 = 7^2$

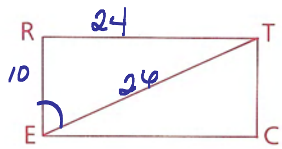
$BC = \sqrt{49-25} = 2\sqrt{6}$

a BC $= 2\sqrt{6}$ **b** $\sin \angle A = \frac{OPP}{HYP} = \frac{2\sqrt{6}}{7}$ **c** $\tan \angle B = \frac{OPP}{ADJ} = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12}$

8 Given: RECT is a rectangle.
ET = 26, RT = 24

Find: **a** $\sin \angle RET$ **b** $\cos \angle RET$

$\frac{OPP}{HYP} = \frac{24}{26} = \frac{12}{13}$ $\frac{ADJ}{HYP} = \frac{10}{26} = \frac{5}{13}$



(ER, 24, 26)
2(5, 12, 13)

Problem Set B

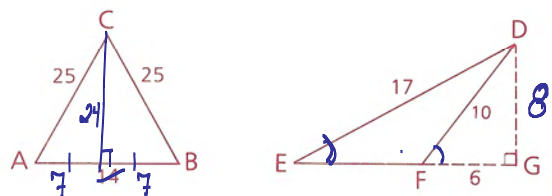
9 Using the given figures, find

a $\cos \angle A = \frac{ADJ}{HYP} = 7/25$

b $\sin \angle E = OPP/HYP = 8/17$

c $\sin \angle DFG$

$\frac{OPP}{HYP} = \frac{4}{5}$

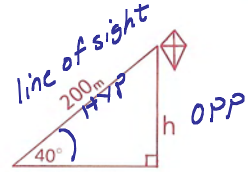


8 2(3, 4, 5)

* test **10** Use the fact that $\boxed{\sin 40° \approx 0.6428}$ to find the height of the kite to the nearest meter.

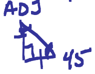
$\sin 40° = \frac{h}{200}$

$200(.6428) = h,\ h \approx 129$ km



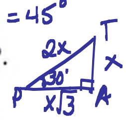
* test **11** **a** If $\tan \angle A = 1$, find $m\angle A$. $= 45°$

11a $\tan\angle = \frac{OPP}{ADJ} = 1$



b If $\sin \angle P = 0.5$, find $m\angle P$.

$\sin\angle P = \frac{1}{2} \rightarrow \frac{x}{2x}$, $\sin\angle = \frac{OPP}{HYP}$

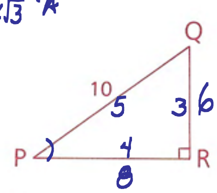


$m\angle P = 30°$

12 Given: $\sin \angle P = \frac{3}{5}$, PQ = 10

Find: $\cos \angle P = \frac{ADJ}{HYP} = \frac{8}{10} = \frac{4}{5}$

$\sin\angle P = \frac{3}{5} = \frac{OPP}{HYP} = \frac{?}{10} \rightarrow \frac{3}{5} = \frac{?}{10}$

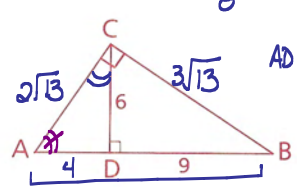


SOH CAH TOA

13 Using the figure, find

a $\tan \angle ACD$

b $\sin \angle A$



AD: $\frac{AD}{6} = \frac{6}{9} \rightarrow AD = 4$

AC: $4^2 + 6^2 = AC^2$
$\sqrt{52} = AC$
$2\sqrt{13} = AC$

$\boxed{a}$ $\tan\angle ACD = \frac{OPP}{ADJ} = \frac{4}{6} = \frac{2}{3}$

$\boxed{b}$ $\sin\angle A = \frac{OPP}{HYP} = \frac{6}{2\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{6\sqrt{13}}{26} = \boxed{\frac{3\sqrt{13}}{13}}$ 13

$BC = \sqrt{6^2 + 9^2}$

$BC = \sqrt{117} = 3\sqrt{13}$

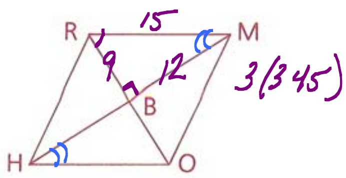
SOH CAH TOA

Problem Set B, *continued*

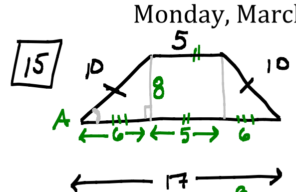
14 Given: RHOM is a rhombus.
RO = 18, HM = 24

Find: **a** cos ∠BRM **b** tan ∠BHO

$\frac{ADJ}{HYP} = \frac{9}{15} = \frac{3}{5}$ $\frac{OPP}{ADJ} = \frac{9}{12} = \frac{3}{4}$



15 Given a trapezoid with sides 5, 10, 17, and 10, find the sine of one of the acute angles.

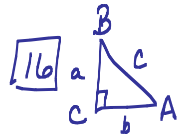


2(3,4,5)

$\sin A = \frac{OPP}{HYP} = \frac{8}{10} = \frac{4}{5}$

16 Given △ABC with ∠C = 90°, indicate whether each statement is true Always (A), Sometimes (S), or Never (N).

a sin ∠A = cos ∠B **b** sin ∠A = tan ∠A **c** sin ∠A = cos ∠A



$\sin\angle A = \cos\angle B$

$\frac{a}{c} = \frac{a}{c}$

ALWAYS

(b) $\sin\angle A = \tan\angle A$

$\frac{a}{c} = \frac{a}{b}$

hyp = leg RT△?

Never

(c) $\sin A = \cos A$

$\frac{a}{c} = \frac{b}{c}$

$leg_1 = leg_2$ in rt△?

Sometimes

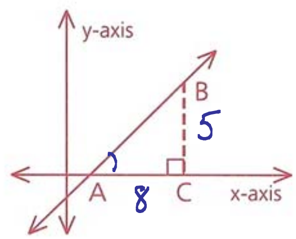
17 If △EQU is equilateral and △RAT is a right triangle with RA = 2, RT = 1, and ∠T = 90°, show that sin ∠E = cos ∠A.

Skip

18 If the slope of $\overleftrightarrow{AB}$ is $\frac{5}{8}$, find the tangent of ∠BAC.

slope: $\frac{rise}{run} = \frac{\Delta Y}{\Delta X} = \frac{5}{8}$

$\tan\angle BAC = 5/8$



Problem Set C

C is for Challenge :)

19 Use the definitions of the trigonometric ratios to verify the following relationships, given △ABC in which ∠C = 90°.

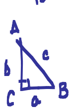
Identities

a $(\sin\angle A)^2 + (\cos\angle A)^2 = 1$ Pyth. Identity.

c $\frac{\sin\angle A}{\cos\angle A} = \tan\angle A$ Quotient Identity

b $\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B}$ Law of Sines

d $\sin\angle A = \cos(90° - \angle A)$



$(\sin\angle A)^2 + (\cos A)^2 = 1$

$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$

$\frac{a^2}{c^2} + \frac{b^2}{c^2}$

PYTH THM
$leg^2 + leg^2 = hyp^2$
$a^2 + b^2 = c^2$

$\frac{a^2+b^2}{c^2}$

$\frac{c^2}{c^2}$

$1 = 1$

$\boxed{b}$ $\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B}$

$\frac{a}{\frac{a}{c}} = \frac{b}{\frac{b}{c}}$

$\cancel{a} \cdot \frac{c}{\cancel{a}} = \cancel{b} \cdot \frac{c}{\cancel{b}}$

$c = c$

$\boxed{c}$ $\frac{\sin\angle A}{\cos\angle A} = \tan A$

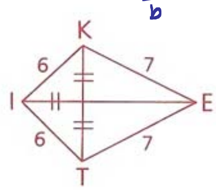
$\frac{\frac{a}{c}}{\frac{b}{c}}$

$\frac{a}{\cancel{c}} \cdot \frac{\cancel{c}}{b}$

$\frac{a}{b} = \tan A$

22 Given: KITE is a kite with sides as marked.

Find: tan ∠KEI



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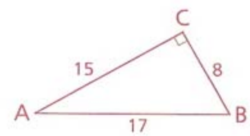
Classwork

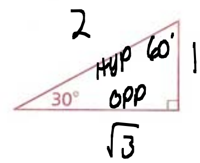
1 Find each ratio.

a sin ∠A
b cos ∠A
c tan ∠A
d sin ∠B
e cos ∠B
f tan ∠B



2 Find each ratio.

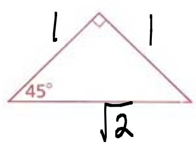
a sin 30°
b cos 30°
c tan 30°
d sin 60°
e cos 60°
f tan 60°



$\sin 60 = \frac{\cancel{x}\sqrt{3}}{2\cancel{x}} = \frac{\sqrt{3}}{2}$

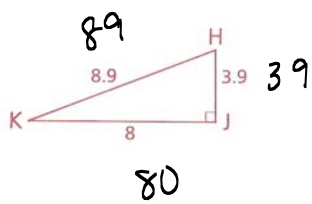
3 Find each ratio.

a sin 45°
b cos 45°
c tan 45°



4 Find each ratio.

a cos ∠H
b tan ∠K



1a	8/17
1b	15/17
1c	8/15
1d	15/17
1e	8/17
1f	15/8
2a	1/2
2b	$\sqrt{3}/2$
2c	$\sqrt{3}/3$
2d	$\sqrt{3}/2$
2e	1/2
2f	$\sqrt{3}$
3a	$\sqrt{2}/2$
3b	$\sqrt{2}/2$
3c	1
4a	39/89
4b	39/80