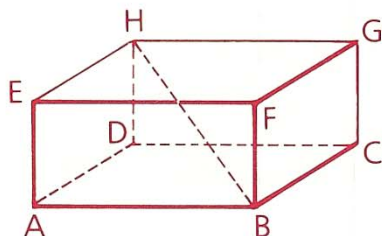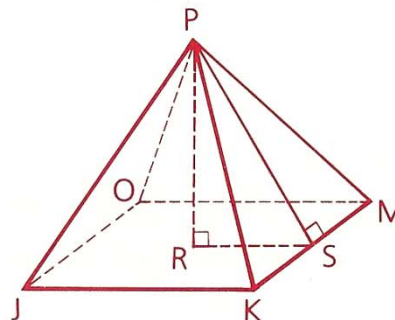## Objective

After studying this section, you will be able to
▪ Apply the Pythagorean Theorem to solid figures

Part One: Introduction

Rectangular Solid

Regular Square Pyramid

Many of the problems in this section will involve the two figures shown above.

In the rectangular solid:

ABFE is one of the 6 rectangular *faces*

$\overline{AB}$ is one of the 12 *edges*

$\overline{HB}$ is one of the 4 *diagonals* of the solid. (The others are $\overline{AG}$, $\overline{CE}$, and $\overline{DF}$.)

In the regular square pyramid:

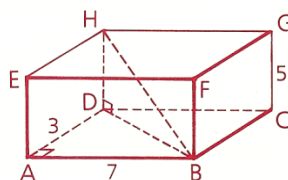JKMO is a square, and it is called the *base*

P is the *vertex*

$\overline{PR}$ is the *altitude* of the pyramid and is perpendicular to the base at its center.

$\overline{PS}$ is called a *slant height* and is perpendicular to a side of the base.

Note   A *cube* is a rectangular solid in which all edges are congruent.

Class Examples

Problem 1 *The dimensions of a rectangular solid are 3, 5, and 7. Find the diagonal.*

Solution It does not matter which edges are given the lengths 3, 5, and 7. Let $AD = 3$, $AB = 7$, and $HD = 5$, and use the Pythagorean Theorem twice.

In $\triangle ABD$,

$$3^2 + 7^2 = (DB)^2$$
$$9 + 49 = (DB)^2$$
$$\sqrt{58} = DB$$

In $\triangle HDB$,

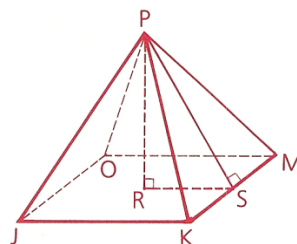$$5^2 + \left(\sqrt{58}\right)^2 = (HB)^2$$
$$25 + 58 = (HB)^2$$
$$\sqrt{83} = HB$$

The measure of the diagonal is $\sqrt{83}$.

Problem 2 *Given: The regular square pyramid shown, with altitude $\overline{PR}$ and slant height $\overline{PS}$, perimeter of JKMO = 40, PK = 13*

Find: **a** JK **b** PS **c** PR

Solution **a** $JK = \frac{1}{4}(40) = 10$

b The slant height of the pyramid is the $\perp$ bis. of $\overline{MK}$, so PSK is a right $\triangle$.

$$(SK)^2 + (PS)^2 = (PK)^2$$
$$5^2 + (PS)^2 = 13^2$$
$$PS = 12$$

c The altitude of a regular pyramid is perpendicular to the base at its center. Thus, $RS = \frac{1}{2}(JK) = 5$, and PRS is a right $\triangle$.

$$(RS)^2 + (PR)^2 = (PS)^2$$
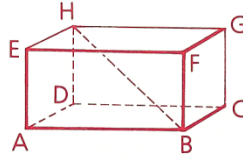$$5^2 + (PR)^2 = 12^2$$
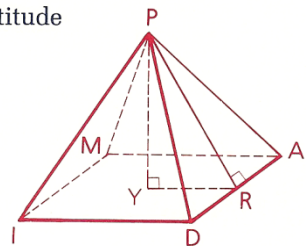$$25 + (PR)^2 = 144$$
$$PR = \sqrt{119}$$

4 Given: The rectangular solid shown,
GC = 8, HG = 12, BC = 9

Find: **a** HB, a diagonal of the solid

      b AG, another diagonal of the solid

15 Find the diagonal of a cube if each edge is 2.

5 Given: The regular square pyramid shown, with altitude $\overline{PY}$ and slant height $\overline{PR}$, ID = 14, PY = 24

Find: **a** AD

b YR

c PR

d The perimeter of base AMID

e A diagonal of the base (not shown in the diagram)
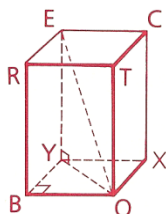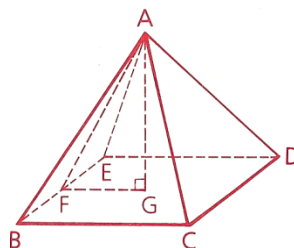
Homework

1 Given: The rectangular solid shown,
     BY = 3, OB = 4, EY = 12

Find: **a** YO, a diagonal of face BOXY

     b EO, a diagonal of the solid

3 Given: Regular square pyramid ABCDE,
     with slant height $\overline{AF}$, altitude $\overline{AG}$,
     and base BCDE;
     perimeter of BCDE = 40,
   ∠AFG = 60°

Find: The altitude and the slant height

11 Given: ⊙P as shown

Find: **a** The coordinates of point E

     b The area of sector EPG to the
        nearest tenth

     c The length of $\overset{\frown}{GE}$ to the nearest
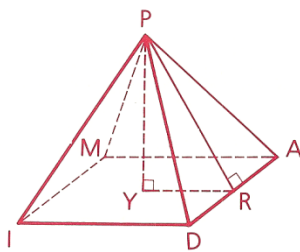   tenth
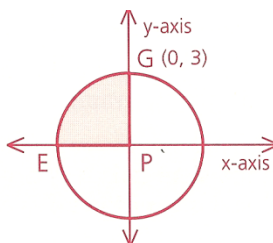
13 ABCDEFGH is a rectangular solid.

a If face diagonal $\overline{CH}$ measures 17, edge $\overline{GH}$ measures 8, and edge $\overline{FG}$ measures 6, how long is diagonal $\overline{AG}$?

b If diagonal $\overline{AG}$ measures 50, edge $\overline{AE}$ measures 40, and edge $\overline{EF}$ measures 3, how long is edge $\overline{FG}$?
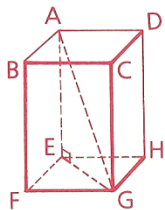
14 PADIM is a regular square pyramid. Slant height $\overline{PR}$ measures 10, and the base diagonals measure $12\sqrt{2}$.

a Find ID.

b Find the altitude of the pyramid.

c Find RD.

d Find PD (length of a lateral edge).

Class Work

2 Find the diagonal of a rectangular solid whose dimensions are 3, 4, and 5.

6 Find the slant height of a regular square pyramid if the altitude is 12 and one of the sides of the square base is 10.

12 Given: Diagram as marked
Find: AB (the length of $\overline{AB}$)