NAME Stu Dent
Adv Geo – 1

AMDG

9.8: The Pythagorean Theorem and Space Figures

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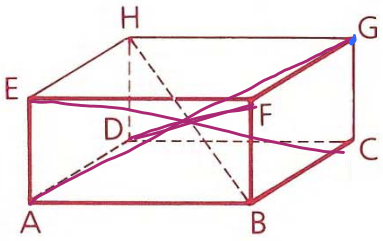
W 19 Mar 14

Objective

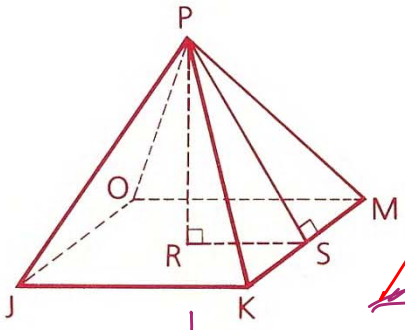
After studying this section, you will be able to

- Apply the Pythagorean Theorem to solid figures

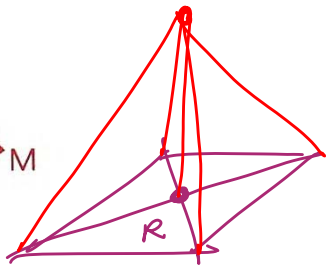
Part One: Introduction



Rectangular Solid



Regular Square Pyramid



Many of the problems in this section will involve the two figures shown above.

In the rectangular solid:

ABFE is one of the 6 rectangular ***faces*** BFGC

$\overline{AB}$ is one of the 12 ***edges*** $\overline{EF}$

$\overline{HB}$ is one of the 4 ***diagonals*** of the solid. (The others are $\overline{AG}$, $\overline{CE}$, and $\overline{DF}$.)

inside

diag of face: AC

In the regular square pyramid:

JKMO is a square, and it is called the ***base***

P is the ***vertex*** top of mtn

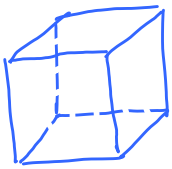
$\overline{PR}$ is the ***altitude*** of the pyramid and is perpendicular to the base at its center.

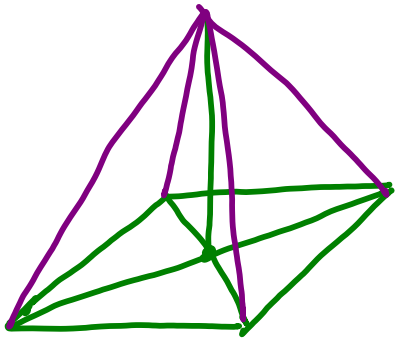
fall straight down

$\overline{PS}$ is called a ***slant height*** and is perpendicular to a side of the base.

slide down the side

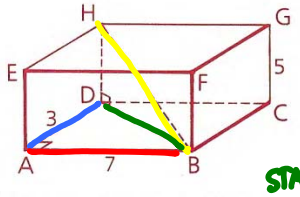
Note A ***cube*** is a rectangular solid in which all edges are congruent.

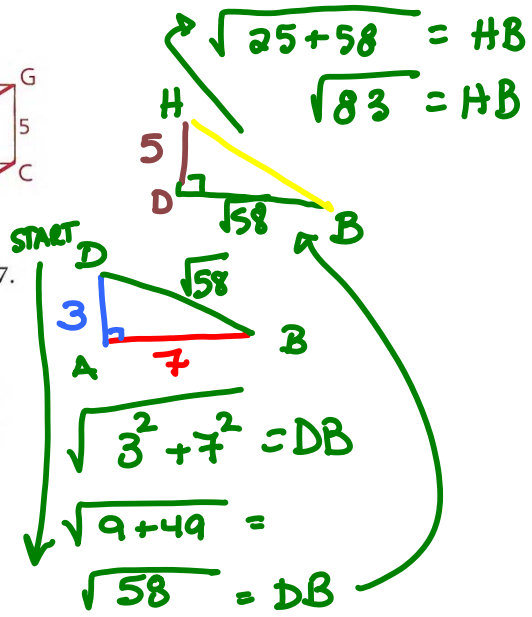




Class Examples

Problem 1 *The dimensions of a rectangular solid are 3, 5, and 7. Find the diagonal.*





Solution It does not matter which edges are given the lengths 3, 5, and 7. Let AD = 3, AB = 7, and HD = 5, and use the Pythagorean Theorem twice.

In △ABD,

$3^2 + 7^2 = (DB)^2$

$9 + 49 = (DB)^2$

$\sqrt{58} = DB$

In △HDB,

$5^2 + (\sqrt{58})^2 = (HB)^2$

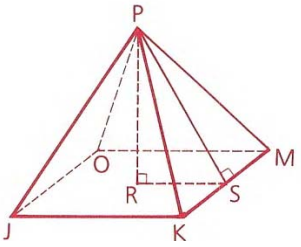
$25 + 58 = (HB)^2$

$\sqrt{83} = HB$

The measure of the diagonal is $\sqrt{83}$.

Problem 2 *Given: The regular square pyramid shown, with altitude* $\overline{PR}$ *and slant height* $\overline{PS}$*, perimeter of* JKMO = 40, PK = 13

Find: **a** JK **b** PS **c** PR



Solution

a $JK = \frac{1}{4}(40) = 10$

b The slant height of the pyramid is the ⊥ bis. of $\overline{MK}$, so PSK is a right △.

$(SK)^2 + (PS)^2 = (PK)^2$

$5^2 + (PS)^2 = 13^2$

$PS = 12$

c The altitude of a regular pyramid is perpendicular to the base at its center. Thus, $RS = \frac{1}{2}(JK) = 5$, and PRS is a right △.

$(RS)^2 + (PR)^2 = (PS)^2$

$5^2 + (PR)^2 = 12^2$

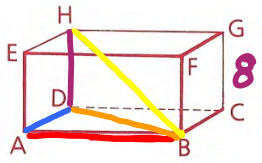
$25 + (PR)^2 = 144$

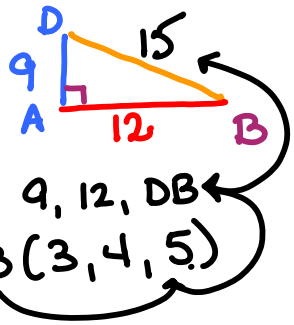
$PR = \sqrt{119}$

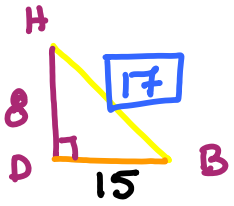
4 Given: The rectangular solid shown,
GC = 8, HG = 12, BC = 9

Find: **a** HB, a diagonal of the solid

b AG, another diagonal of the solid



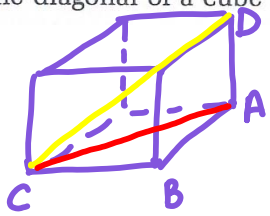




SPECIALS

1. 3, 4, 5
2. 5, 12, 13
3. 7, 24, 25
4. 8, 15, 17
5. 30, 60, 90
   $x, x\sqrt{3}, 2x$
6. 45, 45, 90
   $x, x, x\sqrt{2}$

15 Find the diagonal of a cube if each edge is 2.



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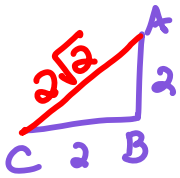
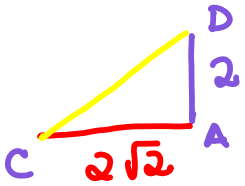


$x, x, x\sqrt{2}$



$DC = \sqrt{2^2 + (2\sqrt{2})^2}$

$= \sqrt{4 + 4 \cdot 2}$

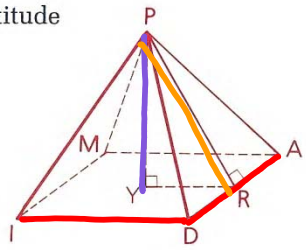
$= \sqrt{4 + 8}$

$= \sqrt{12}$

$= \sqrt{4} \cdot \sqrt{3}$

$DC = 2\sqrt{3}$

5 Given: The regular square pyramid shown, with altitude $\overline{PY}$ and slant height $\overline{PR}$,
ID = 14, PY = 24

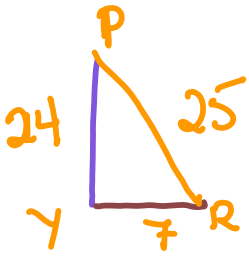


Find: **a** AD

14

b YR 7

c PR



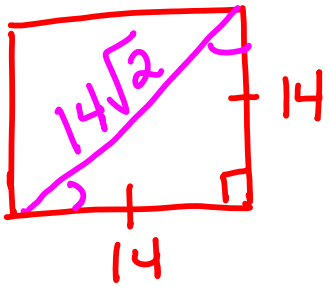
d The perimeter of base AMID

$4 \cong sds$

$4(14) = P$

$4(10+4) = 40+16 = 56$

e A diagonal of the base (not shown in the diagram)



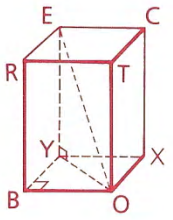
$x, x, x\sqrt{2}$

Homework

1 Given: The rectangular solid shown, BY = 3, OB = 4, EY = 12

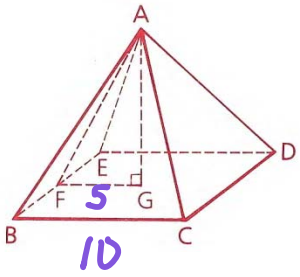
Find: **a** YO, a diagonal of face BOXY

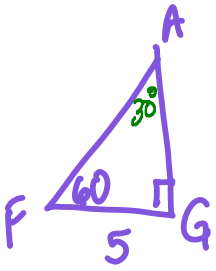
b EO, a diagonal of the solid



3 Given: Regular square pyramid ABCDE, with slant height $\overline{AF}$, altitude $\overline{AG}$, and base BCDE;
perimeter of BCDE = 40,
$\angle AFG = 60°$

Find: The altitude and the slant height





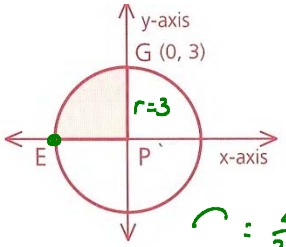
30	60	90
x	$x\sqrt{3}$	2x
5	$5\sqrt{3}$	10
FG	AG	AF
	Alt	Slant

11 Given: ⊙P as shown

Find: **a** The coordinates of point E $(-3, 0)$

b The area of sector EPG to the nearest tenth

c The length of $\overset{\frown}{GE}$ to the nearest tenth



$\frown : \frac{\angle}{360} C_o$

b Sector $= \frac{\angle}{360} A_o$

$A_{SECTOR} = \frac{90}{360} \pi 3^2$

$\frac{1}{4} 9\pi = \frac{9}{4}\pi$

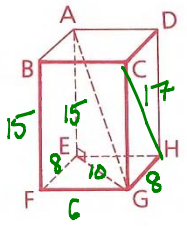
c $\overset{\frown}{GE} = \frac{1}{4} \pi 6$

$= \frac{3}{2}\pi$

13 ABCDEFGH is a rectangular solid.

a If face diagonal $\overline{CH}$ measures 17, edge $\overline{GH}$ measures 8, and edge $\overline{FG}$ measures 6, how long is diagonal $\overline{AG}$?

b If diagonal $\overline{AG}$ measures 50, edge $\overline{AE}$ measures 40, and edge $\overline{EF}$ measures 3, how long is edge $\overline{FG}$?

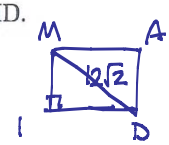


$\triangle EFG = 2(3\ 4\ 5) \therefore EG = 10$

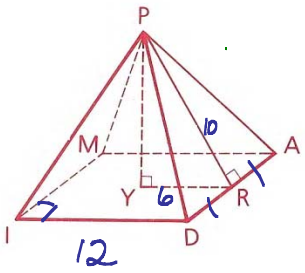
$\triangle AEG = (10, 15, \_)$ reduced $\triangle$ $5(2, 3, \_) \rightarrow \sqrt{2^2+3^2} = \sqrt{13} \therefore AG = 5\sqrt{13}$

14 PADIM is a regular square pyramid. Slant height $\overline{PR}$ measures 10, and the base diagonals measure $12\sqrt{2}$.

a Find ID.



45 45 90
x x x√2
$12\sqrt{2}$



$\triangle PYR$
$(6\ 8\ 10)$
$2(3\ \_\ 5)$

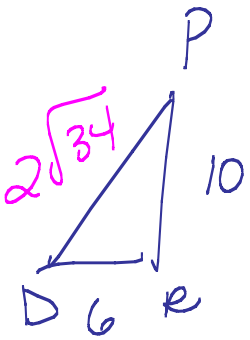
b Find the altitude of the pyramid.

8

c Find RD.

6

d Find PD (length of a lateral edge).



6 10 PD

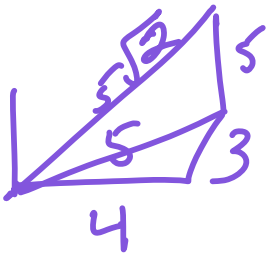
$2(3\ 5\ \_)$

$\sqrt{3^2+5^2}$

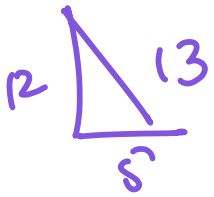
$\sqrt{34}$

Class Work

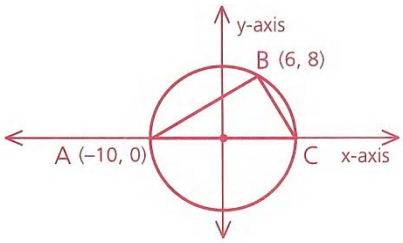
2 Find the diagonal of a rectangular solid whose dimensions are 3, 4, and 5.



6 Find the slant height of a regular square pyramid if the altitude is 12 and one of the sides of the square base is 10.



12 Given: Diagram as marked
Find: AB (the length of $\overline{AB}$)



$$AB = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\sqrt{(6 - -10)^2 + (8 - 0)^2}$$

$$\sqrt{16^2 + 8^2}$$

$$8(1, 2, \sqrt{5})$$

$$8\sqrt{5}$$