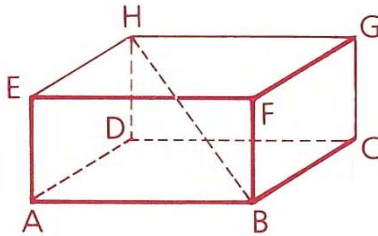


Objective

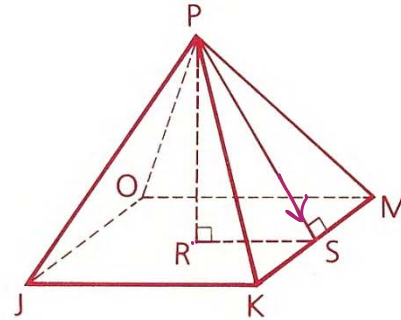
After studying this section, you will be able to

- Apply the Pythagorean Theorem to solid figures

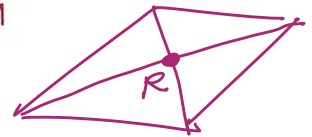
Part One: Introduction



Rectangular Solid



Pract
Regular Square Pyramid



Many of the problems in this section will involve the two figures shown above.

In the rectangular solid:

ABFE is one of the 6 rectangular

faces flat side

\overline{AB} is one of the 12 *edges lines*

\overline{HB} is one of the 4 *diagonals* of the solid. (The others are \overline{AG} , \overline{CE} , and \overline{DF} .)

In the regular square pyramid:

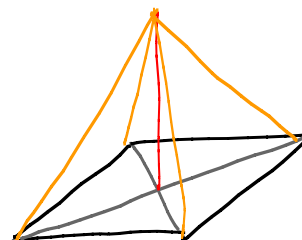
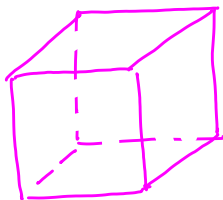
JKMO is a square, and it is called the *base*

P is the *vertex*

\overline{PR} is the *altitude* of the pyramid and is perpendicular to the base at its center. *fall straight*

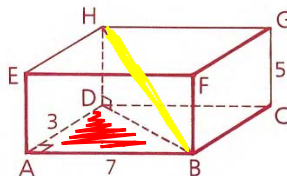
\overline{PS} is called a *slant height* and is perpendicular to a side of the base. *slide*

Note A *cube* is a rectangular solid in which all edges are congruent.



Class Examples

Problem 1 The dimensions of a rectangular solid are 3, 5, and 7. Find the diagonal.



Solution

It does not matter which edges are given the lengths 3, 5, and 7. Let $AD = 3$, $AB = 7$, and $HD = 5$, and use the Pythagorean Theorem twice.

In $\triangle ABD$,

$$3^2 + 7^2 = (DB)^2$$

$$9 + 49 = (DB)^2$$

$$\sqrt{58} = DB$$

In $\triangle HDB$,

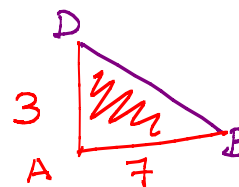
$$5^2 + (\sqrt{58})^2 = (HB)^2$$

$$25 + 58 = (HB)^2$$

$$\sqrt{83} = HB$$

The measure of the diagonal is $\sqrt{83}$.

$$\sqrt{83} = \sqrt{58^2 + 5^2}$$

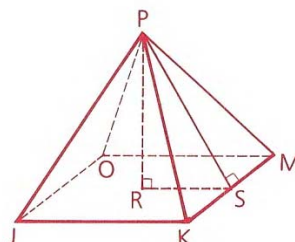


$$\sqrt{3^2 + 7^2} = DB$$

$$\sqrt{58} = DB$$

Problem 2 Given: The regular square pyramid shown, with altitude \overline{PR} and slant height \overline{PS} , perimeter of $JKMO = 40$, $PK = 13$

Find: **a** JK **b** PS **c** PR



Solution

a $JK = \frac{1}{4}(40) = 10$

b The slant height of the pyramid is the \perp bis. of \overline{MK} , so $\triangle PSK$ is a right \triangle .

$$(SK)^2 + (PS)^2 = (PK)^2$$

$$5^2 + (PS)^2 = 13^2$$

$$PS = 12$$

c The altitude of a regular pyramid is perpendicular to the base at its center. Thus, $RS = \frac{1}{2}(JK) = 5$, and $\triangle PRS$ is a right \triangle .

$$(RS)^2 + (PR)^2 = (PS)^2$$

$$5^2 + (PR)^2 = 12^2$$

$$25 + (PR)^2 = 144$$

$$PR = \sqrt{119}$$

NAME

Adv Geo -

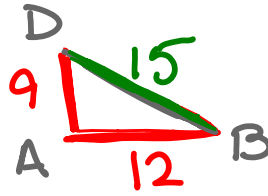
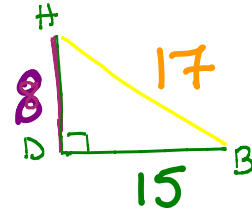
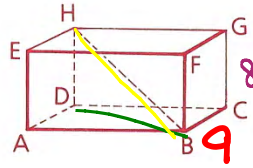
9.8: The Pythagorean Theorem and Space Figures

Ms. Kresovic

W 19 Mar 14

4 Given: The rectangular solid shown,
GC = 8, HG = 12, BC = 9

Find: a HB, a diagonal of the solid
b AG, another diagonal of the solid



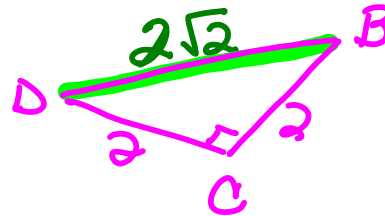
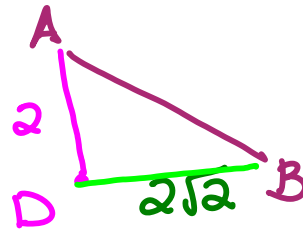
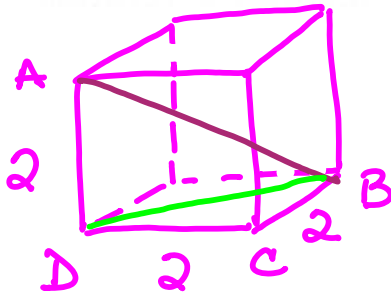
9, 12, DB
3(3, 4, 5)

$$8^2 + 15^2 = 17^2$$

$$64 + 225 = 289$$

$$HB = AG$$

15 Find the diagonal of a cube if each edge is 2.



$$\sqrt{AD^2 + DB^2} = AB$$

$$\sqrt{2^2 + (2\sqrt{2})^2} =$$

$$\sqrt{4 + 4 \cdot 2} =$$

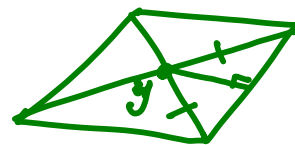
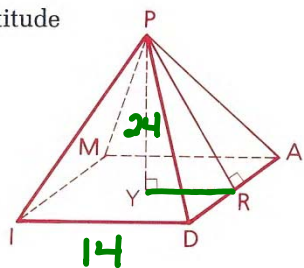
$$\sqrt{4 + 8}$$

$$\sqrt{12}$$

$$\sqrt{4 \cdot 3}$$

$$2\sqrt{3} = AB$$

- 5 Given: The regular square pyramid shown, with altitude \overline{PY} and slant height \overline{PR} ,
 $ID = 14$, $PY = 24$



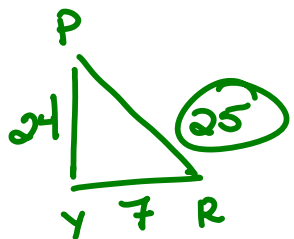
Find: a AD

sq base \rightarrow 4 \cong sds \rightarrow AD = 14

b YR

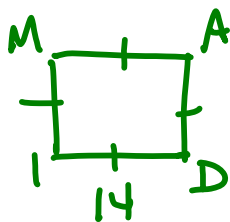
7

c PR



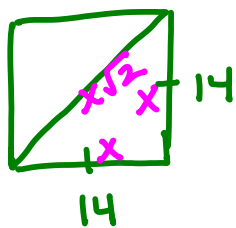
d The perimeter of base AMID

Sum of
sides
4s



$$4(10 + 4) = 40 + 16 = 56$$

e A diagonal of the base (not shown in the diagram)



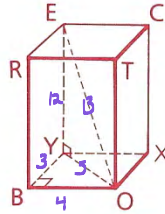
$$14\sqrt{2}$$

Homework

- 1 Given: The rectangular solid shown,
 $BY = 3$, $OB = 4$, $EY = 12$

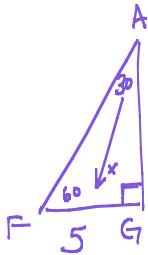
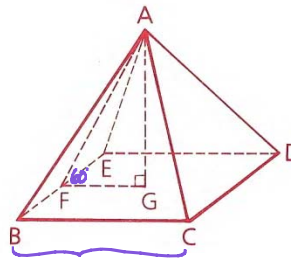
Find: a YO, a diagonal of face BOXY 5

b EO, a diagonal of the solid 13



- 3 Given: Regular square pyramid ABCDE,
 with slant height \overline{AF} , altitude \overline{AG} ,
 and base BCDE;
 perimeter of BCDE = 40,
 $\angle AFG = 60^\circ$

Find: The altitude and the slant height



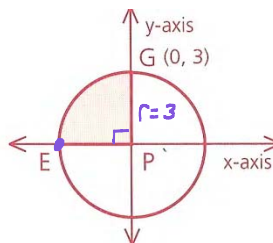
30	60	90
x	$x\sqrt{3}$	$2x$
5	$5\sqrt{3}$	10
FG	AG	AF
	alt	slant

- 11 Given: $\odot P$ as shown

Find: a The coordinates of point E (-3, 0)

b The area of sector EPG to the nearest tenth

c The length of \widehat{GE} to the nearest tenth

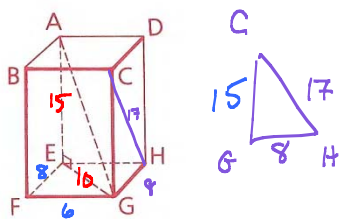


$$\begin{aligned} \boxed{b} \text{ Area} &= \frac{\angle}{360} (\pi r^2) \\ &= \frac{90}{360} (\pi 3^2) \\ &= \frac{9}{4} \pi \end{aligned}$$

$$\begin{aligned} \boxed{c} \text{ Length Arc} &= \frac{\angle}{360} \pi d \\ &= \frac{90}{360} 6 \pi \\ &= \frac{3}{2} \pi \end{aligned}$$

13 ABCDEFGH is a rectangular solid.

- a If face diagonal \overline{CH} measures 17, edge \overline{GH} measures 8, and edge \overline{FG} measures 6, how long is diagonal \overline{AG} ?
- b If diagonal \overline{AG} measures 50, edge \overline{AE} measures 40, and edge \overline{EF} measures 3, how long is edge \overline{FG} ?



$$\triangle EFG = 2(3, 4, -)$$

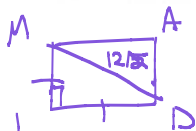
$$EG = 10$$

$$\triangle AEG = 10, 15, AG$$

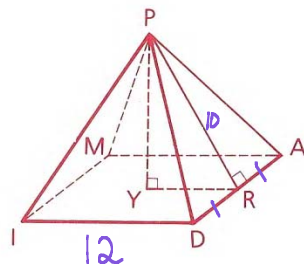
$$5(2, 3, -) \rightarrow \sqrt{2^2 + 3^2} = \sqrt{13} \Rightarrow AG = 5\sqrt{13}$$

14 PADIM is a regular square pyramid. Slant height \overline{PR} measures 10, and the base diagonals measure $12\sqrt{2}$.

- a Find ID.



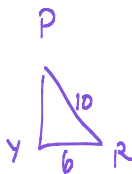
$$\begin{array}{ccc} 45 & 45 & 90 \\ x & x & x\sqrt{2} \\ & & 12\sqrt{2} \end{array}$$



$$(12)$$

- b Find the altitude of the pyramid.

$$PY = 8$$



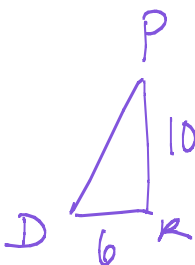
$$2(3, 4, 5)$$

- c Find RD.

$$6$$

$$6, 10, PD = 2\sqrt{34}$$

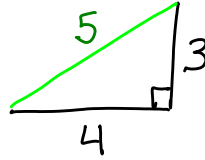
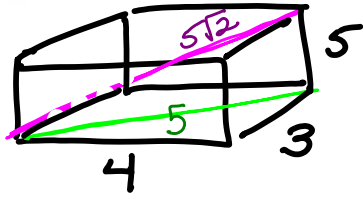
- d Find PD (length of a lateral edge).



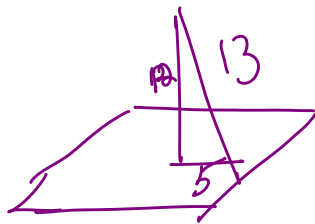
$$2(3, 5, -) \\ \sqrt{3^2 + 5^2} \\ \sqrt{34}$$

Class Work

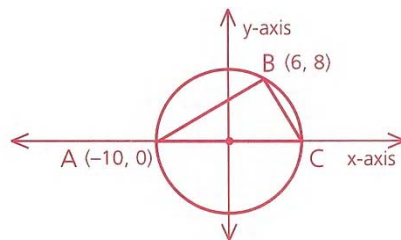
- 2 Find the diagonal of a rectangular solid whose dimensions are 3, 4, and 5.



- 6 Find the slant height of a regular square pyramid if the altitude is 12 and one of the sides of the square base is 10.



- 12 Given: Diagram as marked
Find: AB (the length of \overline{AB})



$$\begin{aligned}
 AB &= \sqrt{\Delta x^2 + \Delta y^2} \\
 &= \sqrt{(-10-6)^2 + (0-8)^2} \\
 &= \sqrt{(-16)^2 + (-8)^2} \\
 &= \sqrt{16^2 + 8^2} \\
 &= 8(1, 2, \text{---}) \\
 &= 8\sqrt{1^2 + 2^2} \\
 &= 8\sqrt{5}
 \end{aligned}$$

