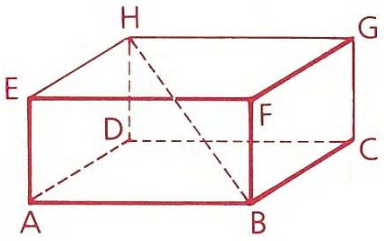
Objective

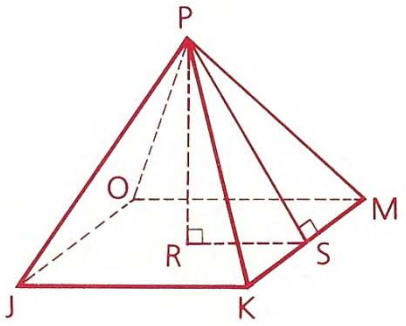
After studying this section, you will be able to

- Apply the Pythagorean Theorem to solid figures

Part One: Introduction



Rectangular Solid



Regular Square Pyramid

Many of the problems in this section will involve the two figures shown above.

In the rectangular solid:

ABFE is one of the 6 rectangular ***faces***

$\overline{AB}$ is one of the 12 ***edges***

$\overline{HB}$ is one of the 4 ***diagonals*** of the solid. (The others are $\overline{AG}$, $\overline{CE}$, and $\overline{DF}$.)

In the regular square pyramid:

JKMO is a square, and it is called the ***base***

P is the ***vertex***

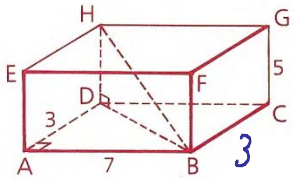
$\overline{PR}$ is the ***altitude*** of the pyramid and is perpendicular to the base at its center.

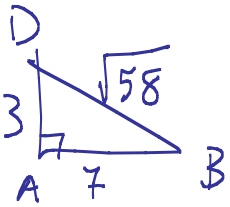
$\overline{PS}$ is called a ***slant height*** and is perpendicular to a side of the base.

Note A ***cube*** is a rectangular solid in which all edges are congruent.

Class Examples

Problem 1 *The dimensions of a rectangular solid are 3, 5, and 7. Find the diagonal.*





Solution It does not matter which edges are given the lengths 3, 5, and 7. Let AD = 3, AB = 7, and HD = 5, and use the Pythagorean Theorem twice.

In △ABD,

$3^2 + 7^2 = (DB)^2$

$9 + 49 = (DB)^2$

$\sqrt{58} = DB$

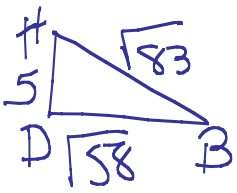
In △HDB,

$5^2 + (\sqrt{58})^2 = (HB)^2$

$25 + 58 = (HB)^2$

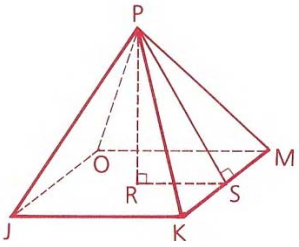
$\sqrt{83} = HB$

The measure of the diagonal is $\sqrt{83}$.



Problem 2 *Given: The regular square pyramid shown, with altitude* $\overline{PR}$ *and slant height* $\overline{PS}$*, perimeter of* JKMO = 40, PK = 13

Find: **a** JK **b** PS **c** PR



Solution

a $JK = \frac{1}{4}(40) = 10$

b The slant height of the pyramid is the ⊥ bis. of $\overline{MK}$, so PSK is a right △.

$(SK)^2 + (PS)^2 = (PK)^2$

$5^2 + (PS)^2 = 13^2$

$PS = 12$

c The altitude of a regular pyramid is perpendicular to the base at its center. Thus, $RS = \frac{1}{2}(JK) = 5$, and PRS is a right △.

$(RS)^2 + (PR)^2 = (PS)^2$

$5^2 + (PR)^2 = 12^2$

$25 + (PR)^2 = 144$

$PR = \sqrt{119}$

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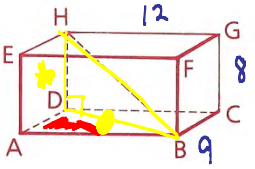
4 Given: The rectangular solid shown,
GC = 8, HG = 12, BC = 9

Find: **a** HB, a diagonal of the solid

b AG, another diagonal of the solid



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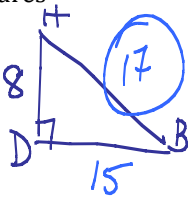
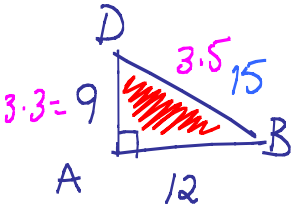




3.4

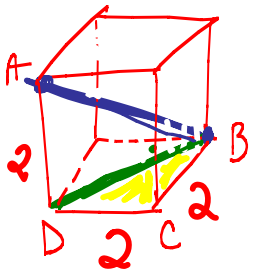
3(3,4,5)

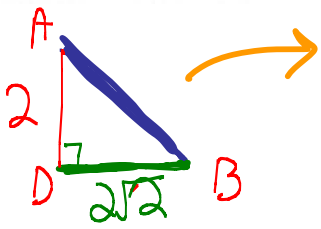
$2\sqrt{2} \cdot 2\sqrt{2} = 4\sqrt{4}$

$4 \cdot 2$

8

15 Find the diagonal of a cube if each edge is 2.





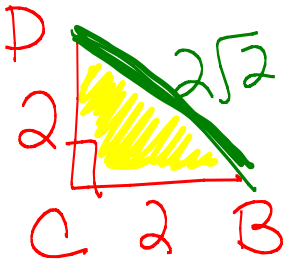
$2^2 + (2\sqrt{2})^2 = AB^2$

$4 + 8 = AB^2$

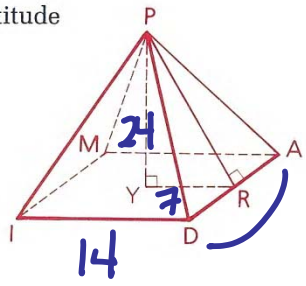
$12 = AB^2$

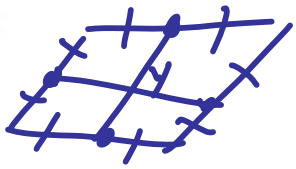
$4 \cdot 3 = AB^2$

$2\sqrt{3} = AB$



5 Given: The regular square pyramid shown, with altitude $\overline{PY}$ and slant height $\overline{PR}$,
ID = 14, PY = 24

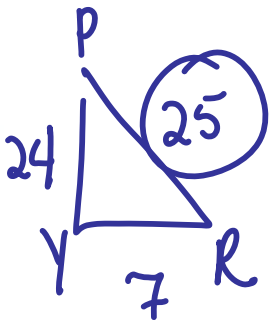




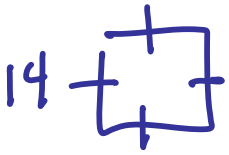
Find: **a** AD 14

b YR $\frac{1}{2}14 = 7$

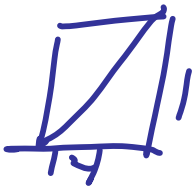
c PR



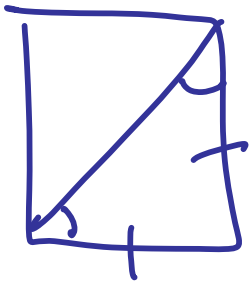
d The perimeter of base AMID $4(14) = 4(10+4) = 40+16 = 56$



e A diagonal of the base (not shown in the diagram)



$= 14\sqrt{2}$



45 45 90

X X X$\sqrt{2}$

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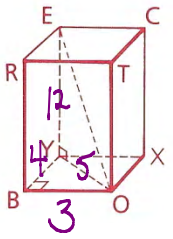
Homework

1 Given: The rectangular solid shown,
BY = 3, OB = 4, EY = 12

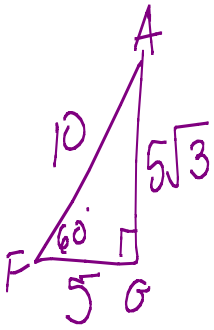
Find: **a** YO, a diagonal of face BOXY 5

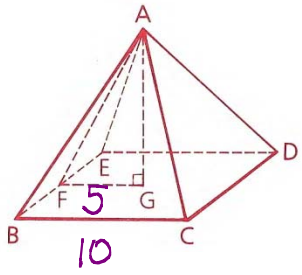
b EO, a diagonal of the solid 13



3 Given: Regular square pyramid ABCDE, with slant height $\overline{AF}$, altitude $\overline{AG}$, and base BCDE;
perimeter of BCDE = 40,
$\angle AFG = 60°$

Find: The altitude and the slant height



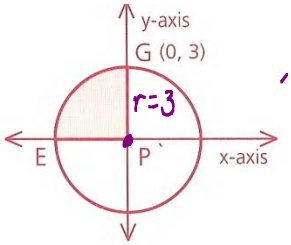


11 Given: ⊙P as shown

Find: **a** The coordinates of point E (-3,0)

b The area of sector EPG ~~to the nearest tenth~~

~~**c** The length of $\overset{\frown}{GE}$ to the nearest tenth~~



b $A_{sector} = \frac{\angle}{360}\pi r^2$

$= \frac{90}{360}\pi 3^2$

$= \frac{1}{4} 9\pi$

$= \frac{9}{4}\pi$

c $Arc\ length = \frac{\angle}{360} C$

$= \frac{90}{360} 6\pi$

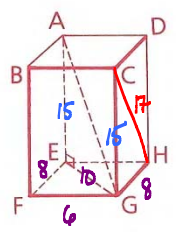
$= \frac{6}{4}\pi$

$= \frac{3}{2}\pi$

13 ABCDEFGH is a rectangular solid.

a If face diagonal $\overline{CH}$ measures 17, edge $\overline{GH}$ measures 8, and edge $\overline{FG}$ measures 6, how long is diagonal $\overline{AG}$?

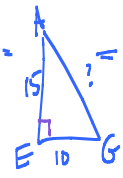
b If diagonal $\overline{AG}$ measures 50, edge $\overline{AE}$ measures 40, and edge $\overline{EF}$ measures 3, how long is edge $\overline{FG}$?



ΔCGH (8 – 17)
CG = 15

CG = AE

ΔAEG = 5(2, 3, ?)



$\sqrt{2^2+3^2}$

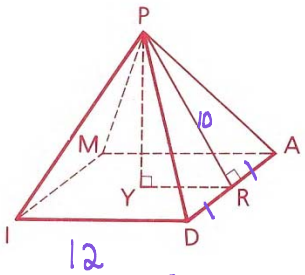
$5\sqrt{13}$

ΔEFG = 2(345) ∴ EG = 10

14 PADIM is a regular square pyramid. Slant height $\overline{PR}$ measures 10, and the base diagonals measure $12\sqrt{2}$.

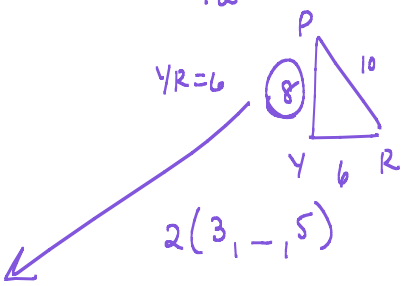
a Find ID.

45 45 90
x x x√2
12√2



12

b Find the altitude of the pyramid.



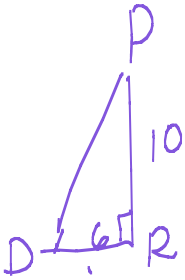
Y/R = 6

2(3, −, 5)

c Find RD.

6

d Find PD (length of a lateral edge).



$10^2 + 6^2 = PD^2$

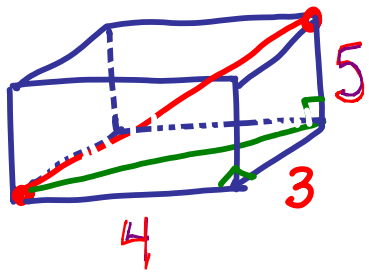
2(3, 5, −)

$2\sqrt{3^2+5^2}$

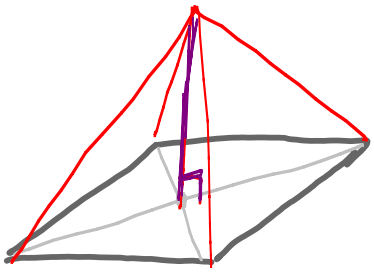
$2\sqrt{34}$

Class Work

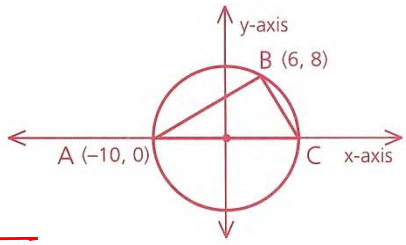
2 Find the diagonal of a rectangular solid whose dimensions are 3, 4, and 5.



6 Find the slant height of a regular square pyramid if the altitude is 12 and one of the sides of the square base is 10.



12 Given: Diagram as marked
Find: AB (the length of $\overline{AB}$)



$$\sqrt{\Delta x^2 + \Delta y^2}$$

$$\sqrt{16^2 + 8^2}$$

$8\left(2, 1, \sqrt{\ }\right)$

$2^2 + 1^2 = h^2$

$8\sqrt{5}$