Name Student
Adv Geo – 8

Ms. Kresovic
18 March 2014

Special Right Triangles (9.7)

Objectives

After studying this section, you will be able to

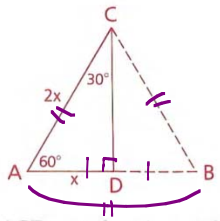
- Identify the ratio of side lengths in a 30°-60°-90° triangle
- Identify the ratio of side lengths in a 45°-45°-90° triangle

The smallest side is ALWAYS opposite the smallest angle.

The largest side is ALWAYS opposite the largest angle.

Theorem 72 ***In a triangle whose angles have the measures 30, 60, and 90, the lengths of the sides opposite these angles can be represented by x, x$\sqrt{3}$, and 2x respectively. (30°-60°-90°-Triangle Theorem)***

Given: $\triangle ABC$ is equilateral.
$\overrightarrow{CD}$ bisects $\angle ACB$.
Prove: $AD:DC:AC = x:x\sqrt{3}:2x$



Proof: Since $\triangle ABC$ is equilateral, $\angle ACD = 30°$, $\angle A = 60°$, $\angle ADC = 90°$, and $AD = \frac{1}{2}(AC)$.
By the Pythagorean Theorem, in $\triangle ADC$,
$$x^2 + (DC)^2 = (2x)^2$$

$$x^2 + DC^2 = 4x^2$$
$$-x^2 \qquad -x^2$$
$$\sqrt{DC^2} = \sqrt{3x^2}$$
$$DC = \sqrt{3}\cdot x \text{ or } x\sqrt{3}$$

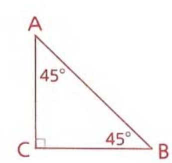
work through the rest now.

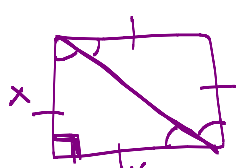
AD : DC : AC =

30°	60°	90°
x	x$\sqrt{3}$	2x

Theorem 73 ***In a triangle whose angles have the measures 45, 45, and 90, the lengths of the sides opposite these angles can be represented by x, x, and x$\sqrt{2}$, respectively. (45°-45°-90°-Triangle Theorem)***

Given: $\triangle ACB$, with $\angle A = 45°$ and $\angle B = 45°$.
Prove: $AC:CB:AB = x:x:x\sqrt{2}$
The proof of this theorem is left to you.





$$x^2 + x^2 = hyp^2$$
$$\sqrt{2x^2} = \sqrt{hyp^2}$$
$$\sqrt{2}\cdot x \text{ or}$$
$$x\sqrt{2} = hyp$$

45	45	90
x	x	x$\sqrt{2}$

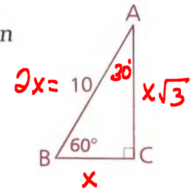
You will see 30°-60°-90° and 45°-45°-90° triangles frequently in this book and in other mathematics courses. Their ratios are worth memorizing now.

Six Common Families of Right Triangles	
* 30°-60°-90° ⇔ (x, x$\sqrt{3}$, 2x)	(5, 12, 13)
* 45°-45°-90° ⇔ (x, x, x$\sqrt{2}$)	(7, 24, 25)
(3, 4, 5)	(8, 15, 17)

*Essential for trig!

Class Examples

Problem 1 Type: *Hypotenuse (2x) known*
Find BC and AC.

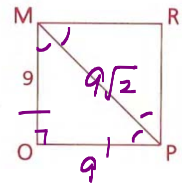


30	60	90
x	$x\sqrt{3}$	$2x$
5	$5\sqrt{3}$	10
BC	AC	AB

If $2x = 10$

$x = 5$

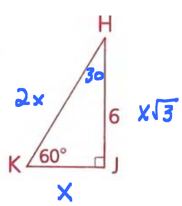
Problem 3 Type: *Leg (x) known*
MOPR is a square.
Find MP.



45 - 45 - 90

x	x	$x\sqrt{2}$
9	9	$9\sqrt{2}$

Problem 2 Type: *Longer leg $(x\sqrt{3})$ known*
Find JK and HK.



30	60	90
x	$x\sqrt{3}$	$2x$
$2\sqrt{3}$	6	$4\sqrt{3}$

If $\frac{x\sqrt{3}}{\sqrt{3}} = \frac{6}{\sqrt{3}}$

divide

$x = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{6\sqrt{3}}{\sqrt{9}}$

$x = \frac{6}{3}\sqrt{3}$

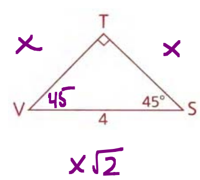
$x = 2\sqrt{3}$

mult $\sqrt{3}\, x\sqrt{3} = 6\sqrt{3}$

$\frac{3x}{3} = \frac{6\sqrt{3}}{3}$

$x = 2\sqrt{3}$

Problem 4 Type: *Hypotenuse $(x\sqrt{2})$ known*
Find ST and TV.



If $\frac{x\sqrt{2}}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{4\sqrt{2}}{2}$

$x = 2\sqrt{2}$

$ST = 2\sqrt{2}$
$TV = 2\sqrt{2}$

AMDG

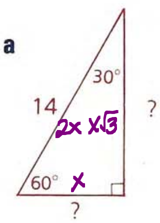
Name Ms. Kresovic

Adv Geo – ***Special Right Triangles (9.7)*** 18 March 2014

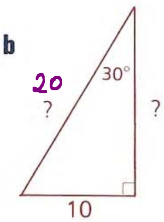
Homework

1 Find the two missing sides in each 30°-60°-90° triangle. Try to do the calculations in your head.

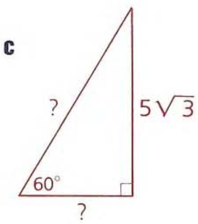
a



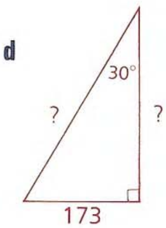
b



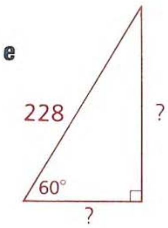
c



d



e



1a 30, 60, 90 ⇒ $\boxed{7,\ 7\sqrt{3}}$, 14

1b 10, $\boxed{10\sqrt{3},\ 20}$

1c 5, $5\sqrt{3}$, 10

1d

1e

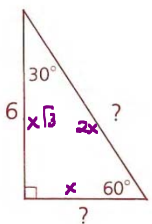
2 Find the two missing sides of each triangle. (Hint: These are a bit harder, and you may want to put x, x$\sqrt{3}$, and 2x on the proper sides as shown in the sample problems.)

$x\sqrt{3} = 6$

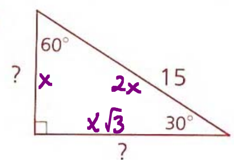
$x = \frac{6}{\sqrt{3}}$

$x = \frac{6\sqrt{3}}{3}$

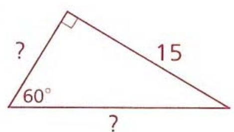
a



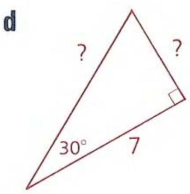
b



c



d



2a $2\sqrt{3}$, 6, $4\sqrt{3}$

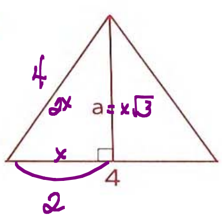
2b $\boxed{\frac{15}{2},\ \frac{15}{2}\sqrt{3}}$, 15

2c

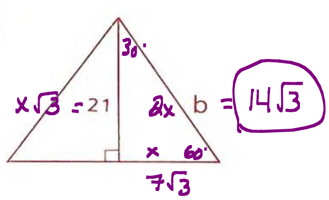
2d

3 Solve for the variable in each of these equilateral triangles.

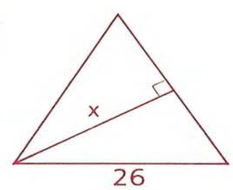
a



b



c



3a

$a = 2\sqrt{3}$

3b

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{21}{\sqrt{3}}$

$x = \frac{21}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{21\sqrt{3}}{3}$

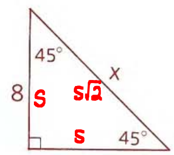
$x = 7\sqrt{3}$

3c

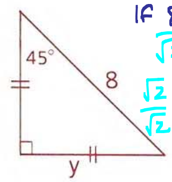
4 Solve for the variable in each of these 45°-45°-90° triangles.

s=side

a



b



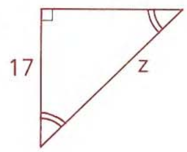
If $8 = x\sqrt{2}$

$\frac{8}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

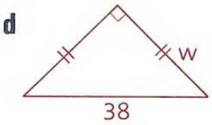
$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{8}{\sqrt{2}} = x$

$4\sqrt{2} = x$

c



d



4a $8\sqrt{2}$

4c

45	45	90
x	x	$x\sqrt{2}$
$4\sqrt{2}$	$4\sqrt{2}$	8

4b

4d

5 The perimeter of a square is 44. Find the length of a diagonal.

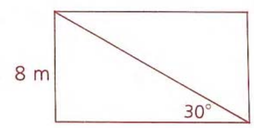
s=11

45-45-90

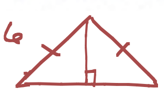
x x $x\sqrt{2}$

11 11 $11\sqrt{2}$

6 Find the length of the diagonal of the rectangle.



7 Find the altitude of an equilateral triangle if a side is 6 mm long.



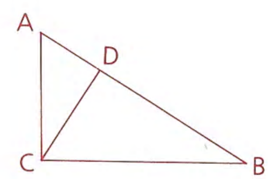
30 60 90

x $x\sqrt{3}$ 2x

3 $\boxed{3\sqrt{3}}$ 6

8 Given: $\overline{AC} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$, $\angle B = 30°$, $BC = 8\sqrt{3}$

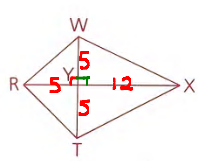
Find: CD



9 Given: TRWX is a kite ($\overline{TR} \cong \overline{WR}$ and $\overline{TX} \cong \overline{XW}$).
RY = 5, TW = 10, YX = 12

Find: **a** TR → 45-45-90 → $5\sqrt{2}$

b WX → 13



YR YT TR

x x $x\sqrt{2}$

5 5 $5\sqrt{2}$

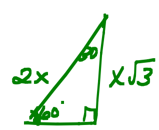
TEST! ALSO, TRIG

$\boxed{\textbf{10 a}}$ Find the ratio of the longer leg to the hypotenuse in a 30°-60°-90° triangle.

$\frac{\text{long leg}}{\text{hyp}} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

30-60-90

x $x\sqrt{3}$ 2x



10

b Find the ratio of one of the legs to the hypotenuse in a 45°-45°-90° triangle.

$\frac{\text{one leg}}{\text{hyp}} = \frac{\cancel{x}\ 1}{\cancel{x}\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

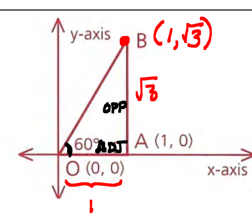
45 45 90

x x $x\sqrt{2}$

12 **a** Find the coordinates of B.

b Find the slope of $\overleftrightarrow{OB}$.

c Find $\frac{AB}{OA}$. (In a trigonometry class, this ratio is called the *tangent* of angle BOA.)



b. Slope OB $= \frac{\Delta y}{\Delta x} = \frac{\sqrt{3} - 0}{1 - 0} = \sqrt{3}$

c. $\frac{AB}{OA} = \frac{\text{OPP. LEG}}{\text{ADJ. LEG}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\tan 60° = \sqrt{3}$