AMDG

Name Student

Adv Geo – 7

Special Right Triangles (9.7)

Ms. Kresovic

18 March 2014

Objectives

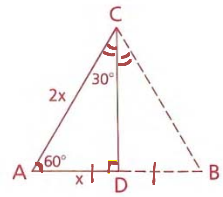
After studying this section, you will be able to
- Identify the ratio of side lengths in a 30°-60°-90° triangle
- Identify the ratio of side lengths in a 45°-45°-90° triangle

The smallest side is ALWAYS opposite the smallest angle.

The largest side is ALWAYS opposite the largest angle.

Theorem 72 ***In a triangle whose angles have the measures 30, 60, and 90, the lengths of the sides opposite these angles can be represented by x, $x\sqrt{3}$, and 2x respectively. (30°-60°-90°-Triangle Theorem)***

Given: $\triangle ABC$ is equilateral.
$\overrightarrow{CD}$ bisects $\angle ACB$.
Prove: $AD:DC:AC = x:x\sqrt{3}:2x$



Proof: Since $\triangle ABC$ is equilateral, $\angle ACD = 30°$, $\angle A = 60°$, $\angle ADC = 90°$, and $AD = \frac{1}{2}(AC)$.
By the Pythagorean Theorem, in $\triangle ADC$,
$x^2 + (DC)^2 = (2x)^2$

$x^2 + DC^2 = 4x^2$

$-x^2 \qquad -x^2$

$\sqrt{DC^2} = \sqrt{3x^2}$

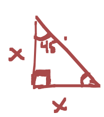
$DC = \sqrt{3} \cdot x$ or $x\sqrt{3}$

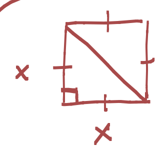
work through the rest now.

AD : DC : AC

$30° : 60° : 90°$

$x : x\sqrt{3} : 2x$





$45° : 45° : 90°$

$x \quad x \quad \boxed{x\sqrt{2}}$

$x^2 + x^2 = hyp^2$

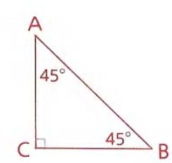
$\sqrt{2x^2} = \sqrt{hyp^2}$

$\sqrt{2}\,x$ or

$x\sqrt{2} = hyp$

Theorem 73 ***In a triangle whose angles have the measures 45, 45, and 90, the lengths of the sides opposite these angles can be represented by x, x, and $x\sqrt{2}$, respectively. (45°-45°-90°-Triangle Theorem)***

Given: $\triangle ACB$, with $\angle A = 45°$ and $\angle B = 45°$.
Prove: $AC:CB:AB = x:x:x\sqrt{2}$
The proof of this theorem is left to you.



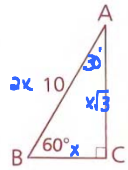
You will see 30°-60°-90° and 45°-45°-90° triangles frequently in this book and in other mathematics courses. Their ratios are worth memorizing now.

Six Common Families of Right Triangles	
* 30°-60°-90° ⟺ $(x, x\sqrt{3}, 2x)$	(5, 12, 13)
* 45°-45°-90° ⟺ $(x, x, x\sqrt{2})$	(7, 24, 25)
(3, 4, 5)	(8, 15, 17)

* Essential for trig!

Class Examples

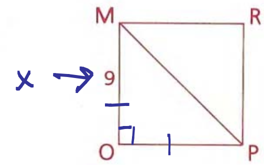
Problem 1 Type: Hypotenuse (2x) known
Find BC and AC.



30	60	90
x	$x\sqrt{3}$	$2x$
$\boxed{5}$	$\boxed{5\sqrt{3}}$	10
BC	AC	AB

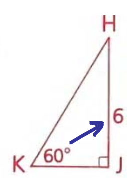
If $2x = 10$
$x = 5$

Problem 3 Type: Leg (x) known
MOPR is a square.
Find MP.



45	45	90
x	x	$x\sqrt{2}$
9	9	$9\sqrt{2}$
MO	OP	MP

Problem 2 Type: Longer leg $(x\sqrt{3})$ known
Find JK and HK.



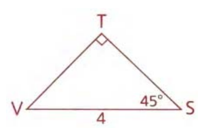
30 -	60	- 90
x	$x\sqrt{3}$	$2x$
$\boxed{2\sqrt{3}}$	6	$\boxed{4\sqrt{3}}$
JK	JH	HK

alt:
If $x\sqrt{3}\cdot\sqrt{3} = 6\sqrt{3}$
$\sqrt{9}\,x$
$\frac{3x}{3} = \frac{6\sqrt{3}}{3}$
$x = 2\sqrt{3}$

or

divide
If $\frac{x\sqrt{3}}{\sqrt{3}} = \frac{6}{\sqrt{3}}$
$x = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
$x = \frac{6\sqrt{3}}{3}$
$x = 2\sqrt{3}$

Problem 4 Type: Hypotenuse $(x\sqrt{2})$ known
Find ST and TV.



45	45	90
x	x	$x\sqrt{2}$
$2\sqrt{2}$	$2\sqrt{2}$	4
TV	ST	SV

If $\frac{x\sqrt{2}}{\sqrt{2}} = \frac{4}{\sqrt{2}}$
$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
$x = \frac{4\sqrt{2}}{\cancel{2}}$
$x = 2\sqrt{2}$

alt
$\sqrt{2}\cdot x\sqrt{2} = 4\sqrt{2}$
$\frac{2x}{2} = \frac{4\sqrt{2}}{2}$
$x = 2\sqrt{2}$

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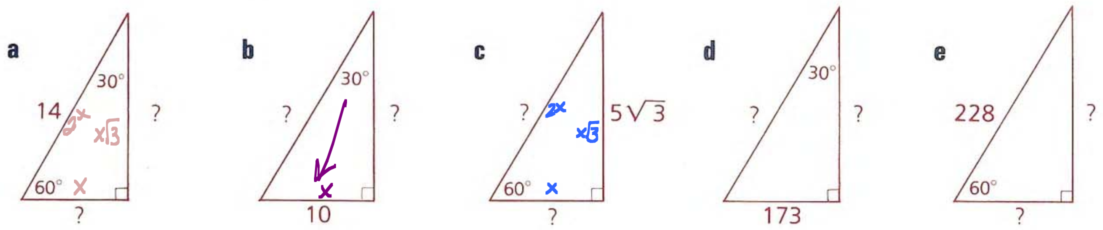
Homework

1 Find the two missing sides in each 30°-60°-90° triangle. Try to do the calculations in your head.



1a $x, x\sqrt{3}, 2x \rightarrow 7, 7\sqrt{3}, 14$

1b $x, x\sqrt{3}, 2x \rightarrow 10, 10\sqrt{3}, 20$

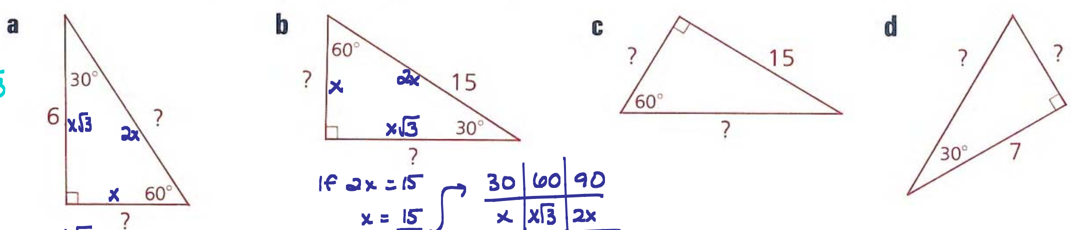
1c $x, x\sqrt{3}, 2x \rightarrow 5, 5\sqrt{3}, 10$

1d ____________________

1e ____________________

2 Find the two missing sides of each triangle. (Hint: These are a bit harder, and you may want to put x, $x\sqrt{3}$, and 2x on the proper sides as shown in the sample problems.)

30	60	90
x	$x\sqrt{3}$	$2x$
$2\sqrt{3}$	6	$4\sqrt{3}$



If $x\sqrt{3} = 6$
$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{6}{\sqrt{3}}$

$x = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow x = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

If $2x = 15$
$x = \frac{15}{2}$

30	60	90
x	$x\sqrt{3}$	$2x$
$\frac{15}{2}$	$\frac{15\sqrt{3}}{2}$	15

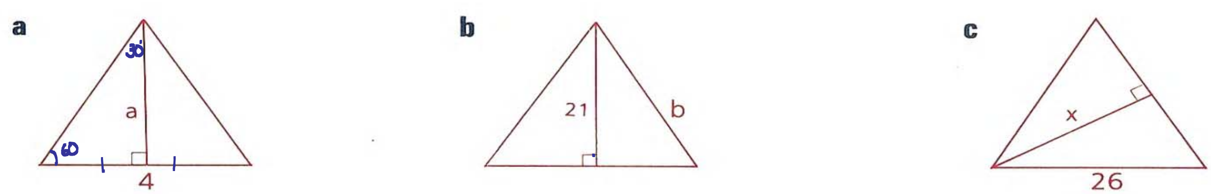
2a $2\sqrt{3}$, 6, $4\sqrt{3}$

2b $15/2$, $15\sqrt{3}/2$, 15

2c ____________________

2d ____________________

3 Solve for the variable in each of these equilateral triangles.



3a

30	60	90
x	$x\sqrt{3}$	$2x$
2	$2\sqrt{3}$	4

3b

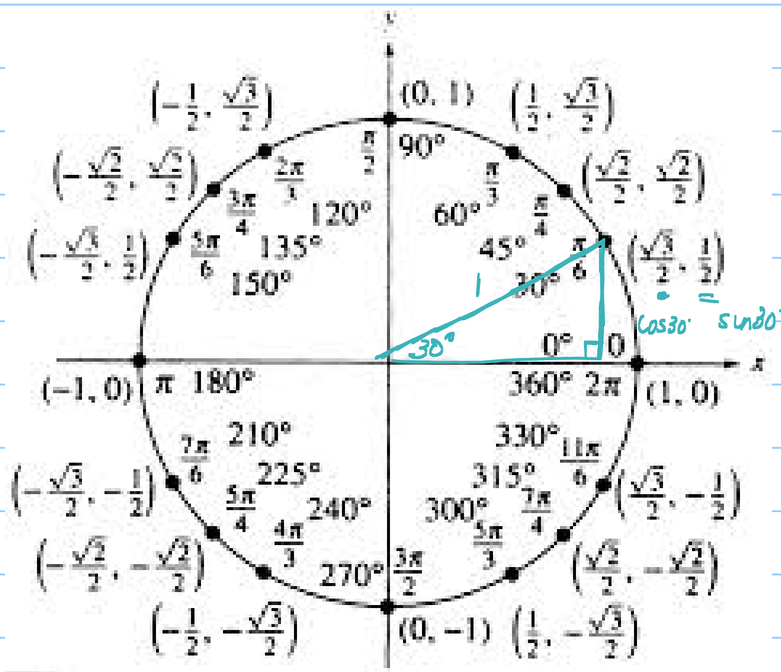
30	60	90
x	$x\sqrt{3}$	$2x$
$7\sqrt{3}$	21	$\boxed{14\sqrt{3}}$

If $\frac{21}{\sqrt{3}} = \frac{x\sqrt{3}}{\sqrt{3}}$

$\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{21}{\sqrt{3}} = x$

$\frac{21\sqrt{3}}{3} = x \rightarrow x = 7\sqrt{3}$

3c

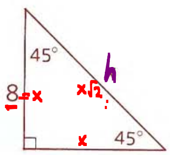


30	60	90
x	x√3	2x
1/2 =	√3/2	1

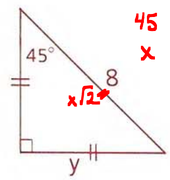
4 Solve for the variable in each of these 45°-45°-90° triangles.

$45\text{-}45\text{-}90$
$x \quad x \quad x\sqrt{2}$
$8 \quad 8 \quad \boxed{8\sqrt{2}}$

a



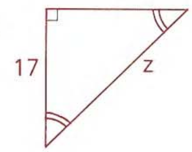
b



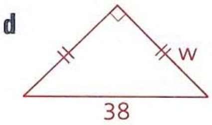
$45 \quad 45 \quad 90$
$x \quad x \quad x\sqrt{2}$
8

$x\sqrt{2} = 8$
$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{8}{\sqrt{2}}$
$x = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$

c



d



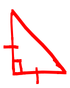
4a ______ $8\sqrt{2}$ ______ 4c ____________

4b ______ $4\sqrt{2}$ ______ 4d ____________

5 The perimeter of a square is 44. Find the length of a diagonal.

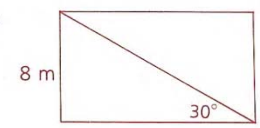
$P = 4s$

Side $= 11$

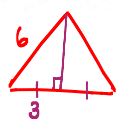


$45 \quad 45 \quad 90$
$x \quad x \quad x\sqrt{2}$
$11 \qquad 11\sqrt{2}$

6 Find the length of the diagonal of the rectangle.



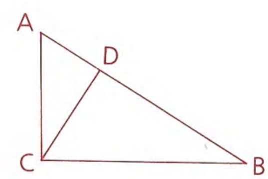
7 Find the altitude of an equilateral triangle if a side is 6 mm long.



$30 - 60 - 90$
$x \quad x\sqrt{3} \quad 2x$
$3 \quad 3\sqrt{3} \quad 6$

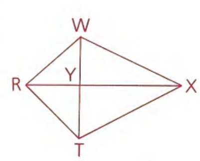
8 Given: $\overline{AC} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$,
$\angle B = 30°$, $BC = 8\sqrt{3}$

Find: CD



9 Given: TRWX is a kite ($\overline{TR} \cong \overline{WR}$ and $\overline{TX} \cong \overline{XW}$).
RY = 5, TW = 10, YX = 12

Find: **a** TR
b WX



10 **a** Find the ratio of the longer leg to the hypotenuse in a 30°-60°-90° triangle.

10

b Find the ratio of one of the legs to the hypotenuse in a 45°-45°-90° triangle.

12 **a** Find the coordinates of B.
b Find the slope of $\overleftrightarrow{OB}$.
c Find $\frac{AB}{OA}$. (In a trigonometry class, this ratio is called the *tangent* of angle BOA.)