AMDG

Name
Adv Geo –

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18 March 2014

Special Right Triangles (9.7)

Objectives

After studying this section, you will be able to

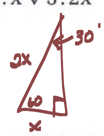
- Identify the ratio of side lengths in a 30°-60°-90° triangle
- Identify the ratio of side lengths in a 45°-45°-90° triangle

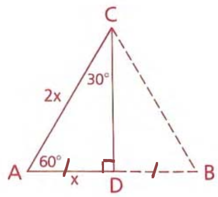
Theorem 72 ***In a triangle whose angles have the measures 30, 60, and 90, the lengths of the sides opposite these angles can be represented by x, $x\sqrt{3}$, and 2x respectively. (30°-60°-90°-Triangle Theorem)***

The <u>smallest</u> side is ALWAYS <u>opposite</u> the <u>smallest</u> angle.

The largest side is ALWAYS opposite the largest angle.

Given: $\triangle ABC$ is equilateral.
$\overrightarrow{CD}$ bisects $\angle ACB$.
Prove: $AD:DC:AC = x:x\sqrt{3}:2x$



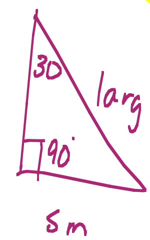


Proof: Since $\triangle ABC$ is equilateral, $\angle ACD = 30°$, $\angle A = 60°$, $\angle ADC = 90°$, and $AD = \frac{1}{2}(AC)$.
By the Pythagorean Theorem, in $\triangle ADC$,

$$x^2 + (DC)^2 = (2x)^2$$

$$x^2 + DC^2 = 4x^2$$
$$-x^2 \qquad -x^2$$
$$\sqrt{DC^2} = \sqrt{3x^2}$$
$$DC = \sqrt{3} \cdot x \quad \text{or} \quad x\sqrt{3}$$

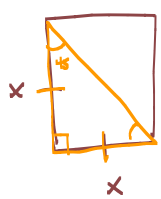
work through the rest now.



AD : DC : AC
x , $x\sqrt{3}$, 2x

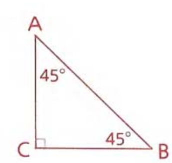
Theorem 73 ***In a triangle whose angles have the measures 45, 45, and 90, the lengths of the sides opposite these angles can be represented by x, x, and $x\sqrt{2}$, respectively. (45°-45°-90°-Triangle Theorem)***

45° : 45° : 90°
x : x : $x\sqrt{2}$



$$x^2 + x^2 = hyp^2$$
$$2x^2 = hyp^2$$
$$\sqrt{2} \cdot x$$
$$\text{or}$$
$$x\sqrt{2} = hyp$$

Given: $\triangle ACB$, with $\angle A = 45°$ and $\angle B = 45°$.
Prove: $AC:CB:AB = x:x:x\sqrt{2}$
The proof of this theorem is left to you.



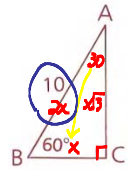
You will see 30°-60°-90° and 45°-45°-90° triangles frequently in this book and in other mathematics courses. Their ratios are worth memorizing now.

Six Common Families of Right Triangles	
* 30°, 60°, 90° ⇔ (x, $x\sqrt{3}$, 2x)	(5, 12, 13)
* 45°, 45°, 90° ⇔ (x, x, $x\sqrt{2}$)	(7, 24, 25)
(3, 4, 5)	(8, 15, 17)

* ESSENTIAL FOR TRIG!

Class Examples

Problem 1 Type: *Hypotenuse (2x) known*
Find BC and AC.

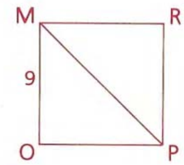


30 :	60	: 90
x :	$x\sqrt{3}$:	$2x$
$\boxed{5}$	$\boxed{5\sqrt{3}}$	10
BC	AC	AB

$\frac{10}{2} = \frac{2x}{2}$

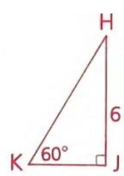
$5 = x$

Problem 3 Type: *Leg (x) known*
MOPR is a square.
Find MP.



45 :	45 :	90
x	x	$x\sqrt{2}$
9		$\boxed{9\sqrt{2}}$
MO	OP	MP

Problem 2 Type: *Longer leg* $(x\sqrt{3})$ *known*
Find JK and HK.



30 :	60 :	90
x	$x\sqrt{3}$	$2x$
$2\sqrt{3}$	6	$4\sqrt{3}$
KJ	HJ	HK

PREFERRED

IF $\frac{x\sqrt{3}}{\sqrt{3}} = \frac{6}{\sqrt{3}}$

$x = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ Rationalize Denominator

$x = \frac{6\sqrt{3}}{3}$

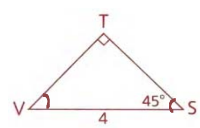
$x = 2\sqrt{3}$

IF $x\sqrt{3} \cdot \sqrt{3} = 6 \cdot \sqrt{3}$

$\frac{x \cdot 3}{3} = \frac{6\sqrt{3}}{3}$

$x = 2\sqrt{3}$

Problem 4 Type: *Hypotenuse* $(x\sqrt{2})$ *known*
Find ST and TV.



45 :	45 :	90
x	x	$x\sqrt{2}$
TV	TS	VS
$\boxed{2\sqrt{2}}$	$\boxed{2\sqrt{2}}$	4

If $\frac{4}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{4}{\sqrt{2}} = x$

$\frac{4\sqrt{2}}{\sqrt{4}}$

$\frac{4\sqrt{2}}{2}$

$2\sqrt{2} = x$

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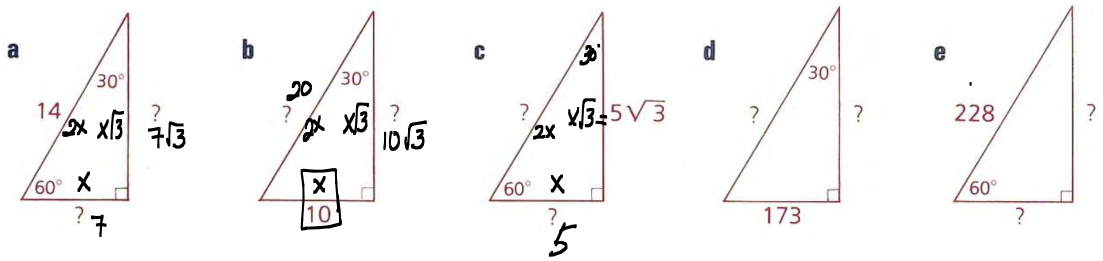
Homework

1 Find the two missing sides in each 30°-60°-90° triangle. Try to do the calculations in your head.



1a 7 & $7\sqrt{3}$ ______________________ 1d ______________________

1b $10\sqrt{3}$ & 20 ______________________ 1e ______________________

1c 5 & 10 ______________________

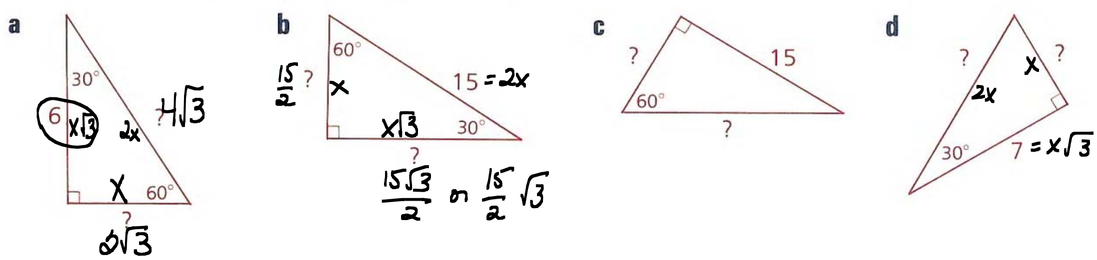
2 Find the two missing sides of each triangle. (Hint: These are a bit harder, and you may want to put x, x$\sqrt{3}$, and 2x on the proper sides as shown in the sample problems.)

$6 = x\sqrt{3}$

$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$

$\frac{6\sqrt{3}}{3} = x$

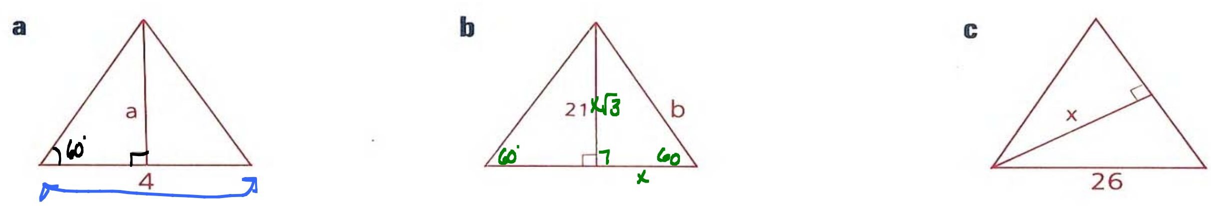
$\boxed{2\sqrt{3} = x}$



2a $2\sqrt{3}$ & $4\sqrt{3}$ ______________________ 2c ______________________

2b $\frac{15}{2}$ & $\frac{15\sqrt{3}}{2}$ ______________________ 2d ______________________

3 Solve for the variable in each of these equilateral triangles.



3a

30	: 60	: 90
x	$x\sqrt{3}$	$2x$
2	$2\sqrt{3}$	

3b

30 :	60 :	90
x	$x\sqrt{3}$	$2x$
$7\sqrt{3}$	21	$14\sqrt{3}$

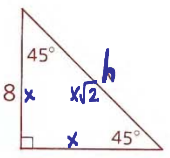
If $\frac{x\sqrt{3}}{\sqrt{3}} = \frac{21}{\sqrt{3}}$

$x = \frac{21}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{21\sqrt{3}}{3} = 7\sqrt{3}$

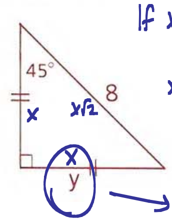
3c

4 Solve for the variable in each of these 45°-45°-90° triangles.

a



b



If $x\sqrt{2} = 8$

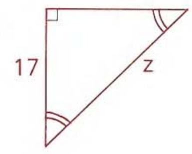
$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{8}{\sqrt{2}}$

$x = \frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

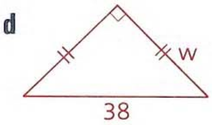
$x = \frac{8\sqrt{2}}{2}$

$\rightarrow x = 4\sqrt{2}$

c



d

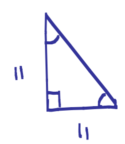


4a $8\sqrt{2}$ ____________________ 4c ____________________

4b $4\sqrt{2}$ ____________________ 4d ____________________

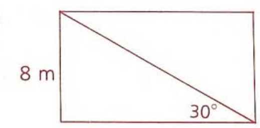
5 The perimeter of a square is 44. Find the length of a diagonal.

side = 11

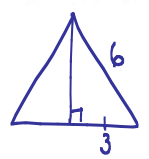


45	45	90
11	11	$11\sqrt{2}$

6 Find the length of the diagonal of the rectangle.



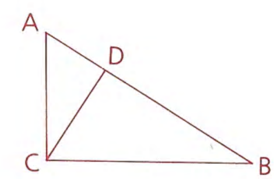
7 Find the altitude of an equilateral triangle if a side is 6 mm long.



30	60	90
x	$x\sqrt{3}$	2x
3	$3\sqrt{3}$	6

8 Given: $\overline{AC} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$, $\angle B = 30°$, $BC = 8\sqrt{3}$

Find: CD

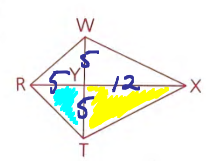


9 Given: TRWX is a kite ($\overline{TR} \cong \overline{WR}$ and $\overline{TX} \cong \overline{XW}$). RY = 5, TW = 10, YX = 12

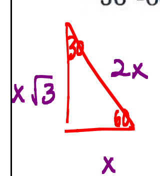
Find: **a** TR → 45, 45, 90 → x, x, $x\sqrt{2}$ → $5\sqrt{2}$

b WX

(5, 12, 13)



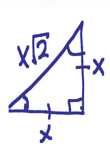
10 a Find the ratio of the longer leg to the hypotenuse in a 30°-60°-90° triangle.



$\frac{\text{long leg}}{\text{hyp}} : \frac{\cancel{x}\sqrt{3}}{2\cancel{x}} = \frac{\sqrt{3}}{2}$

10

b Find the ratio of one of the legs to the hypotenuse in a 45°-45°-90° triangle.



$\frac{\text{leg}}{\text{hyp}} : \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

12 a Find the coordinates of B.

b Find the slope of $\overleftrightarrow{OB}$.

c Find $\frac{AB}{OA}$. (In a trigonometry class, this ratio is called the *tangent* of angle BOA.)