AMDG

Name StuDent Ms. Kresovic

Adv Geo 7 *9.5 Distance Formula (continued) & 9.6 Special Families* Date: Tues., 11 Mar 2014

9.5 The Distance Formula

Objective

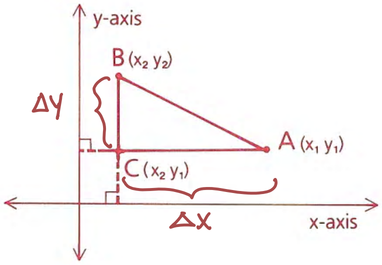
After studying this section, you will be able to

- Use the distance formula to compute lengths of segments in the coordinate plane

Find AB.

$$(OA)^2 + (OB)^2 = (BA)^2$$
$$3^2 + 4^2 = (BA)^2$$
$$25 = (BA)^2$$
$$5 = BA$$

$\sqrt{\Delta x^2 + \Delta y^2}$



To compute any nonvertical, nonhorizontal length, we could draw a right triangle and use the Pythagorean Theorem.

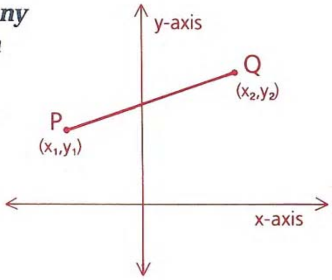
$$(AB)^2 = (CA)^2 + (BC)^2$$
$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\text{or } AB = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

However, it is easier to use the ***distance formula,*** which is derived from the Pythagorean Theorem.

This is not new; this was last night's homework.
See how closely it connects to the Pythagorean Theorem!! ☺

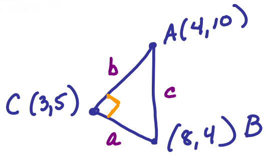
Theorem 71 ***If P = (x_1, y_1) and Q = (x_2, y_2) are any two points, then the distance between them can be found with the formula***

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ or}$$
$$PQ = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$



Class Examples

3 Show that the triangle with vertices at $\overset{B}{(8, 4)}$, $\overset{C}{(3, 5)}$, and $\overset{A}{(4, 10)}$ is a right triangle by using
a The distance formula **b** Slopes



$\boxed{3a}$ $a = \sqrt{5^2+1^2} = \sqrt{26}$
$b = \sqrt{1^2+5^2} = \sqrt{26}$
$c = \sqrt{4^2+6^2} = \sqrt{52}$

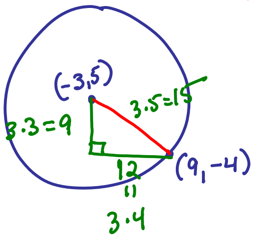
$a^2+b^2 \ \square \ c^2$
$26+26 \ \square \ 52$
$52 = 52$
∴ rt△ by dist/Pyth Thm

$\boxed{3b}$ Slope AC: $\frac{\Delta y}{\Delta x} = \frac{10-5}{4-3} = 5$
Slope BC: $\frac{\Delta y}{\Delta x} = \frac{5-4}{3-8} = \frac{1}{-5}$

OPP RECIP
∴ ⊥ ∴ rt∠ ∴ rt△

5 Find the area of the circle that passes through (9, −4) and whose center is (−3, 5).

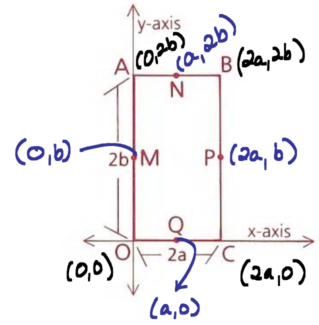
$A_\odot = \pi r^2$, $r = ? = 15$
$= \boxed{225\pi}$



8 Given: Rectangle ABCO

- **a** Find the coordinates of A, B, C, and O.
- **b** Find the coordinates of M, N, P, and Q, the midpoints of the sides.
- **c** Find the slopes of $\overline{MN}$, $\overline{QP}$, $\overline{MQ}$, and $\overline{NP}$. What can we conclude about MNPQ?
- **d** Find the lengths of $\overline{MN}$, $\overline{QP}$, $\overline{MQ}$, and $\overline{NP}$. What can we now conclude about MNPQ?

Slope $\frac{\Delta y}{\Delta x}$



$\boxed{c}$ Slope $\overline{MN} = \frac{\Delta y}{\Delta x} = \frac{2b-b}{a-0} = \frac{b}{a}$

Slope $\overline{PO} = \frac{\Delta y}{\Delta x} = \frac{b-0}{2a-a} = \frac{b}{a}$

Slope $\overline{MQ} = \frac{\Delta y}{\Delta x} = \frac{b-0}{0-a} = \frac{b}{-a}$

Slope $\overline{NP} = \frac{\Delta y}{\Delta x} = \frac{2b-b}{a-2a} = \frac{b}{-a}$

BTH PR Opp SDS ∥ ⇒ ▱

$\boxed{d}$ $MN = \sqrt{a^2+b^2}$

$QP = \sqrt{(2a-a)^2+(b-0)^2} = \sqrt{a^2+b^2}$

$MQ = \sqrt{(0-a)^2+(b-0)^2} = \sqrt{a^2+b^2}$

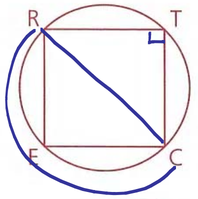
$NP = \sqrt{(a-2a)^2+(2b-b)^2} = \sqrt{a^2+b^2}$

All four sides ▱ are ≅ ⇒

Rhombus

10 In the figure at the right, RECT is a rectangle. Is $\overline{RC}$ a diameter? Why or why not?

→ rt∠



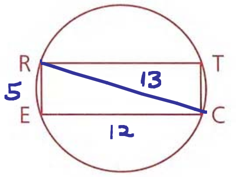
∠T is inscribed (vertex on a circle)

$2\angle T = m\overset{\frown}{REC} = 180^\circ$

$\overline{RC}$ is a diameter, cuts ⊙ in ½

11 In rectangle RECT, RE = 5 and EC = 12.

- **a** Find the circumference of the circle.
- **b** Find the area of the circle to the nearest tenth.



$d = 13$, $r = \frac{13}{2}$

a) $C = \pi d = \boxed{13\pi}$

b) $A = \pi r^2 = \pi\left(\frac{13}{2}\right)^2 = \boxed{\frac{169}{4}\pi}$ or $42\frac{1}{4}\pi$

AMDG

Name
Ms. Kresovic

Adv Geo
9.5 Distance Formula (continued) & 9.6 Special Families
Date: Tues., 11 Mar 2014

9.6: Families of Right Triangles

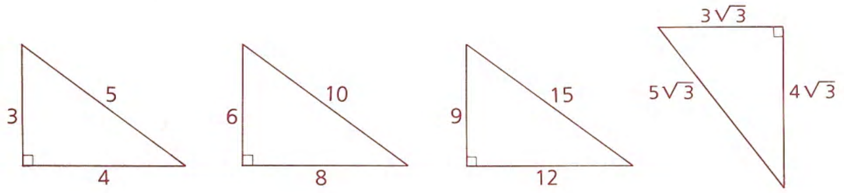
Objectives

After studying this section, you will be able to

- Recognize groups of whole numbers known as Pythagorean triples
- Apply the Principle of the Reduced Triangle

Definition Any three whole numbers that satisfy the equation $a^2 + b^2 = c^2$ form a ***Pythagorean triple.***

Below is a set of right triangles you have encountered many times in this chapter. Do you see how the triangles are related?



(3,4,5) 2(3,4,5) 3(3,4,5) $\sqrt{3}$(3,4,5)

scalar

Other common families are

(5, 12, 13), of which (15, 36, 39) is another member

(7, 24, 25), of which (14, 48, 50) is another member

(8, 15, 17), of which $\left(4, 7\frac{1}{2}, 8\frac{1}{2}\right)$ is another member

There are infinitely many families, including (9, 40, 41), (11, 60, 61), (20, 21, 29), and (12, 35, 37), but most are not used very often.

Know Quest

(3,4,5)
(5,12,13)
(7,24,25)
(8,15,17)
(9,40,41)

★ Principle of the Reduced Triangle ★

1 Reduce the difficulty of the problem by multiplying or dividing the three lengths by the same number to obtain a similar, but simpler, triangle in the same family.
2 Solve for the missing side of this easier triangle.
3 Convert back to the original problem.
Mult Scalar back.

Class Examples will be scattered in the homework.

9.5 Homework

1 Find the distance between each pair of points.

- **a** (4, 0) and (6, 0)
- **b** (2, 3) and (2, − 1)
- **c** (4, 1) and (7, 5)

1a ___________________________________

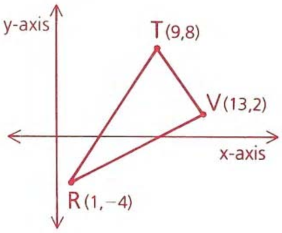
1b ___________________________________

1c ___________________________________

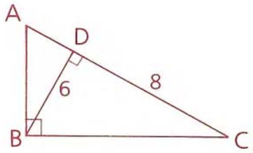
6 Given: △RTV as shown

Find: **a** The length of the median from T

b The length of the segment joining the midpoints of $\overline{RT}$ and $\overline{TV}$



7 Find AD and BC.



AMDG

Name
Adv Geo

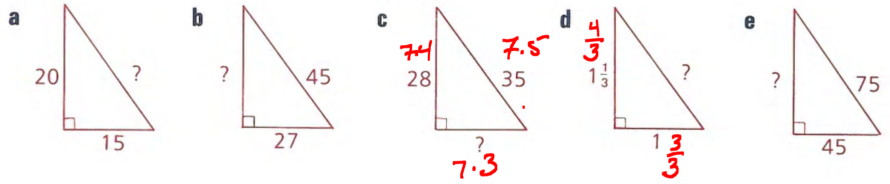
9.5 Distance Formula (continued) & 9.6 Special Families

Ms. Kresovic
Date: Tues., 11 Mar 2014

9.6 Homework

In problems 1–5, find the missing side in each triangle.

1 (3, 4, 5)



1a ______

1b ______

1c $7(3,4,5) \rightarrow 21$

1d $\frac{1}{3}(3,4,5) \rightarrow 5/3$

1e ______

2a ______

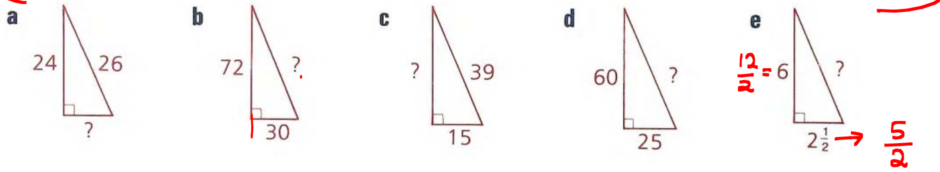
2b ______

2c ______

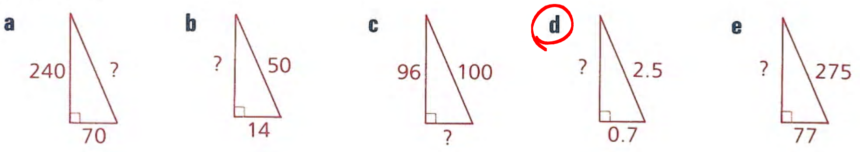
2d ______

2e $\frac{1}{2}(5,12,13) \rightarrow 13/2$

2 (5, 12, 13)



3 (7, 24, 25)



3a $10(7,24,25) \rightarrow 250$

3b ______

3c ______

3d $.1(7,24,25) \rightarrow 2.4$

3e ______

4a ______

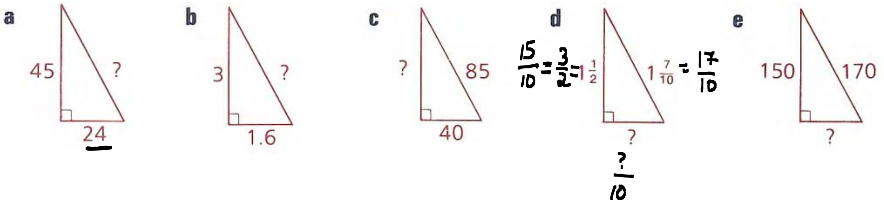
4b ______

4c ______

4d $\frac{1}{10}(8,15,17) \rightarrow \frac{8}{10} = \frac{4}{5}$ Always reduce

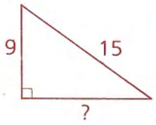
4e ______

4 (8, 15, 17)

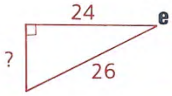


5 Mixed

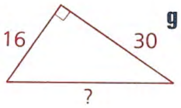
a



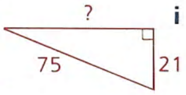
c



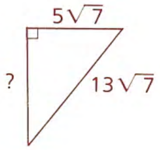
e



g

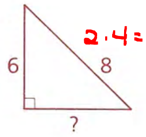


i



TRICK

b

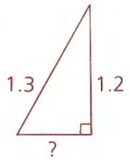


3·2 =

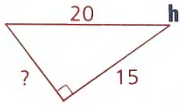
2·4=

2·x

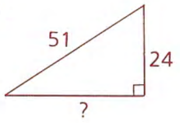
d



f



h



5a

5b $2(3,x,4) \rightarrow 3^2+x^2=4^2 \rightarrow x^2=16-9 \rightarrow x^2=7 \rightarrow x\sqrt{7} \longrightarrow \boxed{2\sqrt{7}}$

5c

5d

5e

5f

5g $3(7,24,25) = \boxed{72}$

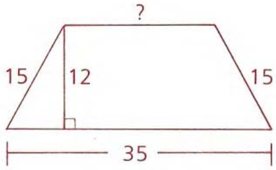
5h

5i

6 Find the diagonal of a rectangle whose sides are 20 and 48.

7 Find the perimeter of an isosceles triangle whose base is 16 dm and whose height is 15 dm.

8 Find the length of the upper base of the isosceles trapezoid.



AMDG

Name Ms. Kresovic

Adv Geo *9.5 Distance Formula (continued) & 9.6 Special Families* Date: Tues., 11 Mar 2014

Classwork

In the problems below, find the missing side of each triangle. Hand in immediately after completion and begin working on the homework assignment.

1 Find the distance between each pair of points.

d (−2, −4) and (−8, 4)

e The origin and (2, 5)

f (2, 1) and (6, 3)

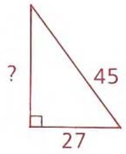
1d ______________________________

1e ______________________________

1f______________________________

1 (3, 4, 5)

b

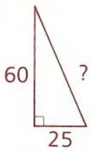


1b ____

2d ____

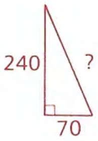
2 (5, 12, 13)

d



3 (7, 24, 25)

a



3a ____