AMDG

Name **_9.4: The Pythagorean Theorem,_** Ms. Kresovic

Adv Geo – **_Geometry's Most Elegant Theorem_** Date

Objective: After studying this section, you will be able to apply the Pythagorean Theorem and its converse.

Prior knowledge:

- Triangle Inequality Theorem (chapter 1): The third side of a triangle must be
  - Smaller than the sum of the other two sides, and
  - Larger than the difference.
- Used the Pythagorean Theorem before.

$(5-2) < x < (5+2)$

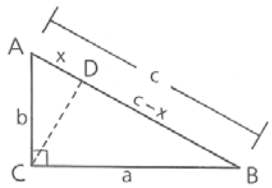
$3 < x < 7$

Theorem 69 ***The square of the measure of the hypotenuse of a right triangle is equal to the sum of the squares of the measures of the legs. (Pythagorean Theorem)***

Given: $\triangle ACB$ is a right $\triangle$ with right $\angle ACB$.

Prove: $a^2 + b^2 = c^2$

Proof:

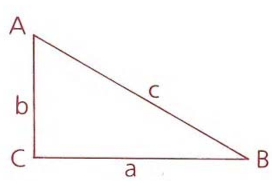


If rt$\triangle$ then $sm^2 + md^2 = lg^2$

1 $\angle ACB$ is a right $\angle$.	1 Given
2 Draw $\overline{CD} \perp$ to $\overline{AB}$.	2 From a point outside a line, only one $\perp$ can be drawn to the line.
3 $\overline{CD}$ is an altitude.	3 A segment drawn from a vertex of a $\triangle$ $\perp$ to the opposite side is an altitude.
4 $a^2 = (c - x)c$	4 In a right $\triangle$ with an altitude drawn to the hypotenuse, $(\text{leg})^2 = (\text{adjacent seg.})(\text{hypot.})$.
5 $a^2 = c^2 - cx$	5 Distributive Property
6 $b^2 = xc$	6 Same as 4
7 $a^2 + b^2 = c^2 - cx + cx$	7 Addition Property
8 $a^2 + b^2 = c^2$	8 Algebra

Theorem 70 ***If the square of the measure of one side of a triangle equals the sum of the squares of the measures of the other two sides, then the angle opposite the longest side is a right angle.***

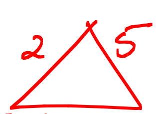
If $a^2 + b^2 = c^2$, then $\triangle ACB$ is a right $\triangle$ and $\angle C$ is the right $\angle$.



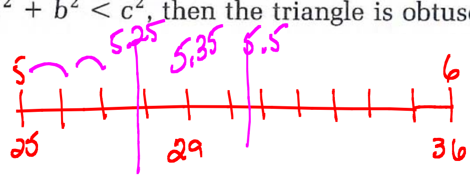
If $sm^2 + md^2 = lg^2$ then rt$\triangle$

If, in the diagram above, we increased c while keeping a and b the same, $\angle C$ would become larger. Try it. Thus, a valuable extension of Theorem 70 can be stated:

* If c is the length of the longest side of a triangle, and
- * $a^2 + b^2 > c^2$, then the triangle is acute
- * $a^2 + b^2 = c^2$, then the triangle is right
- * $a^2 + b^2 < c^2$, then the triangle is obtuse



$3 < x < 7$



$2^2 + 5^2$

$4 + 25 = 29$

$> \Rightarrow$ acute

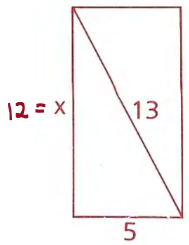
If rt then $x = \sqrt{29} \approx 5.38$

$< \Rightarrow$ obtuse

Class Examples

Problem 2 Find the perimeter of the rectangle shown.

→ sum sides



$5^2 + x^2 = 13^2$

$x^2 = 169 - 25$

$x^2 = 144$

$x = 12$

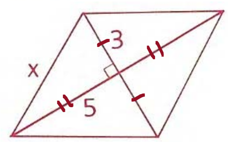
why not ±? No neg lengths!! :)

(5, 12, 13) All whole numbers! Pyth. Triples

$P_{rect}: 2(l+w) = 2(17) = \boxed{34}$

Problem 3 Find the perimeter of a rhombus with diagonals of 6 and 10.

→ ≅ sds → ⊥ & bis



$3^2 + 5^2 = x^2$

$9 + 25 = x^2$

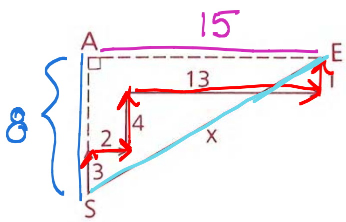
$34 = x^2$

$\sqrt{34} = x$

$P_{RHOM} = 4x = 4(\sqrt{34}) = \boxed{4\sqrt{34}}$

Problem 4 Nadia skipped 3 m north, 2 m east, 4 m north, 13 m east, and 1 m north. How far is Nadia from where she started?

Test 2/6 Quiz



$8^2 + 15^2 = x^2$

$64 + 225 = x^2$

$289 = x^2$

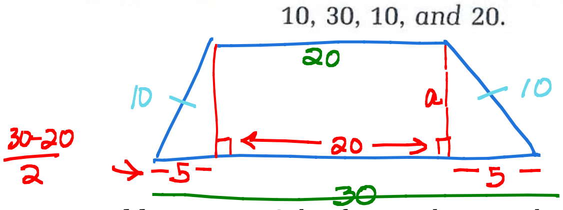
$17 = x$

(8, 15, 17) Another Pyth. Triple!! :)

All whole numbers again! WOW

Problem 5 Find the altitude of an isosceles trapezoid whose sides have lengths of 10, 30, 10, and 20.

→ ⊥



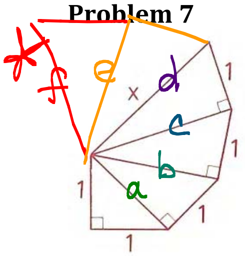
$5^2 + a^2 = 10^2$

$a^2 = 100 - 25$

$a^2 = 75$

$a = 5\sqrt{3}$

Problem 7 Solve for *x* in the partial spiral.



$1^2 + 1^2 = a^2$

$\sqrt{2} = a$

$a^2 + 1^2 = b^2$

$2 + 1 = b^2$

$\sqrt{3} = b$

$b^2 + 1^2 = c^2$

$3 + 1 = c^2$

$\sqrt{4} = c$

$2 = c$

$c^2 + 1^2 = d^2$

$4 + 1 = d^2$

$\sqrt{5} = d$

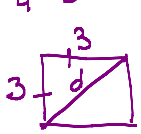
$\sqrt{6} = e$

$\sqrt{7} = f$

2 Find the length of the diagonal of a square with perimeter 12 cm.

→ 4 sides, ≅ sds, rt ∠s

$P = 12 \Rightarrow s = \frac{12}{4} = 3$



$3^2 + 3^2 = d^2$

$9 + 9 = d^2$

$9 \cdot 2 = d^2$

$3\sqrt{2} = d$

What if: $3(1, 1, \underline{x})$

$1^2 + 1^2 = x^2$

$\sqrt{2} = x$

$3\sqrt{2} = \text{diag.}$

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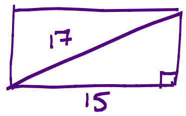
4 Find the perimeter of a rectangle whose diagonal is 17 mm long and whose base is 15 mm long.

$17^2 = 15^2 + h^2$

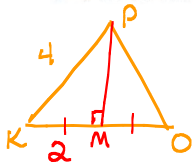
$289 - 225 = h^2$

$64 = h^2$, $8 = h$



$h = 8$mm

6 $\overline{PM}$ is an altitude of equilateral triangle PKO. If PK = 4, find PM.



If eq lat or isos △ then alt also median

3 ≅ sds

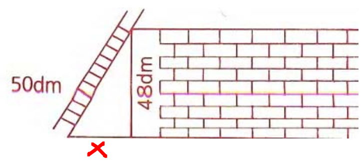
$2^2 + a^2 = 4^2$

$a^2 = 16 - 4$

$a^2 = 12$

$a = \boxed{2\sqrt{3}}$

8 How far is the foot of the ladder from the wall?



(x, 48, 50)

2 (d, 24, 25)

$d^2 + 24^2 = 25^2$

$d^2 + 576 = 625$

$d^2 = 49$

$d = 7$

$2d = x$

$2(7) = x$

$14 = x$

$\boxed{14\text{ dm}}$

(7, 24, 25) Another Triple

9 $\overline{AC} \parallel$ y-axis and $\overline{CB} \parallel$ x-axis.

a Find the coordinates of C. $(x_A, y_B) = (2, 3)$

b Find AC and CB. AC = 8 & BC = 6

c Find AB. = 10

d Is AB = $\sqrt{(8-2)^2 + (11-3)^2}$? Yes!!

$\sqrt{\Delta x^2 + (\Delta y)^2}$

$\sqrt{6^2 + 8^2}$

$\sqrt{100}$

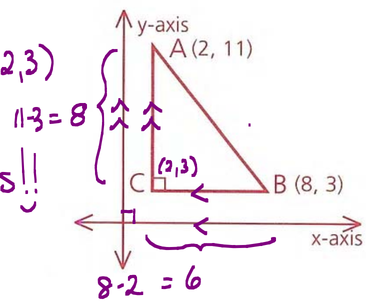
10

$6^2 + 8^2 = AB^2$

$36 + 64 =$

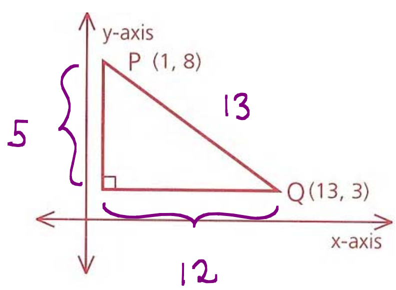
$100 = AB^2$

$10 = AB$



10 Use the method suggested by part **d** of problem 9 to find PQ.

dist. form.



$\sqrt{\Delta x^2 + \Delta y^2}$

$\sqrt{(13-1)^2 + (3-8)^2}$

$\sqrt{12^2 + (-5)^2}$

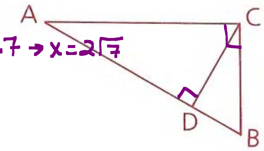
$\sqrt{144 + 25}$

$\sqrt{169}$

13

(5, 12, 13) A Triple!!!

12 ∠ACB is a right angle and $\overline{CD} \perp \overline{AB}$.



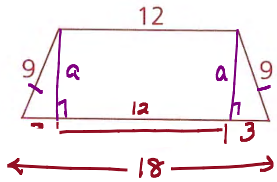
a If AD = 7 and BD = 4, find CD. $\frac{7}{x}=\frac{x}{4} \rightarrow x^2 = 4 \cdot 7 \rightarrow x = 2\sqrt{7}$

b If CD = 8 and DB = 6, find CB.

c If BC = 8 and BD = 2, find AB.

d If AC = 21 and AB = 29, find CB.

14 Find the altitude (length of a segment perpendicular to both bases) of the isosceles trapezoid shown.

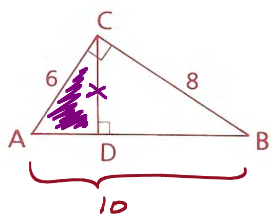


$$3^2 + a^2 = 9^2$$
$$9 + a^2 = 81$$
$$a^2 = 72$$
$$a^2 = 9 \cdot 4 \cdot 2 \rightarrow a = 6\sqrt{2}$$

$$18 - 12 = 6 \qquad 6 \div 2 = 3$$

16 Given: Diagram as shown

Find: CD

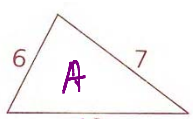


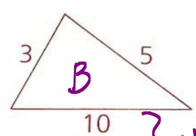
$2(3,4,5) = 6, 8, 10$

$\frac{\text{leg}}{\text{hyp}}$ SΔ LΔ $\frac{x}{6} = \frac{8}{10}$

$10x = 48 \rightarrow x = 4.8$

22 Classify the triangles.





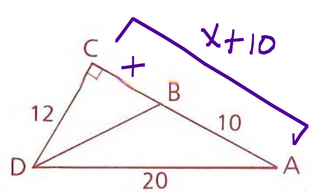
$$6^2 + 7^2 = \text{hyp}^2$$
$$36 + 49 =$$
$$85 = \text{hyp}^2$$
$$\sqrt{85} = \text{hyp}$$

$1 < a < 13$

Obtuse

$2 < x < 8$ } NOT POSSIBLE

24 Find the perimeter of △DBC.



$$\text{leg}^2 + \text{leg}^2 = \text{hyp}^2$$
$$12^2 + (x+10)^2 = 20^2$$
$$144 + x^2 + 20x + 100 = 400$$
$$x^2 + 20x + 244 = 400$$
$$x^2 + 20x - 156 = 0 \rightarrow (x+26)(x-6) = 0$$

$x = -26$ nonnegative lengths

$\boxed{x = 6}$

156 → 3, 52 → 2, 26

26 The perimeter of an isosceles triangle is 32, and the length of the altitude to its base is 8. Find the length of a leg.

P = sum sides = 32

$2y + 2x = 32$

$y + x = 16$

$y = 16 - x$

$x^2 + 8^2 = y^2$ Pyth. Thm.

$x^2 + 64 = (16 - x)^2$ Substitute

$x^2 + 64 = 256 - 32x + x^2$ Mult.

$+32x \; -x^2 - 64 \quad -64 + 32x - x^2$

$32x = 192$

$x = 6$

& $y = 10$

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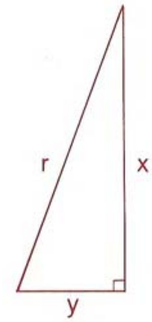
Homework

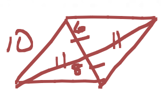
1. Solve for the third side. Let x & y be the legs of a right triangle, and r be the hypotenuse.



	x	y	r	work
a.	4	5	$\sqrt{41}$	$\sqrt{16+25}=$
b.	15	8	17	
c.	$3\cdot4$ 12	$3\cdot3$ 9	$3\cdot5$ 15	$3(3,4,5)$
d.	12	5	13	$12^2+y^2=13^2$ $y^2=169-144$ $y^2=25$
e.	5	$5\sqrt{3}$	10	
f.	5	2	$\sqrt{29}$	$5^2+y^2=\sqrt{29}^2$ $y^2=29-25$
g.	$2\sqrt{5}$	$3\sqrt{2}$	$\sqrt{38}$	$(2\sqrt{5})^2+y^2=\sqrt{38}^2$ $20+y^2=38$ $y^2=18$

→ ≅ sides　　→ ⊥ bis

3 Find the perimeter of a rhombus with diagonals 12 km and 16 km.



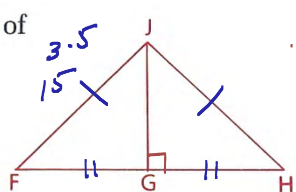
$2(3,4,5)$　　$4(10)=40$

$6^2+8^2=36+64=100$

5 Given: $\overline{JG}$ is the altitude to base $\overline{FH}$ of isosceles triangle JFH.
FJ = 15, FH = 24

Find: JG

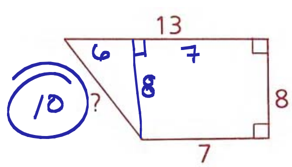
$3\cdot3=9$



7 Find the missing length in the trapezoid.

$(6,8,\_\_)$

$2(3,4,5)$



$AB^2 = AC^2 + CB^2$

$AB = \sqrt{AC^2 + CB^2}$

PEMDAS

$\sqrt[2]{x} = x^{\frac{1}{2}}$

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$\sqrt{x^2+y^2}$

let $x = 2$ & 3

$x + y \neq \sqrt{x^2+y^2}$

$2 + 3 \neq \sqrt{4+9}$

$5 \neq \sqrt{13}$

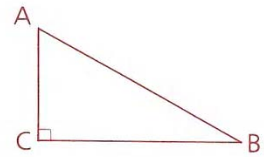
11 Find the missing length in terms of the variable(s) provided.

a AC = x, BC = y, AB = __?__ $\sqrt{x^2+y^2}$

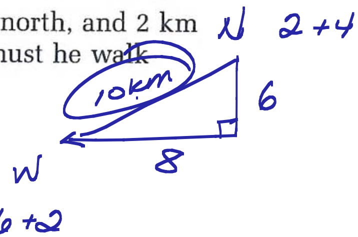
b AC = 2, BC = x, AB = __?__ $\sqrt{4+x^2}$

c AC = 3a, BC = 4a, AB = __?__ $5a$

d AB = 13c, AC = 5c, BC = __?__ $12c$

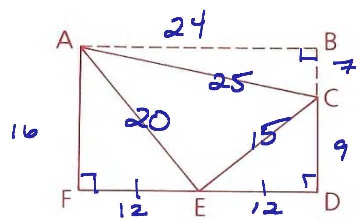


13 Al Capone walked 2 km north, 6 km west, 4 km north, and 2 km west. If Big Al decides to "go straight," how far must he walk across the fields to his starting point?

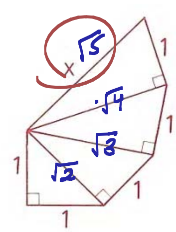


15 A piece broke off rectangle ABDF, leaving trapezoid ACDF. If BD = 16, BC = 7, FD = 24, and E is the midpoint of $\overline{FD}$, what is the perimeter of $\triangle ACE$?

60



17 Solve for x in the partial spiral to the right.



19 Woody Woodpecker pecked at a 17-m wooden pole until it cracked and the upper part fell, with the top hitting the ground 10 m from the foot of the pole. Since the upper part had not completely broken off, Woody pecked away where the pole had cracked. How far was Woody above the ground?

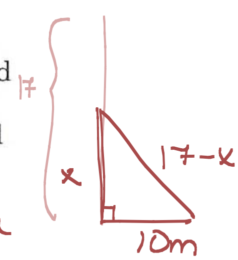
$10^2 + x^2 = (17-x)^2$

$100 + x^2 = 289 - 34x + x^2$

$+34x \; -100 \; -x^2 \quad -100 \; +34x \; -x^2$

$34x = 189$

$x = \frac{189}{34} = 5\frac{19}{34}$ m



21 The lengths of the diagonals of a rhombus are in the ratio 2:1. If the perimeter of the rhombus is 20, find the sum of the lengths of the diagonals.

sum of diags $= 6x$

$\boxed{6\sqrt{5}}$

$P = 20$

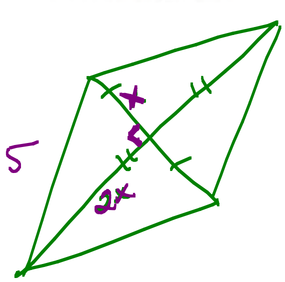
$s = \frac{20}{4} = 5$

$x^2 + (2x)^2 = 5^2$

$5x^2 = 25$

$x^2 = 5$

$x = \sqrt{5}$



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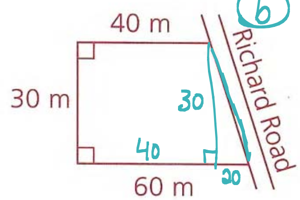
23 George and Diane bought a plot of land along Richard Road with the dimensions shown.

a Find the area of the plot.

b Find, to the nearest meter, the length of frontage on Richard Road.



$10(2, 3, \sqrt{13})$

4 ± 9

$10\sqrt{13}$

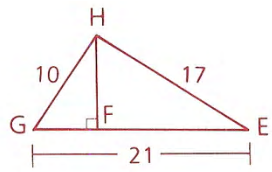
23a $A_{TRAP} = (\text{AVE BASES})\ \text{HEIGHT}$

$\left(\frac{40+60}{2}\right) \cdot 30$

$50 \cdot 30 = 1500\ m^2$

25 **a** Find HF. 7

b Is △EHF similar to △HGF?



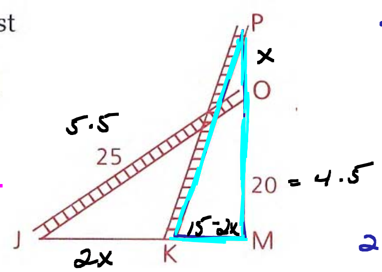
$10^2 + 17^2 \quad \square \quad 21^2$

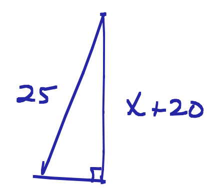
$100 + 289 \quad \square \quad 441$

$389 < 441$

Not Rt △ ∴ Not ~ △

27 A ladder 25 ft long (JO) is leaning against a wall, reaching a point 20 ft above the ground (MO). The ladder is then moved so that JK = 2(PO). Find KM.





$(x+20)(x+20)$

$x^2 + 20x + 20x + 400$

$x^2 + 40x + 400$

$(15-2x)(15-2x)$

$225 - 30x - 30x + 4x^2$

$225 - 60x + 4x^2$

$25^2 = (15-2x)^2 + (x+20)^2$

$625 = 225 - 60x + 4x^2 + x^2 + 40x + 400$

$-625 \quad -625$

$0 = 5x^2 - 20x$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x = 0$ No sense

If $x = 4$ then

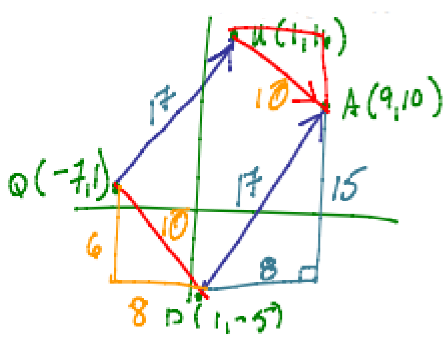
$KM = 15 - 2(4) = 7\text{ ft}$

31 Quadrilateral QUAD has vertices at Q = (−7, 1), U = (1, 16), A = (9, 10), and D = (1, −5).

a Plot the figure and indicate what type of quadrilateral QUAD is.

b Find the perimeter of QUAD.

(Hint: Use the properties of quadrilaterals that you learned in chapter 5.)



$m\,UA = \frac{\Delta y}{\Delta x} = \frac{16-10}{1-9} = \frac{6}{-8} = -\frac{3}{4}$

$m\,QD = \frac{\Delta y}{\Delta x} = \frac{1+5}{-7-1} = \frac{6}{-8} = -\frac{3}{4}$

∥

$m\,QU = \frac{\Delta y}{\Delta x} = \frac{16-1}{1+7} = \frac{15}{8}$

$m\,DA = \frac{\Delta y}{\Delta x} = \frac{-5-10}{1-9} = \frac{-15}{-8} = \frac{15}{8}$

∥

NOT OPP RECIP

∴ ▱

$P = \underbrace{10+10}_{20} + \underbrace{17+17}_{34}$

$= 54$