AMDG

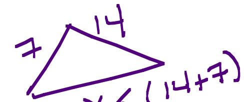
Name **_9.4: The Pythagorean Theorem,_** Ms. Kresovic

Adv Geo – **_Geometry's Most Elegant Theorem_** Date

Objective: After studying this section, you will be able to apply the Pythagorean Theorem and its converse.

Prior knowledge:

- Triangle Inequality Theorem (chapter 1): The third side of a triangle must be
  - o Smaller than the sum of the other two sides, and
  - o Larger than the difference.
- Used the Pythagorean Theorem before.



$(14-7) < x < (14+7)$

$7 < x < 21$

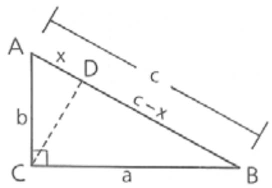
Theorem 69 ***The square of the measure of the hypotenuse of a right triangle is equal to the sum of the squares of the measures of the legs. (Pythagorean Theorem)***

In rt Δ, $sm^2 + md^2 = large^2$

Given: $\triangle ACB$ is a right $\triangle$ with right $\angle ACB$.

Prove: $a^2 + b^2 = c^2$

Proof:



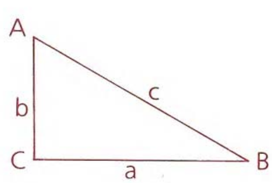
1 $\angle ACB$ is a right $\angle$.	1 Given
2 Draw $\overline{CD} \perp$ to $\overline{AB}$.	2 From a point outside a line, only one $\perp$ can be drawn to the line.
3 $\overline{CD}$ is an altitude.	3 A segment drawn from a vertex of a $\triangle$ $\perp$ to the opposite side is an altitude.
4 $a^2 = (c - x)c$	4 In a right $\triangle$ with an altitude drawn to the hypotenuse, $(\text{leg})^2 = (\text{adjacent seg.})(\text{hypot.})$.
5 $a^2 = c^2 - cx$	5 Distributive Property
6 $b^2 = xc$	6 Same as 4
7 $a^2 + b^2 = c^2 - cx + cx$	7 Addition Property
8 $a^2 + b^2 = c^2$	8 Algebra

Theorem 70 ***If the square of the measure of one side of a triangle equals the sum of the squares of the measures of the other two sides, then the angle opposite the longest side is a right angle.***

$sm^2 + md^2 = large^2 \Rightarrow$ rt Δ

"OR" $leg^2 + leg^2 = hyp^2$

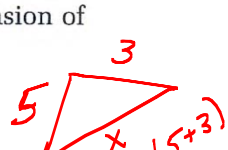
If $a^2 + b^2 = c^2$, then $\triangle ACB$ is a right $\triangle$ and $\angle C$ is the right $\angle$.



If, in the diagram above, we increased c while keeping a and b the same, $\angle C$ would become larger. Try it. Thus, a valuable extension of Theorem 70 can be stated:

If c is the length of the longest side of a triangle, and

- * $a^2 + b^2 > \boxed{c^2}$, then the triangle is acute
- * $a^2 + b^2 = c^2$, then the triangle is right
- * $a^2 + b^2 < c^2$, then the triangle is obtuse



$(5-3) < x < (5+3)$

$2 < x < 8$

If rt Δ

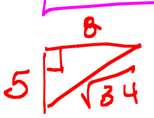
$3^2 + 5^2 = x^2$

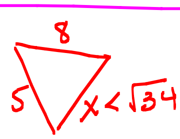
$9 + 25 = x^2$

$34 = x^2$

$\sqrt{34} = x$

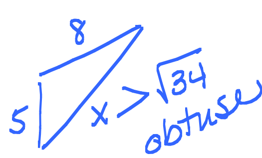
$5.8 \approx x$





$\rightarrow$ Acute

$c^2 < a^2 + b^2$

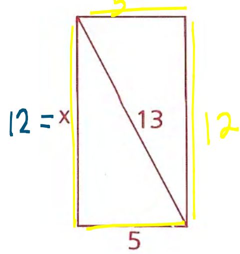


$c^2 > a^2 + b^2$

Class Examples

Problem 2 *Find the perimeter of the rectangle shown.*

sum sides $= 2(5+12) = \boxed{34}$



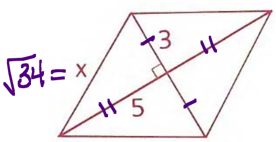
$5^2 + x^2 = 13^2$
$x^2 = 169 - 25$
$x^2 = 144$
$x = 12$

Why not $\pm$? Not going to have a negative length!!

All whole numbers!! ⇒ Pyth. Triple
(5, 12, 13)
Interesting

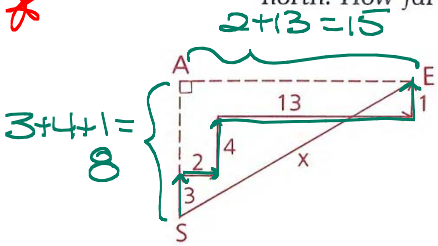
Problem 3 *Find the perimeter of a rhombus with diagonals of 6 and 10.*

sum sides ↳ 4 ≅ sds ↳ ⊥ bis



$3^2 + 5^2 = x^2$
$9 + 25 = x^2$
$34 = x^2 \rightarrow x = \sqrt{34}$ then Perimeter $= 4x = \boxed{4\sqrt{34}}$

Problem 4 Nadia skipped 3 m north, 2 m east, 4 m north, 13 m east, and 1 m north. How far is Nadia from where she started?

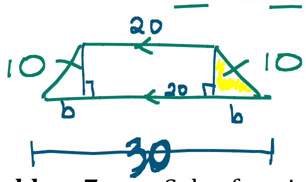


$8^2 + 15^2 = \text{distance}^2$
$64 + 225 = \text{distance}^2$
$289 = \text{distance}^2$
$17\text{m} = \text{distance}$

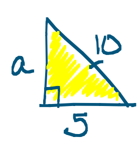
Another Triple: WOW
(8, 15, 17)

Problem 5 Find the altitude of an isosceles trapezoid whose sides have lengths of 10, 30, 10, and 20.

→ ⊥ → 2 ≅ sds → bases ∥

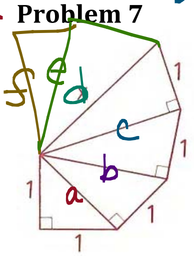


$b = \frac{30-20}{2}$



$a^2 + 5^2 = 10^2$
$a^2 = 100 - 25 = 75$
$a^2 = 25 \cdot 3$
$a = 5\sqrt{3}$

Problem 7 Solve for *x* in the partial spiral.



$1^2 + 1^2 = a^2$
$2 = a^2$
$\sqrt{2} = a$

$a^2 + 1^2 = b^2$
$2 + 1 = b^2$
$\sqrt{3} = b$

$b^2 + 1^2 = c^2$
$3 + 1 = c^2$
$\sqrt{4} = c$
$2 = c$

$c^2 + 1^2 = d^2$
$4 + 1 = d^2$
$\sqrt{5} = d$

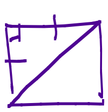
$d^2 + 1^2 = e^2$
$\sqrt{6} = e$

$\sqrt{7} = f$

2 Find the length of the diagonal of a square with perimeter 12 cm.

↳ 4 ≅ sds & 4 rt ∠s

$P = 12$
$s = \frac{12}{4} = 3\text{cm}$



$3^2 + 3^2 = d^2$
$9 + 9 = d^2$
$9 \cdot 2 = d^2$
$3\sqrt{2} = d$

$(3, 3, d)$
→ FACTOR THE SCALAR $3(1, 1, x)$
$1^2 + 1^2 = x^2$
$\sqrt{2} = x$
BUT $d = 3x \rightarrow$ SO $d = 3\sqrt{2}$

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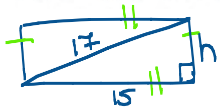
$\to$ 4sds, 4 rt∠s

4 Find the $\boxed{\text{perimeter}}$ of a rectangle whose diagonal is 17 mm long and whose base is 15 mm long.



$h^2 + 15^2 = 17^2$

$h^2 = 289 - 225$

$h^2 = 64$

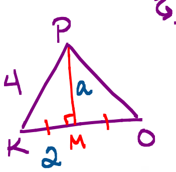
$h = 8$

(8, 15, 17) A TRIPLE !!

$P = 2(l+w) = 2(23) = \boxed{46}$

6 $\overline{PM}$ is an altitude of equilateral triangle PKO. If PK = 4, find PM.

⊥ & median



$2^2 + a^2 = 4^2$

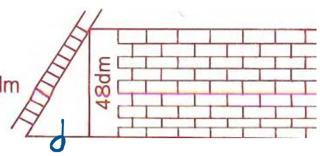
$a^2 = 16 - 4$

$a^2 = 12$

$a^2 = 4 \cdot 3$

$\to a = 2\sqrt{3}$

8 How far is the foot of the ladder from the wall?



(d, 48, 50)

$2\underbrace{(x, 24, 25)}$

$x^2 + 576 = 625$

$x^2 = 49$

$x = 7 \longrightarrow d = 2x$ so $d = 14$dm

9 $\overline{AC} \parallel$ y-axis and $\overline{CB} \parallel$ x-axis.

a Find the coordinates of C. $(x_A, y_B) : (2, 3)$

b Find AC and CB. 8 & 6 respectively

c Find AB.

d Is AB $= \sqrt{(8-2)^2 + (11-3)^2}$?

$\sqrt{6^2 + 8^2}$

$\sqrt{36+64}$

$\sqrt{100}$

10

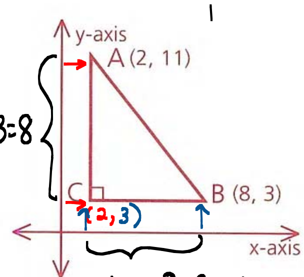
"distance formula"

$6^2 + 8^2 = AB^2$

$36 + 64 = AB^2$

$100 = AB^2$

$10 = AB$



$\Delta Y = 11 - 3 = 8$

$\Delta X = 8 - 2 = 6$

$2(3,4,5) \Rightarrow 6, 8, 10$

TRIPLES

(5, 12, 13)

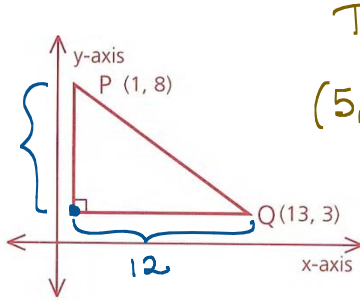
10 Use the method suggested by part **d** of problem 9 to find PQ.

$\sqrt{5^2 + 12^2}$

$\sqrt{25 + 144}$

$\sqrt{169}$

13



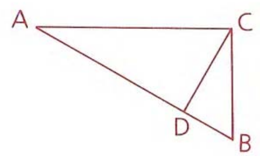
12 $\angle ACB$ is a right angle and $\overline{CD} \perp \overline{AB}$.

a If AD = 7 and BD = 4, find CD. $2\sqrt{7}$

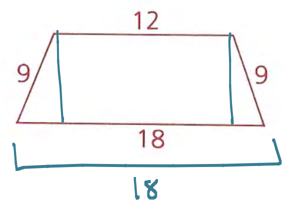
b If CD = 8 and DB = 6, find CB. 10

c If BC = 8 and BD = 2, find AB. 32

d If AC = 21 and AB = 29, find CB. 20

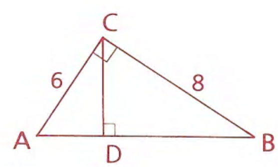


14 Find the altitude (length of a segment perpendicular to both bases) of the isosceles trapezoid shown.

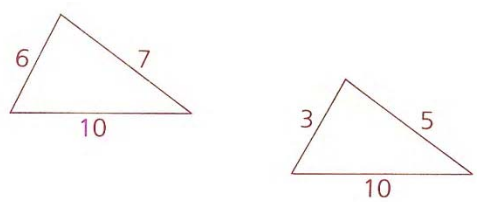


16 Given: Diagram as shown

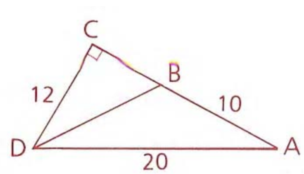
Find: CD



22 Classify the triangles.



24 Find the perimeter of $\triangle DBC$.



26 The perimeter of an isosceles triangle is 32, and the length of the altitude to its base is 8. Find the length of a leg.

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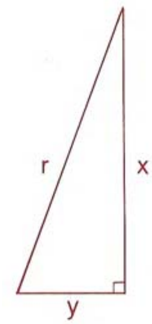
Homework

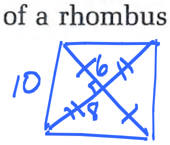
1. Solve for the third side. Let *x* & *y* be the legs of a right triangle, and *r* be the hypotenuse.



	x	*y*	*r*	work
a.	4	5	$\sqrt{41}$	
b.	15	8	17	$y^2+15^2=17^2$ $y^2+225=289$ $y^2=64$ $y=8$
c.	12	3·3 9	3·5 15	
d.	12	5	13	$12^2+y^2=13^2$ $y^2=169-144$
e.	5	$5\sqrt{3}$	10	
f.	5	2	$\sqrt{29}$	$5^2+y^2=\sqrt{29}^2 \rightarrow 25+y^2=29$ $y^2=4$
g.	$2\sqrt{5}$	$3\sqrt{2}$	$\sqrt{38}$	$(2\sqrt{5})^2+y^2=\sqrt{38}^2$ $20+y^2=38 \rightarrow y^2=18$ $y=3\sqrt{2}$

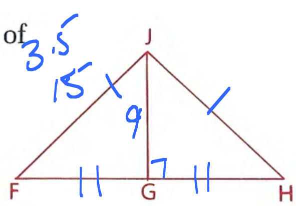
3 Find the perimeter of a rhombus with diagonals 12 km and 16 km.

→ ⊥ bis



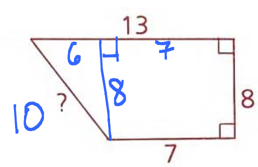
$4(10)=40$ km

5 Given: $\overline{JG}$ is the altitude to base $\overline{FH}$ of isosceles triangle JFH.
FJ = 15, FH = 24
Find: JG



9

7 Find the missing length in the trapezoid.

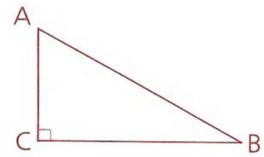


$2(3,4,5)$

$AC^2 + BC^2 = AB^2$

$\sqrt{AC^2 + BC^2} = AB$

11 Find the missing length in terms of the variable(s) provided.



a AC = x, BC = y, AB = ___?___ $\sqrt{x^2 + y^2}$

b AC = 2, BC = x, AB = ___?___ $\sqrt{4 + x^2}$

$a(3,4,5)$ **c** AC = 3a, BC = 4a, AB = ___?___ $\sqrt{9a^2+16a^2} = \sqrt{25a^2} = 5a$

d AB = 13c, AC = 5c, BC = ___?___ $12c$

$AC^2 + BC^2 = AB^2$

$(5c)^2 + BC^2 = (13c)^2$

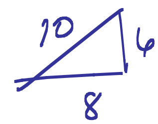
$c(5,12,13)$

13 Al Capone walked 2 km north, 6 km west, 4 km north, and 2 km west. If Big Al decides to "go straight," how far must he walk across the fields to his starting point?

N: $2 + 4 = 6$

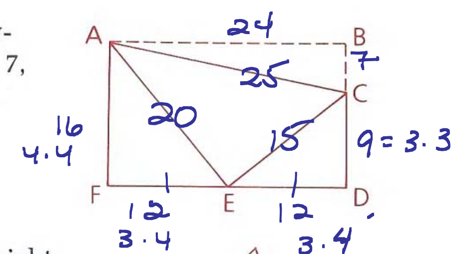
W: $6 + 2 = 8$

$2(3\ 4\ 5) \rightarrow$ 10 km

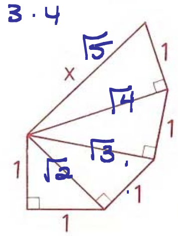


15 A piece broke off rectangle ABDF, leaving trapezoid ACDF. If BD = 16, BC = 7, FD = 24, and E is the midpoint of $\overline{FD}$, what is the perimeter of $\triangle ACE$?

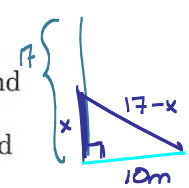
60



17 Solve for x in the partial spiral to the right.



19 Woody Woodpecker pecked at a 17-m wooden pole until it cracked and the upper part fell, with the top hitting the ground 10 m from the foot of the pole. Since the upper part had not completely broken off, Woody pecked away where the pole had cracked. How far was Woody above the ground?



$10^2 + x^2 = (17-x)^2$

$100 + x^2 = 289 - 34x + x^2$

$-289 \quad -x^2 \quad -289 \quad -x^2$

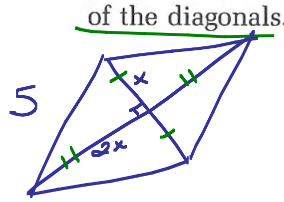
$-189 = -34x$

$\frac{189}{34} = x$

$5\frac{19}{34}$ m

21 The lengths of the diagonals of a rhombus are in the ratio 2:1. If the perimeter of the rhombus is 20, find the sum of the lengths of the diagonals.

$6x = 6\sqrt{5}$



$P = 20$

$S = 5$

$x^2 + (2x)^2 = 5^2$

$5x^2 = 25$

$x^2 = 5$

$x = \sqrt{5}$

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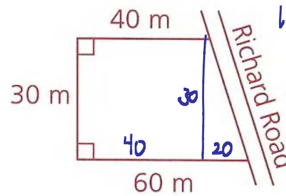
23 George and Diane bought a plot of land along Richard Road with the dimensions shown.

a Find the area of the plot.

b Find, to the nearest meter, the length of frontage on Richard Road.



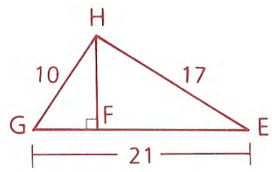
10 (2, 3, 4+9, 6 $10\sqrt{13}$

a A_{TRAP} = AVE BASES · HEIGHT

$$\left(\frac{40+60}{2}\right)(30) = 50 \cdot 30 = 1500\ m^2$$

25 **a** Find HF. 7

b Is △EHF similar to △HGF?



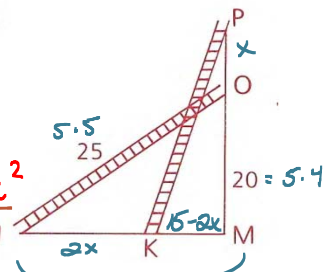
$10^2 + 17^2 \;\boxed{\phantom{<}}\; 21^2$

$100 + 289 \;\boxed{\phantom{<}}\; 441$

$389 \;\boxed{<}\; 441$

OBTUSE

27 A ladder 25 ft long (JO) is leaning against a wall, reaching a point 20 ft above the ground (MO). The ladder is then moved so that JK = 2(PO). Find KM.



$25^2 = (20+x)^2 + (15-2x)^2$

$625 = 400 + 40x + x^2 + 225 - 60x + 4x^2$

$625 = 625 - 20x + 5x^2$

$0 = -20x + 5x^2$

$0 = -4x + x^2 \rightarrow x(x-4) = 0$

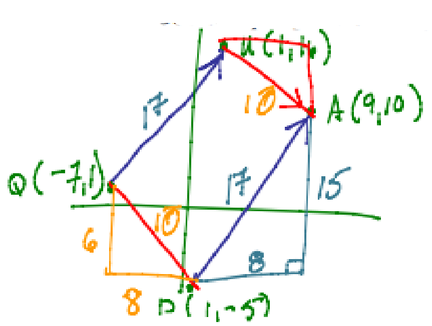
$\cancel{x=0}$ $x = 4$ Then $KM = 15 - 2(4) = 7\ ft$

31 Quadrilateral QUAD has vertices at Q = (−7, 1), U = (1, 16), A = (9, 10), and D = (1, −5).

a Plot the figure and indicate what type of quadrilateral QUAD is.

b Find the perimeter of QUAD.

(Hint: Use the properties of quadrilaterals that you learned in chapter 5.)



$m\,UA = \frac{\Delta Y}{\Delta X} = \frac{16-10}{1-9} = \frac{6}{-8} = -\frac{3}{4}$

$m\,QD = \frac{\Delta Y}{\Delta X} = \frac{1+5}{-7-1} = \frac{6}{-8} = -\frac{3}{4}$ ∥

$m\,QU = \frac{\Delta Y}{\Delta X} = \frac{16-1}{1+7} = \frac{15}{8}$

$m\,DA = \frac{\Delta Y}{\Delta X} = \frac{-5-10}{1-9} = \frac{-15}{-8} = \frac{15}{8}$ ∥

NOT OPP RECIP

∴ ▱

$P = \underbrace{10+10}_{20} + \underbrace{17+17}_{34} = 54$