AMDG

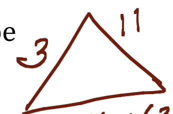
Name **9.4: The Pythagorean Theorem,** Ms. Kresovic

Adv Geo – ***Geometry's Most Elegant Theorem*** Date

Objective: After studying this section, you will be able to apply the Pythagorean Theorem and its converse.

Prior knowledge:

- Triangle Inequality Theorem (chapter 1): The third side of a triangle must be
  - Smaller than the sum of the other two sides, and
  - Larger than the difference.
- Used the Pythagorean Theorem before.



$(11-3) < x < (3+11)$

$8 < x < 14$

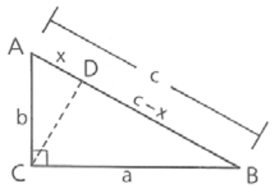
Theorem 69 ***The square of the measure of the hypotenuse of a right triangle is equal to the sum of the squares of the measures of the legs. (Pythagorean Theorem)***

If rt$\triangle$, then $leg^2 + leg^2 = hyp^2$

or $sm^2 + med^2 = long^2$

Given: $\triangle ACB$ is a right $\triangle$ with right $\angle ACB$.

Prove: $a^2 + b^2 = c^2$



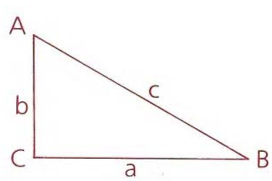
Proof:

1 $\angle ACB$ is a right $\angle$.	1 Given
2 Draw $\overline{CD} \perp$ to $\overline{AB}$.	2 From a point outside a line, only one $\perp$ can be drawn to the line.
3 $\overline{CD}$ is an altitude.	3 A segment drawn from a vertex of a $\triangle$ $\perp$ to the opposite side is an altitude.
4 $a^2 = (c - x)c$	4 In a right $\triangle$ with an altitude drawn to the hypotenuse, $(\text{leg})^2 = (\text{adjacent seg.})(\text{hypot.})$.
5 $a^2 = c^2 - cx$	5 Distributive Property
6 $b^2 = xc$	6 Same as 4
7 $a^2 + b^2 = c^2 - cx + cx$	7 Addition Property
8 $a^2 + b^2 = c^2$	8 Algebra

Theorem 70 ***If the square of the measure of one side of a triangle equals the sum of the squares of the measures of the other two sides, then the angle opposite the longest side is a right angle.***

If $sm^2 + med^2 = long^2$ then rt$\triangle$

If $a^2 + b^2 = c^2$, then $\triangle ACB$ is a right $\triangle$ and $\angle C$ is the right $\angle$.



If, in the diagram above, we increased c while keeping a and b the same, $\angle C$ would become larger. Try it. Thus, a valuable extension of Theorem 70 can be stated:

Classify given lengths $(3, 11, \sqrt{130})$

$3^2 + 11^2 = \sqrt{130}^2$

$9 + 121 = 130 \therefore$ Rt$\triangle$

If c is the length of the longest side of a triangle, and

- $a^2 + b^2 > c^2$, then the triangle is acute
- $a^2 + b^2 = c^2$, then the triangle is right
- $a^2 + b^2 < c^2$, then the triangle is obtuse

Given sides $(3, 11, 10) =$

$3^2 + 11^2 \;\; 10^2$

$130 > 100 \therefore$ Acute

Given lengths $(3, 11, 12)$

$3^2 + 11^2 \;\; 12^2$

$130 < 144 \therefore$

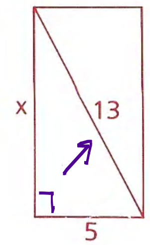
Obtuse

∵ ⇒ bc ∴ ⇒ therefore

Class Examples

Problem 2 Find the perimeter of the rectangle shown.

↳ sum sides ↳ 4 rt∠s → rt△



Pyth. Thm

$5^2 + x^2 = 13^2$

$25 + x^2 = 169$

$x^2 = 144$

$x = \boxed{12}$

Why not ±12?

∵ can not have a negative length

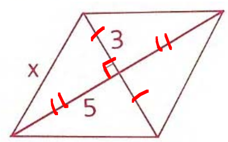
(5, 12, 13) → Interesting

ALL Whole Numbers?!

Problem 3 Find the perimeter of a rhombus with diagonals of 6 and 10.

↳ sum sides

↳ 4 ≅ sds ↳ ⊥ & bis



$3^2 + 5^2 = x^2$

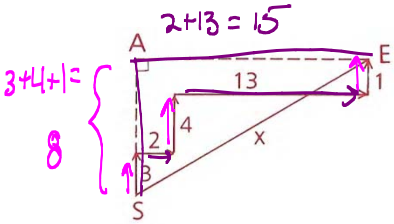
$9 + 25 = x^2$

$34 = x^2$

$\sqrt{34} = x$ the $P = 4x = \boxed{4\sqrt{34}}$

Problem 4 Nadia skipped 3 m north, 2 m east, 4 m north, 13 m east, and 1 m north. How far is Nadia from where she started?

★★



$8^2 + 15^2 = \text{Distance}^2$

$64 + 225 = \text{Distance}^2$

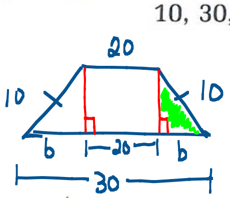
$289 = \text{Distance}^2$

$17 = \text{Distance}$

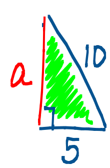
Interesting! (8, 15, 17) another Pythagorean Triple! WOW

Problem 5 Find the altitude of an isosceles trapezoid whose sides have lengths of 10, 30, 10, and 20.

↳ ⊥ → rt∠ ↳ 2 ≅ sds ↳ bases ∥



$b = \frac{30-20}{2}$



$a^2 + 5^2 = 10^2$

$a^2 = 100 - 25 = 75$

$a^2 = 25 \cdot 3$

$a = 5\sqrt{3}$

Problem 7 Solve for d in the partial spiral.

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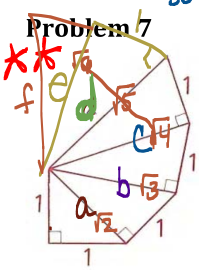


$1^2 + 1^2 = a^2$

$1 + 1 = a^2$

$2 = a^2$

$\sqrt{2} = a$

$1^2 + a^2 = b^2$

$1^2 + \sqrt{2}^2 = b^2$

$1 + 2 = b^2$

$3 = b^2$

$\sqrt{3} = b$

$b^2 + 1^2 = c^2$

$\sqrt{3}^2 + 1^2 = c^2$

$3 + 1 = c^2$

$\sqrt{4} = c$

$2 = c$

$c^2 + 1^2 = d^2$

$\sqrt{4}^2 + 1^2 = d^2$

$4 + 1 = d^2$

$\sqrt{5} = d$

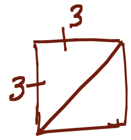
$e = \sqrt{6}$

$f = \sqrt{7}$

2 Find the length of the diagonal of a square with perimeter 12 cm.

↳ 4 ≅ sds & 4 rt∠s

$P = 12 \Rightarrow s = \frac{12}{4} = 3$



$3^2 + 3^2 = \text{diag}^2$

$9 + 9 = \text{diag}^2$

$9 \cdot 2 = \text{diag}^2$

$3\sqrt{2} = \text{diag}$

factor a scalar:

(3, 3, d) ← longest leg

3 (1, 1, x)

$1^2 + 1^2 = x^2$

$2 = x^2 \rightarrow x = \sqrt{2}$ But $3x = d$ so $3\sqrt{2} = d$

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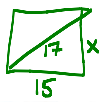
4 Find the perimeter of a rectangle whose diagonal is 17 mm long and whose base is 15 mm long.

$(8, 15, 17)$



$x^2 + 15^2 = 17^2$

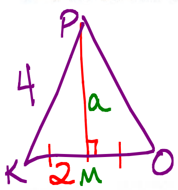
$x = 8$ → Then $p = 2l + 2w$

$2(l+w)$

$2(23) = \boxed{46}$

6 $\overline{PM}$ is an altitude of equilateral triangle PKO. If PK = 4, find PM.

also median



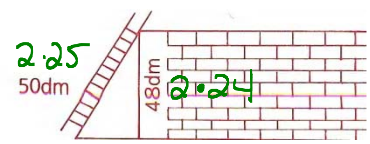
$2^2 + a^2 = 4^2$

$a^2 = 16 - 4$

$a^2 = 12 = 4 \cdot 3$ → $a\ 2\sqrt{3}$

$(x, x\sqrt{3}, 2x)$

8 How far is the foot of the ladder from the wall?



$(d, 48, 50) \Rightarrow 2(x, 24, 25)$

$x^2 + 576 = 625$

$x^2 = 49$

$x = 7$

then $d = 2x = 14$ dm

$2\underbrace{(7, 24, 25)}_{\text{another triple!!!}}$

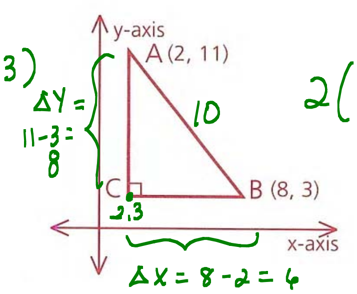
9 $\overline{AC} \parallel$ y-axis and $\overline{CB} \parallel$ x-axis.

a Find the coordinates of C. $(x_A, y_B) : (2, 3)$

b Find AC and CB. 8 & 6

c Find AB. $= 10$

d Is AB $= \sqrt{(8-2)^2 + (11-3)^2}$? Yes!



$2(3, 4, 5)$

$\Delta x^2 + \Delta y^2$

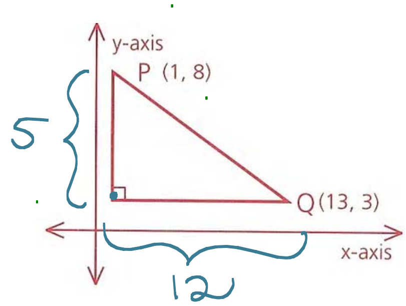
$\sqrt{6^2 + 8^2}$

$\sqrt{36 + 64}$

$\sqrt{100}$

10

10 Use the method suggested by part **d** of problem 9 to find PQ.



$\sqrt{\Delta x^2 + \Delta y^2}$

$\sqrt{5^2 + 12^2}$

$\sqrt{25 + 144}$

13

Review of Alt Hyp

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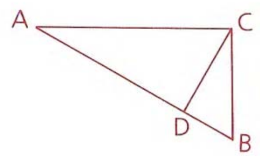
12 ∠ACB is a right angle and $\overline{CD} \perp \overline{AB}$.



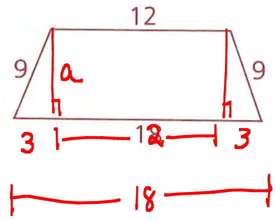
a If AD = 7 and BD = 4, find CD.

b If CD = 8 and DB = 6, find CB.

c If BC = 8 and BD = 2, find AB.

d If AC = 21 and AB = 29, find CB.

14 Find the altitude (length of a segment perpendicular to both bases) of the isosceles trapezoid shown.



$3^2 + a^2 = 9^2$

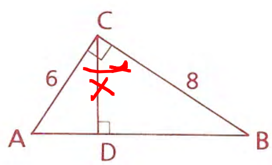
$a^2 = 81 - 9 = 72$

$a^2 = 9 \cdot 4 \cdot 2$

$a = 6\sqrt{2}$

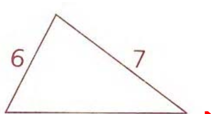
16 Given: Diagram as shown

Find: CD



Review

22 Classify the triangles.



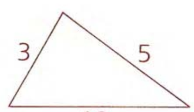
$(7-6) < 10 < (7+6)$

$1 < 10 < 13$

$6^2 + 7^2 \quad 10^2$

$36 + 49 \quad 100$

$85 < 100 \therefore$ obtuse

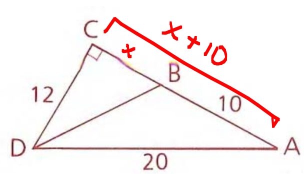


$(5-3) < 10 < (5+3)$

$2 < 10 \not< 8$

Impossible

24 Find the perimeter of △DBC.



156

3 52

2 26

$12^2 + (x+10)^2 = 20^2$

$144 + x^2 + 20x + 100 = 400$

$x^2 + 20x + 244 = 400$

$x^2 + 20x - 156 = 0$

$(x+26)(x-6) = 0$

no neg. lengths → $\boxed{6}$

26 The perimeter of an isosceles triangle is 32, and the length of the altitude to its base is 8. Find the length of a leg.

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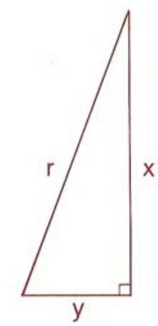
Homework

1. Solve for the third side. Let *x* & *y* be the legs of a right triangle, and *r* be the hypotenuse.

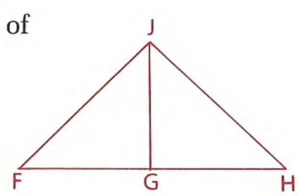


	x	y	r	work
a.	4	5		
b.	15		17	
c.		9	15	
d.	12		13	
e.	5	$5\sqrt{3}$		
f.	5		$\sqrt{29}$	
g.	$2\sqrt{5}$		$\sqrt{38}$	

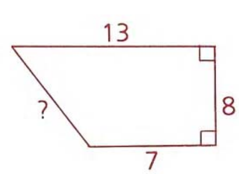
3 Find the perimeter of a rhombus with diagonals 12 km and 16 km.

5 Given: $\overline{JG}$ is the altitude to base $\overline{FH}$ of isosceles triangle JFH.
FJ = 15, FH = 24

Find: JG



7 Find the missing length in the trapezoid.



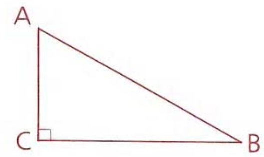
11 Find the missing length in terms of the variable(s) provided.

a AC = x, BC = y, AB = ___?___

b AC = 2, BC = x, AB = ___?___

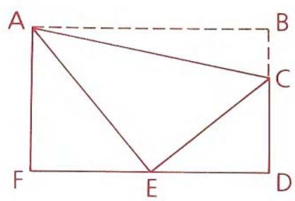
c AC = $3a$, BC = $4a$, AB = ___?___

d AB = $13c$, AC = $5c$, BC = ___?___

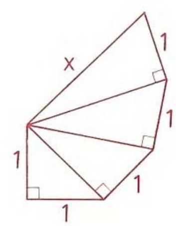


13 Al Capone walked 2 km north, 6 km west, 4 km north, and 2 km west. If Big Al decides to "go straight," how far must he walk across the fields to his starting point?

15 A piece broke off rectangle ABDF, leaving trapezoid ACDF. If BD = 16, BC = 7, FD = 24, and E is the midpoint of $\overline{FD}$, what is the perimeter of $\triangle ACE$?



17 Solve for x in the partial spiral to the right.



19 Woody Woodpecker pecked at a 17-m wooden pole until it cracked and the upper part fell, with the top hitting the ground 10 m from the foot of the pole. Since the upper part had not completely broken off, Woody pecked away where the pole had cracked. How far was Woody above the ground?

21 The lengths of the diagonals of a rhombus are in the ratio 2:1. If the perimeter of the rhombus is 20, find the sum of the lengths of the diagonals.

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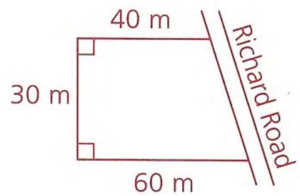
23 George and Diane bought a plot of land along Richard Road with the dimensions shown.

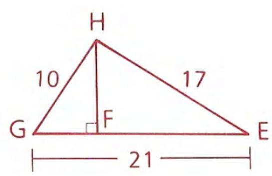
a Find the area of the plot.

b Find, to the nearest meter, the length of frontage on Richard Road.

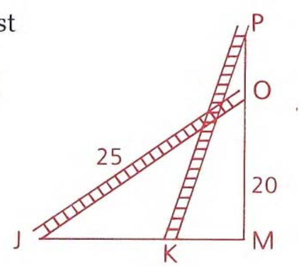


25 **a** Find HF.

b Is $\triangle EHF$ similar to $\triangle HGF$?



27 A ladder 25 ft long (JO) is leaning against a wall, reaching a point 20 ft above the ground (MO). The ladder is then moved so that JK = 2(PO). Find KM.



31 Quadrilateral QUAD has vertices at Q = (−7, 1), U = (1, 16), A = (9, 10), and D = (1, −5).

a Plot the figure and indicate what type of quadrilateral QUAD is.

b Find the perimeter of QUAD.

(Hint: Use the properties of quadrilaterals that you learned in chapter 5.)