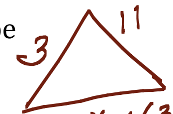
Name | *9.4: The Pythagorean Theorem,* | Ms. Kresovic

Adv Geo – | *Geometry's Most Elegant Theorem* | Date

Objective: After studying this section, you will be able to apply the Pythagorean Theorem and its converse.

Prior knowledge:

- Triangle Inequality Theorem (chapter 1): The third side of a triangle must be
  - Smaller than the sum of the other two sides, and
  - Larger than the difference.
- Used the Pythagorean Theorem before.



$(11-3) < x < (3+11)$

$8 < x < 14$

Theorem 69 ***The square of the measure of the hypotenuse of a right triangle is equal to the sum of the squares of the measures of the legs. (Pythagorean Theorem)***

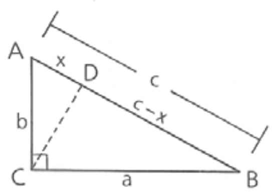
If rtΔ, then $leg^2 + leg^2 = hyp^2$

or $sm^2 + med^2 = long^2$

Given: $\triangle ACB$ is a right $\triangle$ with right $\angle ACB$.

Prove: $a^2 + b^2 = c^2$

Proof:

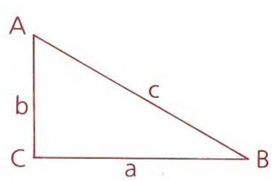


1 $\angle ACB$ is a right $\angle$.	1 Given
2 Draw $\overline{CD} \perp$ to $\overline{AB}$.	2 From a point outside a line, only one $\perp$ can be drawn to the line.
3 $\overline{CD}$ is an altitude.	3 A segment drawn from a vertex of a $\triangle$ $\perp$ to the opposite side is an altitude.
4 $a^2 = (c - x)c$	4 In a right $\triangle$ with an altitude drawn to the hypotenuse, $(\text{leg})^2 = (\text{adjacent seg.})(\text{hypot.})$.
5 $a^2 = c^2 - cx$	5 Distributive Property
6 $b^2 = xc$	6 Same as 4
7 $a^2 + b^2 = c^2 - cx + cx$	7 Addition Property
8 $a^2 + b^2 = c^2$	8 Algebra

Theorem 70 ***If the square of the measure of one side of a triangle equals the sum of the squares of the measures of the other two sides, then the angle opposite the longest side is a right angle.***

If $sm^2 + med^2 = long^2$ then rtΔ

If $a^2 + b^2 = c^2$, then $\triangle ACB$ is a right $\triangle$ and $\angle C$ is the right $\angle$.



If, in the diagram above, we increased c while keeping a and b the same, $\angle C$ would become larger. Try it. Thus, a valuable extension of Theorem 70 can be stated:

If c is the length of the longest side of a triangle, and

- $a^2 + b^2 > c^2$, then the triangle is acute
- $a^2 + b^2 = c^2$, then the triangle is right
- $a^2 + b^2 < c^2$, then the triangle is obtuse

Classify given lengths $(3, 11, \sqrt{130})$

$3^2 + 11^2 = \sqrt{130}^2$

$9 + 121 = 130 \therefore$ RtΔ

Given sides $(3, 11, 10) =$

$3^2 + 11^2 \; 10^2$

$130 > 100 \therefore$ Acute

Given lengths $(3, 11, 12)$

$3^2 + 11^2 \; 12^2$

$130 < 144 \therefore$

Obtuse

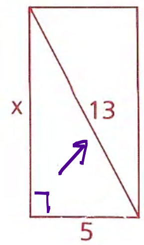
$\because \Rightarrow$ bc $\therefore \Rightarrow$ therefore

Class Examples

Problem 2 Find the perimeter of the rectangle shown.

→ sum sides → rt∠s → rt△

Pyth. Thm



$5^2+x^2=13^2$

$25+x^2=169$

$x^2=144$

$x=\boxed{12}$

Why not ± 12?

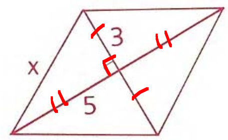
$\because$ can not have a negative length

(5, 12, 13) → Interesting
ALL Whole Numbers?!

→ sum sides

Problem 3 Find the perimeter of a rhombus with diagonals of 6 and 10.

↳ 4 ≅ sds ↳ ⊥ & bis



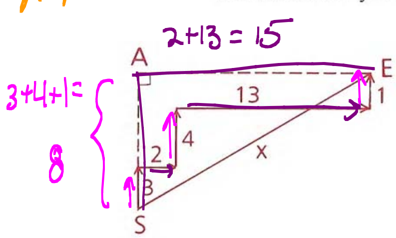
$3^2+5^2=x^2$

$9+25=x^2$

$34=x^2$

$\sqrt{34}=x$ the $P=4x=\boxed{4\sqrt{34}}$

Problem 4 Nadia skipped 3 m north, 2 m east, 4 m north, 13 m east, and 1 m north. How far is Nadia from where she started?



$8^2+15^2=\text{Distance}^2$

$64+225=\text{Distance}^2$

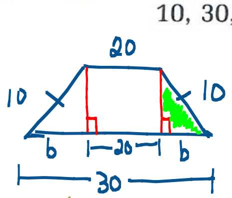
$289=\text{Distance}^2$

$17=\text{Distance}$

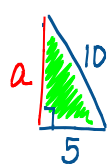
Interesting: (8, 15, 17) another Pythagorean Triple! WOW

→ ⊥ → rt∠ → 2 ≅ sds → bases ∥

Problem 5 Find the altitude of an isosceles trapezoid whose sides have lengths of 10, 30, 10, and 20.



$b=\frac{30-20}{2}$



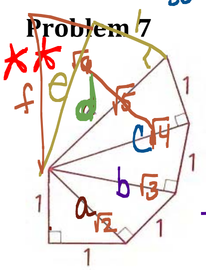
$a^2+5^2=10^2$

$a^2=100-25=75$

$a^2=25\cdot 3$

$a=5\sqrt{3}$

Problem 7 Solve for d in the partial spiral.



$1^2+1^2=a^2$

$1+1=a^2$

$2=a^2$

$\sqrt{2}=a$

$1^2+a^2=b^2$

$1^2+\sqrt{2}^2=b^2$

$1+2=b^2$

$3=b^2$

$\sqrt{3}=b$

$b^2+1^2=c^2$

$\sqrt{3}^2+1^2=c^2$

$3+1=c^2$

$\sqrt{4}=c$

$2=c$

$c^2+1^2=d^2$

$\sqrt{4}^2+1^2=d^2$

$4+1=d^2$

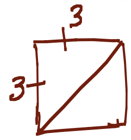
$\sqrt{5}=d$

$e=\sqrt{6}$

$f=\sqrt{7}$

2 Find the length of the diagonal of a square with perimeter 12 cm.

$P=12 \Rightarrow s=\frac{12}{4}=3$



↳ 4 ≅ sds & 4 rt∠s

$3^2+3^2=\text{diag}^2$

$9+9=\text{diag}^2$

$9\cdot 2=\text{diag}^2$

$3\sqrt{2}=\text{diag}$

factor a scalar:

(3, 3, d) ← longest leg

3 (1, 1, x)

$1^2+1^2=x^2$

$2=x^2 \rightarrow x=\sqrt{2}$ But $3x=d$ so $3\sqrt{2}=d$

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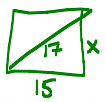
$(8, 15, 17)$

/

4 Find the perimeter of a rectangle whose diagonal is 17 mm long and whose base is 15 mm long.



$x^2 + 15^2 = 17^2$

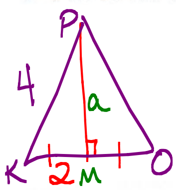
$x = 8$ → Then $p = 2l + 2w$

$2(l + w)$

$2(23) = \boxed{46}$

6 $\overline{PM}$ is an altitude of equilateral triangle PKO. If PK = 4, find PM.

also median



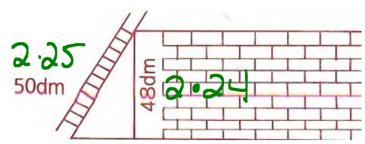
$2^2 + a^2 = 4^2$

$a^2 = 16 - 4$

$a^2 = 12 = 4 \cdot 3$ → $a\ 2\sqrt{3}$

$(x, x\sqrt{3}, 2x)$

8 How far is the foot of the ladder from the wall?



$(d, 48, 50) \Rightarrow 2(x, 24, 25)$

$x^2 + 576 = 625$

$x^2 = 49$

$x = 7$

then $d = 2x = 14$ dm

$2(\underbrace{7, 24, 25})$ another triple!!!

9 $\overline{AC} \parallel$ y-axis and $\overline{CB} \parallel$ x-axis.

a Find the coordinates of C. $(x_A, y_B) : (2, 3)$

b Find AC and CB. 8 & 6

c Find AB. = 10

d Is AB = $\sqrt{(8-2)^2 + (11-3)^2}$? yes!

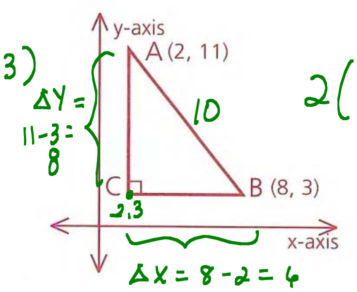
$\sqrt{\Delta x^2 + \Delta y^2}$

$\sqrt{6^2 + 8^2}$

$\sqrt{36 + 64}$

$\sqrt{100}$

10



$2(3, 4, 5)$

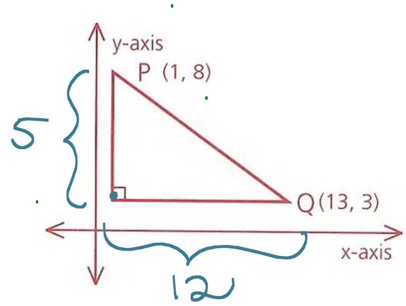
10 Use the method suggested by part **d** of problem 9 to find PQ.

$\sqrt{\Delta x^2 + \Delta y^2}$

$\sqrt{5^2 + 12^2}$

$\sqrt{25 + 144}$

13



Review of Alt Hyp

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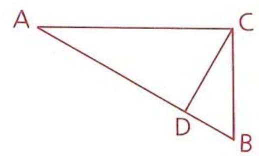
12 ∠ACB is a right angle and $\overline{CD} \perp \overline{AB}$.

a If AD = 7 and BD = 4, find CD.

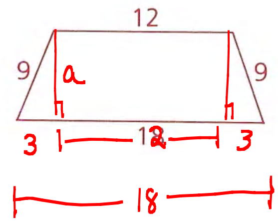
b If CD = 8 and DB = 6, find CB.

c If BC = 8 and BD = 2, find AB.

d If AC = 21 and AB = 29, find CB.



14 Find the altitude (length of a segment perpendicular to both bases) of the isosceles trapezoid shown.



$3^2 + a^2 = 9^2$

$a^2 = 81 - 9 = 72$

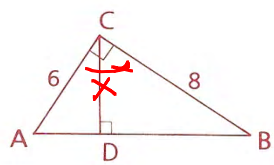
$a^2 = 9 \cdot 4 \cdot 2$

$a = 6\sqrt{2}$

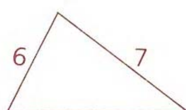
16 Given: Diagram as shown

Find: CD

Review



22 Classify the triangles.



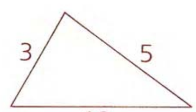
$(7-6) < 10 < (7+6)$

$1 < 10 < 13$

$6^2 + 7^2 \quad 10^2$

$36 + 49 \quad 100$

$85 < 100 \therefore$ obtuse

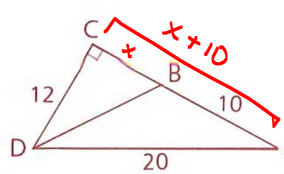


$(5-3) < 10 < (5+3)$

$2 < 10 \not< 8$

Impossible

24 Find the perimeter of △DBC.



156

3 52

2 26

$12^2 + (x+10)^2 = 20^2$

$144 + x^2 + 20x + 100 = 400$

$x^2 + 20x + 244 = 400$

$x^2 + 20x - 156 = 0$

$(x+26)(x-6) = 0$

no neg. lengths → $\boxed{6}$

26 The perimeter of an isosceles triangle is 32, and the length of the altitude to its base is 8. Find the length of a leg.

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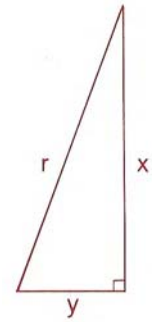
Homework

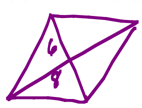
1. Solve for the third side. Let x & y be the legs of a right triangle, and r be the hypotenuse.



	x	y	r	work
a.	4	5	$\sqrt{41}$	
b.	15	8	17	
c.	12	9	15	
d.	12	5	13	
e.	5	$5\sqrt{3}$	10	
f.	5	2	$\sqrt{29}$	
g.	$2\sqrt{5}$	$3\sqrt{2}$	$\sqrt{38}$	$(2\sqrt{5})^2 + y^2 = \sqrt{38}^2$ $20 + y^2 = 38 \rightarrow y^2 = 18$ $y = 3\sqrt{2}$

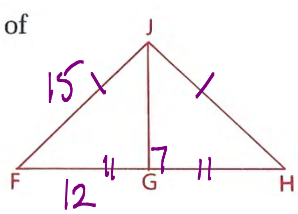
→ ⊥bis

3 Find the perimeter of a rhombus with diagonals 12 km and 16 km.



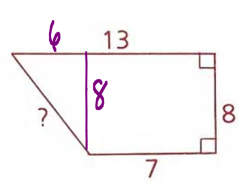
40km

5 Given: $\overline{JG}$ is the altitude to base $\overline{FH}$ of isosceles triangle JFH.
FJ = 15, FH = 24
Find: JG



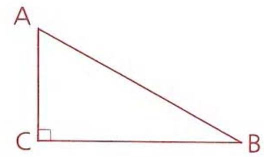
9

7 Find the missing length in the trapezoid.



10

11 Find the missing length in terms of the variable(s) provided.



a AC = x, BC = y, AB = __?__ $x^2+y^2=AB^2 \rightarrow AB=\sqrt{x^2+y^2}$

b AC = 2, BC = x, AB = __?__ $AB^2=4+x^2 \rightarrow AB\ \sqrt{4+x^2}$

c AC = 3a, BC = 4a, AB = __?__ 5a

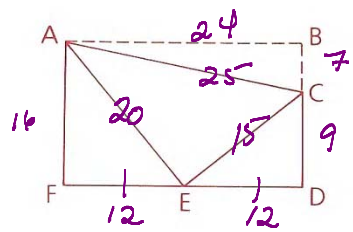
d AB = 13c, AC = 5c, BC = __?__ 12c

13 Al Capone walked 2 km north, 6 km west, 4 km north, and 2 km west. If Big Al decides to "go straight," how far must he walk across the fields to his starting point?

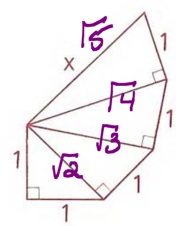
10km

15 A piece broke off rectangle ABDF, leaving trapezoid ACDF. If BD = 16, BC = 7, FD = 24, and E is the midpoint of $\overline{FD}$, what is the perimeter of $\triangle ACE$?

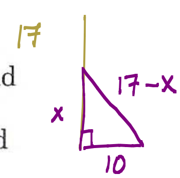
60



17 Solve for x in the partial spiral to the right.



19 Woody Woodpecker pecked at a 17-m wooden pole until it cracked and the upper part fell, with the top hitting the ground 10 m from the foot of the pole. Since the upper part had not completely broken off, Woody pecked away where the pole had cracked. How far was Woody above the ground?



$10^2+x^2=(17-x)^2$

$100+x^2=289-34x+x^2$

$34x\ -100\quad -x^2\quad -100\quad +34x\quad -x^2$

$34x=189$

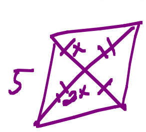
$x=\frac{189}{34}=5\frac{19}{34}$ m

→ this

21 The lengths of the diagonals of a rhombus are in the ratio 2:1. If the perimeter of the rhombus is 20, find the sum of the lengths of the diagonals.

$P=20$

$S=\frac{20}{4}=5$



$x^2+(2x)^2=5^2$

$x^2+4x^2=25$

$5x^2=25$

$x^2=5$

$x=\sqrt{5}$

Sum of diago = 6x

$6\sqrt{5}$

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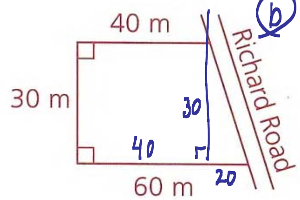
23 George and Diane bought a plot of land along Richard Road with the dimensions shown.

a Find the area of the plot.

b Find, to the nearest meter, the length of frontage on Richard Road.



b) 10 $(2^2 + 3^2 = f^2)$

$4 + 9 = f^2$

13

$10\sqrt{13} \approx 36$ m

a) A_{TRAP} = AVE BASES · HEIGHT

$\left(\frac{40+60}{2}\right) \cdot 30 = 1500\ m^2$

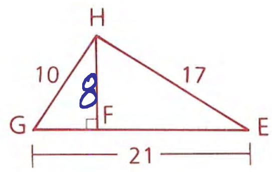
25 **a** Find HF.

b Is △EHF similar to △HGF?

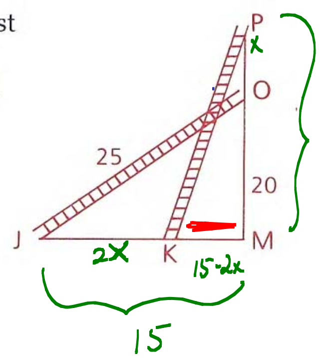
No ∡ EHG is obtuse

$10^2 + 17^2 \ \square\ 21^2$

$389 \ \boxed{<}\ 441$



27 A ladder 25 ft long (JO) is leaning against a wall, reaching a point 20 ft above the ground (MO). The ladder is then moved so that JK = 2(PO). Find KM.



$20x = 5x^2$

$4x = x^2$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x = 0$ or 4

$25^2 = (20+x)^2 + (15-2x)^2$

$625 = 400 + 40x + x^2 + 225 - 60x + 4x^2$

$625 = 625 - 20x + 5x^2$

$-625 + 40x \quad -625 + 20x$

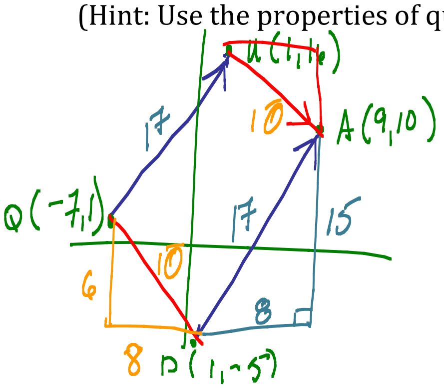
$15 - 2(4) = \boxed{7\ ft}$

31 Quadrilateral QUAD has vertices at Q = (−7, 1), U = (1, 16), A = (9, 10), and D = (1, −5).

a Plot the figure and indicate what type of quadrilateral QUAD is.

b Find the perimeter of QUAD.

(Hint: Use the properties of quadrilaterals that you learned in chapter 5.)



$mUA = \frac{\Delta Y}{\Delta X} = \frac{16-10}{1-9} = \frac{6}{-8} = -\frac{3}{4}$

$mQD = \frac{\Delta Y}{\Delta X} = \frac{1+5}{-7-1} = \frac{6}{-8} = -\frac{3}{4}$

∥

NOT OPP RECIP

∴ ▱

$mQU = \frac{\Delta Y}{\Delta X} = \frac{16-1}{1+7} = \frac{15}{8}$

$mDA = \frac{\Delta Y}{\Delta X} = \frac{-5-10}{1-9} = \frac{-15}{-8} = \frac{15}{8}$

∥

$P = \underbrace{10+10}_{20} + \underbrace{17+17}_{34} = 54$