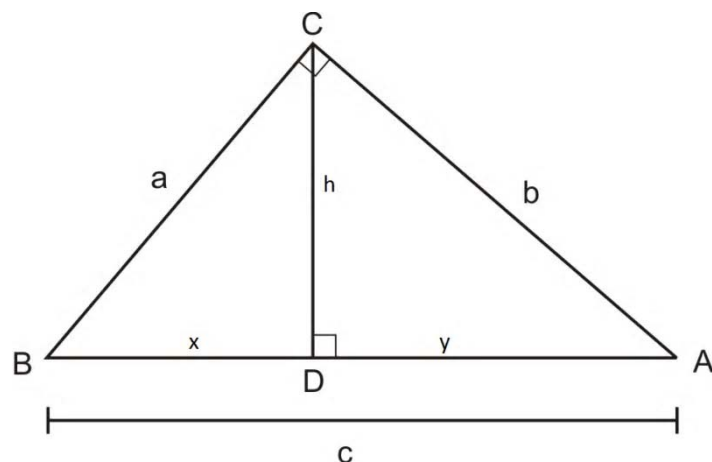


Objective: After studying this section, you will be able to identify the relationships between the parts of a right triangle when an altitude is drawn to the hypotenuse.



Prior Knowledge: Pythagorean Theorem, as $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ where a & b are legs and c is the hypotenuse. In our worksheet, we used similar triangles to observe that the altitude is the geometric mean of the hypotenuse parts, that is $h^2 = xy$. Some of those exercises were leading us to observe two more theorems: $a^2 = xc$ and $b^2 = yc$.

Compare this diagram to the one in our book (below) and see how the formulas are similar. Can you come up with a more generalized (verbal) formula?

Theorem 68 *If an altitude is drawn to the hypotenuse of a right triangle, then*

a *The two triangles formed are similar to the given right triangle and to each other*

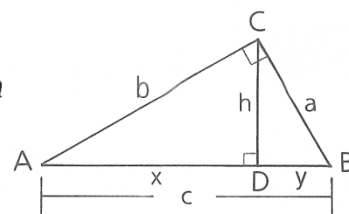
$$\triangle ADC \sim \triangle ACB \sim \triangle CDB$$

b *The altitude to the hypotenuse is the mean proportional between the segments of the hypotenuse*

$$\frac{x}{h} = \frac{h}{y}, \text{ or } h^2 = xy$$

c *Either leg of the given right triangle is the mean proportional between the hypotenuse of the given right triangle and the segment of the hypotenuse adjacent to that leg (i.e., the projection of that leg on the hypotenuse)*

$$\frac{y}{a} = \frac{a}{c}, \text{ or } a^2 = yc; \text{ and } \frac{x}{b} = \frac{b}{c}, \text{ or } b^2 = xc$$



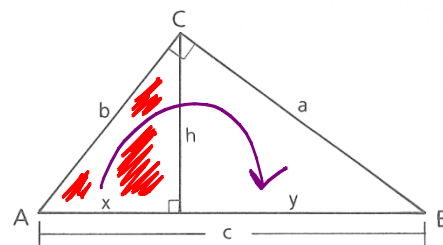
Parts **b** and **c** of Theorem 68 can be summarized as follows.

hyp
leg!

$$\frac{b}{x} = \frac{c}{b} \rightarrow b^2 = xc$$

↑

$$\begin{aligned} h^2 &= x \cdot y \\ b^2 &= x \cdot c \\ a^2 &= y \cdot c \end{aligned}$$

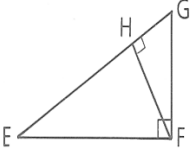
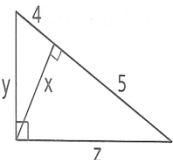
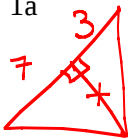
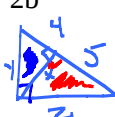
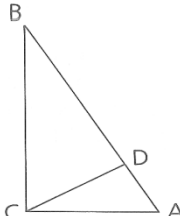
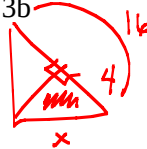


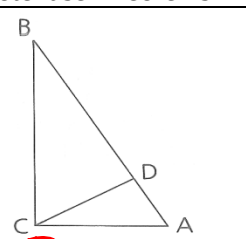
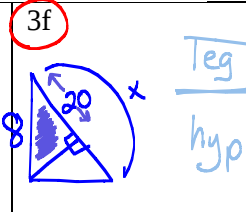
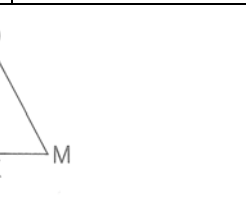
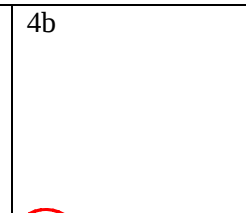
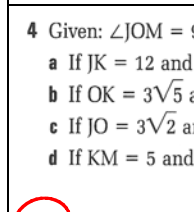
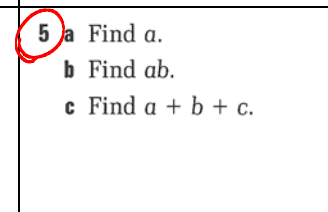
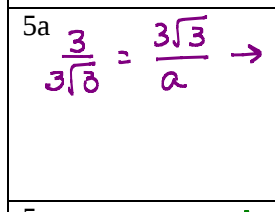
$$\frac{x}{h} = \frac{h}{y} \rightarrow h^2 = xy$$

geo. mean.

9.3: Altitude Hypotenuse Theorems

9.3: 377/ 1-8 all, 14, 16, 17, 21

1 a If $EH = 7$ and $HG = 3$, find HF. b If $EH = 7$ and $HG = 4$, find EF. c If $GF = 6$ and $EG = 9$, find HG.		2 a Find $2x$. b Find $\frac{1}{2}y$. c Find $z + 8$. 									
1a  $\frac{7}{x} = \frac{x}{3} \rightarrow x^2 = 21$ $x = \sqrt{21}$	2a $\frac{4}{x} = \frac{x}{5} \rightarrow x = 2\sqrt{5}$ then $2x = 4\sqrt{5}$										
1b	2b  $\frac{\text{leg}}{\text{hyp}}: \frac{4}{y} = \frac{y}{9} \rightarrow y^2 = 36$ $y = 6$ then $\frac{1}{2}y = 3$										
1c	2c <u>$z+8$</u> $\frac{\text{leg}}{\text{hyp}}: \frac{5}{z} = \frac{z}{9} \rightarrow z^2 = 9 \cdot 5$ $z = 3\sqrt{5}$ $z+8 = \boxed{8+3\sqrt{5}}$										
3 Given: $\overline{AC} \perp \overline{CB}$, $\overline{CD} \perp \overline{AB}$ a If $AD = 4$ and $BD = 9$, find CD. b If $AD = 4$ and $AB = 16$, find AC. c If $BD = 6$ and $AB = 8$, find BC. d If $CD = 8$ and $BD = 16$, find AD. e If $AD = 3$ and $BD = 24$, find AC. f If $BC = 8$ and $BD = 20$, find AB.											
3a	3b  <table data-bbox="997 1520 1297 1698"><tr><th></th><th>little Δ</th><th>big Δ</th></tr><tr><td>leg</td><td>4</td><td>x</td></tr><tr><td>hyp</td><td>x</td><td>16</td></tr></table> $x^2 = 4 \cdot 16$ $\rightarrow x = 2 \cdot 4 = 8$		little Δ	big Δ	leg	4	x	hyp	x	16	
	little Δ	big Δ									
leg	4	x									
hyp	x	16									
3c	3d										

3 Given: $\overline{AC} \perp \overline{CB}$, $\overline{CD} \perp \overline{AB}$ a If $AD = 4$ and $BD = 9$, find CD. b If $AD = 4$ and $AB = 16$, find AC. c If $BD = 6$ and $AB = 8$, find BC. d If $CD = 8$ and $BD = 16$, find AD. e If $AD = 3$ and $BD = 24$, find AC. f If $BC = 8$ and $BD = 20$, find AB.													
3e	3f  <table border="1" data-bbox="1053 428 1479 638"><thead><tr><th></th><th>Big Δ</th><th>Little Δ</th><th>Note</th></tr></thead><tbody><tr><td>leg</td><td>8</td><td>20</td><td>← leg must be smaller than</td></tr><tr><td>hyp</td><td>x</td><td>8</td><td>← hyp</td></tr></tbody></table>		Big Δ	Little Δ	Note	leg	8	20	← leg must be smaller than	hyp	x	8	← hyp
	Big Δ	Little Δ	Note										
leg	8	20	← leg must be smaller than										
hyp	x	8	← hyp										
4 Given: $\angle JOM = 90^\circ$; $\overline{OK}$ is an altitude. a If $JK = 12$ and $KM = 5$, find OK. b If $OK = 3\sqrt{5}$ and $JK = 9$, find KM. c If $JO = 3\sqrt{2}$ and $JK = 3$, find JM. d If $KM = 5$ and $JK = 6$, find OM.													
4a	4b												
4 Given: $\angle JOM = 90^\circ$; $\overline{OK}$ is an altitude. a If $JK = 12$ and $KM = 5$, find OK. b If $OK = 3\sqrt{5}$ and $JK = 9$, find KM. c If $JO = 3\sqrt{2}$ and $JK = 3$, find JM. d If $KM = 5$ and $JK = 6$, find OM.	4c  leg: $\frac{3}{3\sqrt{2}} = \frac{3\sqrt{2}}{x} \rightarrow 3x = (3\sqrt{2})^2$ $\downarrow = 9 \cdot 4$ $3x = 18$ $x = 6$												
4d  leg: $\frac{5}{x} = \frac{x}{11} \rightarrow x^2 = 55$ $x = \sqrt{55}$	5 a Find a. b Find ab. c Find $a + b + c$. 												
5a $\frac{3}{3\sqrt{3}} = \frac{3\sqrt{3}}{a} \rightarrow 3a = 9\sqrt{9}$ $\downarrow = 9 \cdot 3$ $3a = 27$ $a = 9$	5b $leg^2 + leg^2 = hyp^2$ $3 + (3\sqrt{3})^2 = b^2$ $9 + 9 \cdot 9 = b^2$ $\downarrow 9 \cdot 3$ $36 = b^2$ $6 = b$ Then $a \cdot b = (9)(6) = 54$												
5c  rat Δ: $leg^2 + leg^2 = hyp^2$ $6^2 + 9^2 = c^2$ $c^2 = 144 - 36$ $c^2 = 108$ $c = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}$ $c = 2 \cdot 3 \sqrt{3}$ $c = 6\sqrt{3}$	5d												

GEO
MEAN

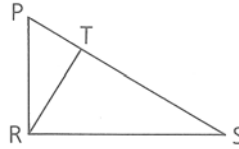
1. AH
2. PYTH

108
54
2 53 12 3

Then $a + b + c = 9 + 6 + 6\sqrt{3} = 15 + 6\sqrt{3}$

6 Given: $\overline{RT}$ is an altitude. $\angle PRS$ is a right $\angle$.

Conclusion: $\frac{PR}{RS} = \frac{RT}{ST}$

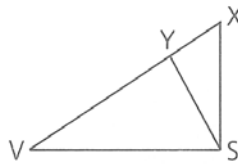


Statements

Reasons

7 Given: $\overline{SY}$ is an altitude. $\angle VSX$ is a right $\angle$.

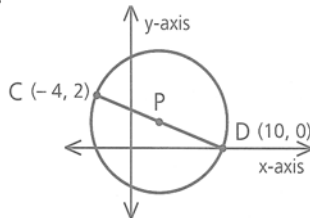
Prove: $XY \cdot SV = XS \cdot YS$



Statements

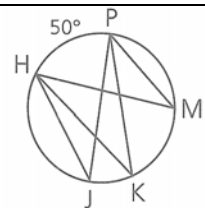
Reasons

8 Find the coordinates of P, the center of the circle.

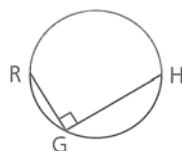


9 Given: Diagram as marked

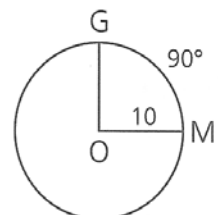
Find: $m\angle HJP$, $m\angle HKP$, and $m\angle HMP$



10 Find the measure of $\widehat{RH}$.



11 Find the area of sector MOG.



NAME _____

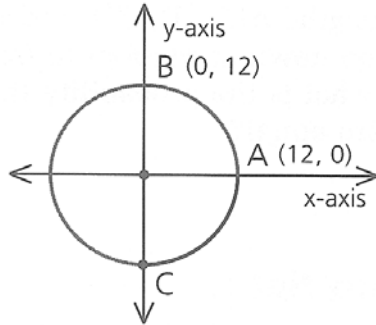
Adv. Geo. – _____

9.3: Altitude Hypotenuse Theorems

Ms. Kresovic

Tuesday, March 04, 2014

- 12** **a** Find the coordinates of point C.
b Find the measure of the arc from A to B to C ($m\widehat{ABC}$).
c Find the length of $\widehat{ABC}$.

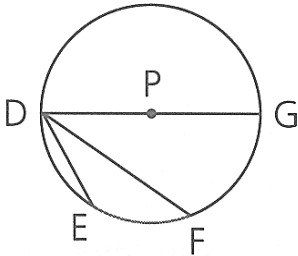


12a

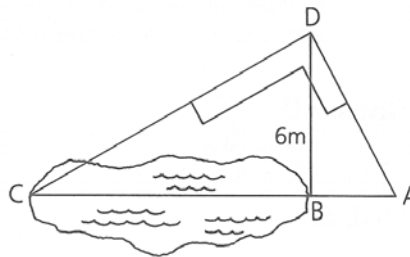
12b

12c

- 13** In $\odot P$, $m\widehat{FG} = 80$ and $m\widehat{DE} = 40$. Find $m\widehat{EF}$ and $m\angle EDF$.



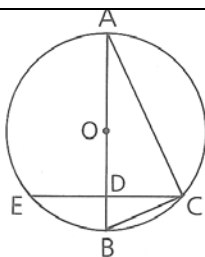
- 14** As Slarpy stood at B, the foot of a 6-m pole, he asked Carpy how far it was across the pond from B to C. Carpy got his carpenter's square and climbed the pole. Using his lines of sight, he set up the figure shown. When Slarpy found that $AB = 3$ m, Carpy knew the answer. What was it?



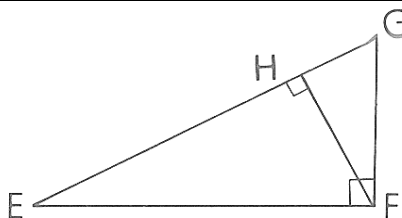
- 15** Given: $\odot O$, $\overline{CD} \perp \overline{AB}$;
 $\angle ACB$ is a right $\angle$.

Conclusions: **a** $\frac{AD}{CD} = \frac{CD}{BD}$

b $\frac{AD}{ED} = \frac{ED}{BD}$



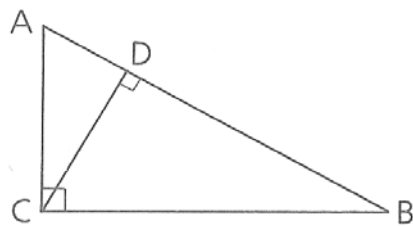
- 16 a** If $HG = 4$ and $EF = 3\sqrt{5}$, find EH .
b If $GF = 6$ and $EH = 9$, find EG .



16a

16b

- 17 a** If $AD = 7$ and $AB = 11$, find CD .
b If $CD = 8$ and $AD = 6$, find AB .
c If $AB = 12$ and $AD = 4$, find BC .
d If $AC = 7$ and $AB = 12$, find BD .



17a

NAME

Adv. Geo. –

9.3: Altitude Hypotenuse Theorems

Ms. Kresovic

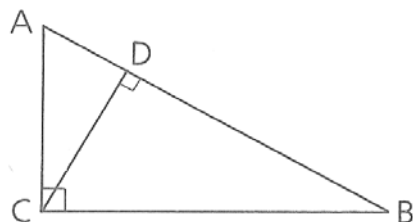
Tuesday, March 04, 2014

17 a If $AD = 7$ and $AB = 11$, find CD .

b If $CD = 8$ and $AD = 6$, find AB .

c If $AB = 12$ and $AD = 4$, find BC .

d If $AC = 7$ and $AB = 12$, find BD .



17b

17c

17d

21 Given: $\overline{AD} \perp \overline{CD}$,
 $\overline{BD} \perp \overline{AC}$,
 $BC = 5$, $AD = 6$
 Find: BD

