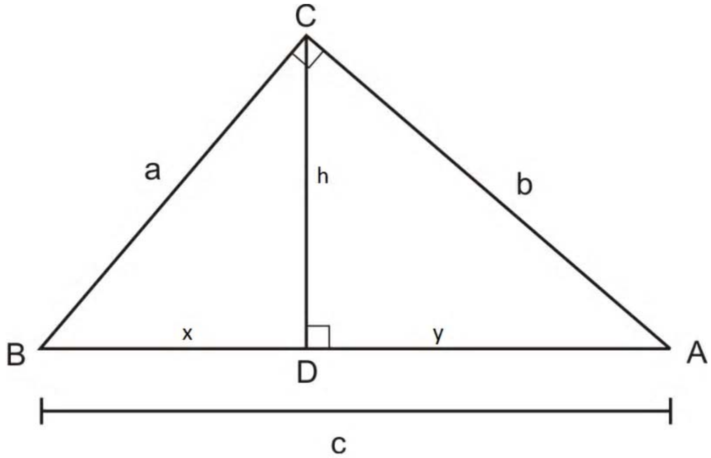
AMDG

NAME | Ms. Kresovic

Adv. Geo. – | 9.3: Altitude Hypotenuse Theorems | Tuesday, March 04, 2014

Objective: After studying this section, you will be able to identify the relationships between the parts of a right triangle when an altitude is drawn to the hypotenuse.



Prior Knowledge: Pythogrean Theorem, as $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$ where a & b are legs and c is the hypotenuse. In our worksheet, we used similar triangles to observe that the altitude is the geometric mean of the hypotenuse parts, that is $h^2 = xy$. Some of those exericses were leading us to observe two more theorems: $a^2 = xc$ and $b^2 = yc$.

Compare this diagram to the one in our book (below) and see how the formulas are similar. Can you come up with a more generalized (verbal) formula?

Theorem 68 ***If an altitude is drawn to the hypotenuse of a right triangle, then***

a ***The two triangles formed are similar to the given right triangle and to each other***

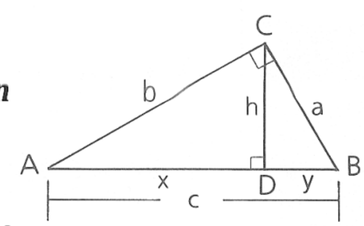
$$\triangle ADC \sim \triangle ACB \sim \triangle CDB$$

b ***The altitude to the hypotenuse is the mean proportional between the segments of the hypotenuse***

$$\frac{x}{h} = \frac{h}{y}, \text{ or } h^2 = xy$$

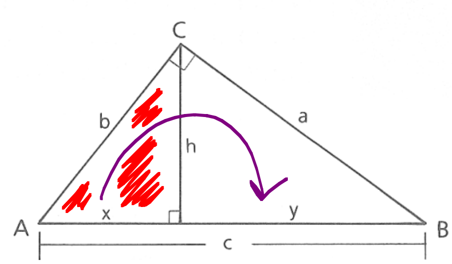
c ***Either leg of the given right triangle is the mean proportional between the hypotenuse of the given right triangle and the segment of the hypotenuse adjacent to that leg (i.e., the projection of that leg on the hypotenuse)***

$$\frac{y}{a} = \frac{a}{c}, \text{ or } a^2 = yc; \text{ and } \frac{x}{b} = \frac{b}{c}, \text{ or } b^2 = xc$$



Parts **b** and **c** of Theorem 68 can be summarized as follows.

$$h^2 = x \cdot y$$
$$\boxed{\begin{aligned} b^2 &= x \cdot c \\ a^2 &= y \cdot c \end{aligned}}$$



$\boxed{\dfrac{\text{hyp}}{\text{leg}}}$!

$\dfrac{b}{x} = \dfrac{c}{b} \rightarrow b^2 = xc$

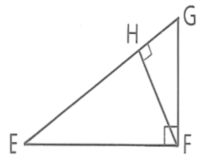
$\dfrac{x}{h} = \dfrac{h}{y} \rightarrow h^2 = xy$

geo. mean.

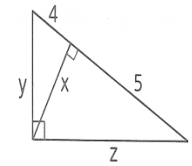
9.3: Altitude Hypotenuse Theorems

9.3: 377/ 1-8 all, 14, 16, 17, 21

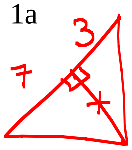
1 **a** If EH = 7 and HG = 3, find HF.
b If EH = 7 and HG = 4, find EF.
c If GF = 6 and EG = 9, find HG.



2 **a** Find 2x. **b** Find $\frac{1}{2}$y. **c** Find z + 8.



1a



$\frac{7}{x} = \frac{x}{3} \rightarrow x^2 = 21$

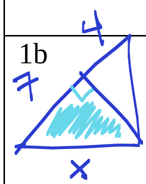
$x = \sqrt{21}$

2a

$\frac{4}{x} = \frac{x}{5} \rightarrow x = 2\sqrt{5}$

then $2x = 4\sqrt{5}$

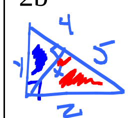
1b



$\frac{\text{leg}}{\text{hyp}}$ $\quad \underset{M\triangle}{\frac{7}{x}} = \underset{L\triangle}{\frac{x}{11}} \rightarrow x^2 = 77$

$\boxed{x = \sqrt{77}}$

2b



$\frac{\text{leg}}{\text{hyp}}: \frac{4}{y} = \frac{y}{9} \rightarrow y^2 = 36$

$y = 6$

then $\frac{1}{2}y = 3$

1c

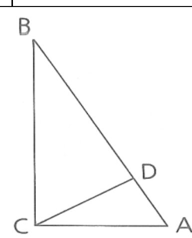
2c z+8

$\frac{\text{leg}}{\text{hyp}}: \frac{5}{z} = \frac{z}{9} \rightarrow z^2 = 9 \cdot 5$

$z = 3\sqrt{5}$

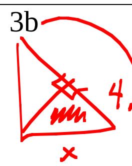
$z + 8 = \boxed{8 + 3\sqrt{5}}$

3 Given: $\overline{AC} \perp \overline{CB}$, $\overline{CD} \perp \overline{AB}$
a If AD = 4 and BD = 9, find CD.
b If AD = 4 and AB = 16, find AC.
c If BD = 6 and AB = 8, find BC.
d If CD = 8 and BD = 16, find AD.
e If AD = 3 and BD = 24, find AC.
f If BC = 8 and BD = 20, find AB.



3a

3b



	little△	big△
leg	4	x
hyp	x	16

$\frac{4}{x} = \frac{x}{16} \rightarrow x = 2 \cdot 4 = 8$

$x^2 = 4 \cdot 16$

3c

3d

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3 Given: $\overline{AC} \perp \overline{CB}$, $\overline{CD} \perp \overline{AB}$

a If AD = 4 and BD = 9, find CD.
b If AD = 4 and AB = 16, find AC.
c If BD = 6 and AB = 8, find BC.
d If CD = 8 and BD = 16, find AD.
e If AD = 3 and BD = 24, find AC.
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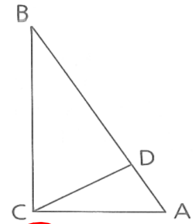
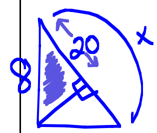


3e

3f



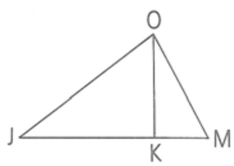
	Big△	Little△
leg	8	20
hyp	x	8

$\frac{8}{x} = \frac{20}{8}$

No Sol !!
← leg must be smaller than hyp

4 Given: $\angle JOM = 90°$; $\overline{OK}$ is an altitude.

a If JK = 12 and KM = 5, find OK.
b If OK = $3\sqrt{5}$ and JK = 9, find KM.
c If JO = $3\sqrt{2}$ and JK = 3, find JM.
d If KM = 5 and JK = 6, find OM.



4a

4b

4 Given: $\angle JOM = 90°$; $\overline{OK}$ is an altitude.

a If JK = 12 and KM = 5, find OK.
b If OK = $3\sqrt{5}$ and JK = 9, find KM.
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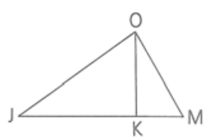
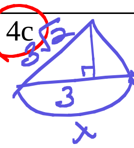


4c



S△ L△

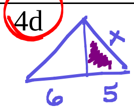
$\frac{\text{leg}}{\text{hyp}}: \frac{3}{3\sqrt{2}} = \frac{3\sqrt{2}}{x} \rightarrow 3x = (3\sqrt{2})^2$

$= 9\sqrt{4}$

$3x = 18$

$x = 6$

4d

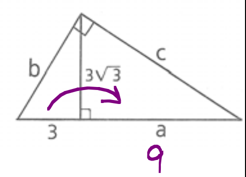


S△ L△

$\frac{\text{leg}}{\text{hyp}} \quad \frac{5}{x} = \frac{x}{11}, \quad x^2 = 55$

$x = \sqrt{55}$

5 **a** Find *a*.
b Find *ab*.
c Find *a* + *b* + *c*.



GEO MEAN

5a

$\frac{3}{3\sqrt{3}} = \frac{3\sqrt{3}}{a} \rightarrow 3a = 9\sqrt{9}$

$= 9 \cdot 3$

$3a = 27$

$a = 9$

5b

$\text{leg}^2 + \text{leg}^2 = \text{hyp}^2$

$3 + (3\sqrt{3})^2 = b^2$

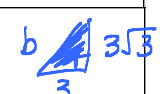
$9 + 9\sqrt{9} = b^2$

$9 \cdot 3$

$9 + 27 = b^2$

$36 = b^2$

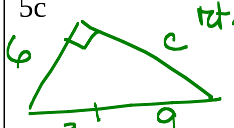
$6 = b$



Then $a \cdot b = (9)(6) = 54$

1. AH
2. Pyth

5c



rt△: $\text{leg}^2 + \text{leg}^2 = \text{hyp}^2$

$6^2 + c^2 = 12^2$

$c^2 = 144 - 36$

$c^2 = 108$

$c = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}$

$c = 2 \cdot 3\sqrt{3}$

$c = 6\sqrt{3}$

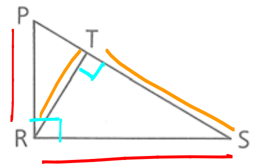
5d

108 / 2 54 / 2 27 / 3 9 / 3 3

Then $a + b + c = 9 + 6 + 6\sqrt{3} = \boxed{15 + 6\sqrt{3}}$

6 Given: $\overline{RT}$ is an altitude. $\angle PRS$ is a right $\angle$.

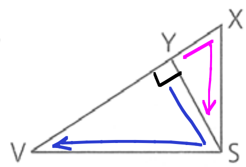
Conclusion: $\frac{PR}{RS} = \frac{RT}{ST}$



Statements	Reasons
1. $\overline{RT}$ alt & $\angle PRS$ rt$\angle$	1. Given
2. $\triangle PRS$ rt$\triangle$	2. rt$\angle \Rightarrow$ rt$\triangle$
3. $\triangle PRS \sim \triangle RTS$	3. alt-hyp
4. $\frac{PR}{RS} = \frac{RT}{ST}$	4. $\sim\triangle \Rightarrow$ corr sds proportional

7 Given: $\overline{SY}$ is an altitude. $\angle VSX$ is a right $\angle$.

Prove: $XY \cdot SV = XS \cdot YS$



Statements	Reasons
1. $\overline{SY}$ alt & $\angle VSX$ rt$\angle$	1. Given
2. $\triangle VSX$ rt$\triangle$	2. rt$\angle \Rightarrow$ rt$\triangle$
3. $\triangle YXS \sim \triangle YSV$	3. Alt-Hyp
4. $\frac{XY}{XS} = \frac{YS}{SV}$	4. $\sim\triangle$s $\Rightarrow$ CORR SDS PROP.
5. $XY \cdot SV = XS \cdot YS$	5. Means Extremes Prod

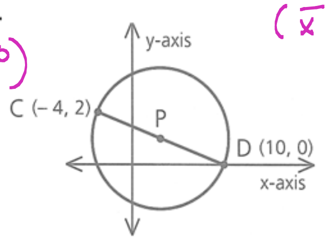
8 Find the coordinates of P, the center of the circle.

$(\bar{x}, \bar{y})$

$\left(\frac{-4+10}{2}, \frac{2+0}{2}\right)$

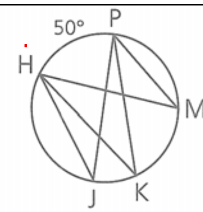
$\left(\frac{6}{2}, \frac{2}{2}\right)$

$(3, 1)$



9 Given: Diagram as marked

Find: $m\angle HJP$, $m\angle HKP$, and $m\angle HMP$



Vertex ON $\angle J = \frac{1}{2} m\overset{\frown}{HP}$

$= \frac{1}{2} 50°$

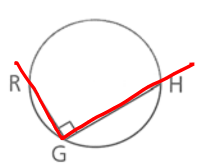
$m\angle M = m\angle K = m\angle J = 25°$

10 Find the measure of $\overset{\frown}{RH}$.

$m\angle = \frac{1}{2} \frown$

$90° = \frac{1}{2} \overset{\frown}{RH}$

$180° = \overset{\frown}{RH}$

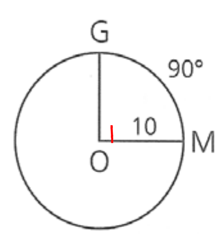


11 Find the area of sector MOG.

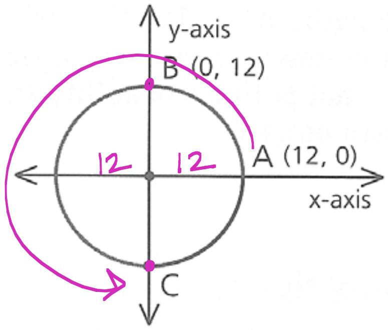
$A_{\odot} = \pi r^2$, $r = 10$

$\frac{90}{360} \cdot \pi(10)^2$

$\frac{1}{4}\ 100\pi = 25\pi$



12 **a** Find the coordinates of point C.
b Find the measure of the arc from A to B to C ($m\widehat{ABC}$).
c Find the length of $\widehat{ABC}$.



12a

$(0, -12)$

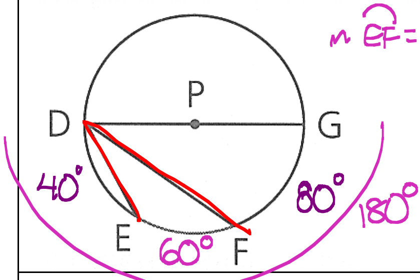
12b

$m\widehat{ABC} = 270°$

12c

$\text{length } \widehat{ABC} = \frac{m\frown}{360} \cdot C$
$\cdot \pi d, \; d = 2r$
$= \frac{270}{360} 24\pi$
$= \frac{3}{\cancel{4}_1} \cancel{24}^6 \pi$
$= \boxed{18\pi}$

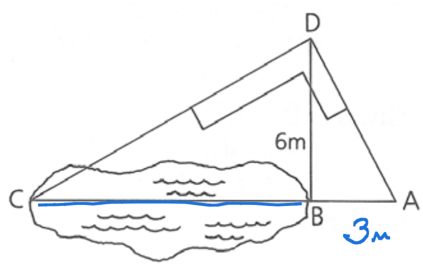
13 In ⊙P, $m\widehat{FG} = 80$ and $m\widehat{DE} = 40$. Find $m\widehat{EF}$ and $m\angle EDF$.



$m\widehat{EF} = 180 - (80 + 40) = 60°$

$m\angle EDF = \frac{1}{2}\widehat{EF}$
$\frac{1}{2} 60°$
$= 30°$

14 As Slarpy stood at B, the foot of a 6-m pole, he asked Carpy how far it was across the pond from B to C. Carpy got his carpenter's square and climbed the pole. Using his lines of sight, he set up the figure shown. When Slarpy found that AB = 3 m, Carpy knew the answer. What was it?

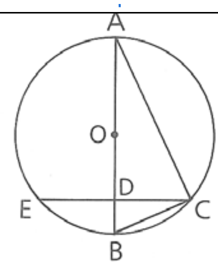


$\frac{\text{pond}}{6} = \frac{6}{3} \rightarrow$ pond 12m

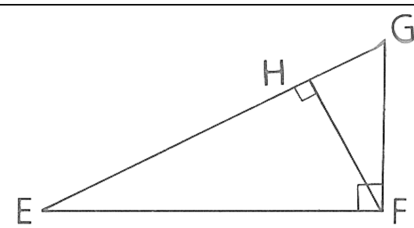
15 Given: $\odot O$, $\overline{CD} \perp \overline{AB}$;
$\angle ACB$ is a right $\angle$.

Conclusions: **a** $\frac{AD}{CD} = \frac{CD}{BD}$

b $\frac{AD}{ED} = \frac{ED}{BD}$



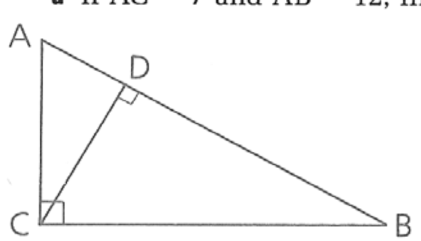
16 **a** If $HG = 4$ and $EF = 3\sqrt{5}$, find EH.
b If $GF = 6$ and $EH = 9$, find EG.



16a

16b

17 **a** If $AD = 7$ and $AB = 11$, find CD.
b If $CD = 8$ and $AD = 6$, find AB.
c If $AB = 12$ and $AD = 4$, find BC.
d If $AC = 7$ and $AB = 12$, find BD.



17a

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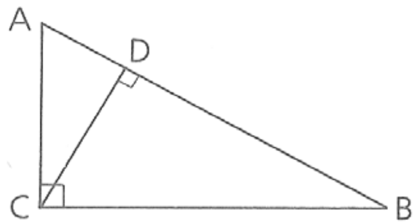
17 **a** If AD = 7 and AB = 11, find CD.

b If CD = 8 and AD = 6, find AB.

c If AB = 12 and AD = 4, find BC.

d If AC = 7 and AB = 12, find BD.



17b

17c

17d

21 Given: $\overline{AD} \perp \overline{CD}$,
$\overline{BD} \perp \overline{AC}$,
BC = 5, AD = 6

Find: BD