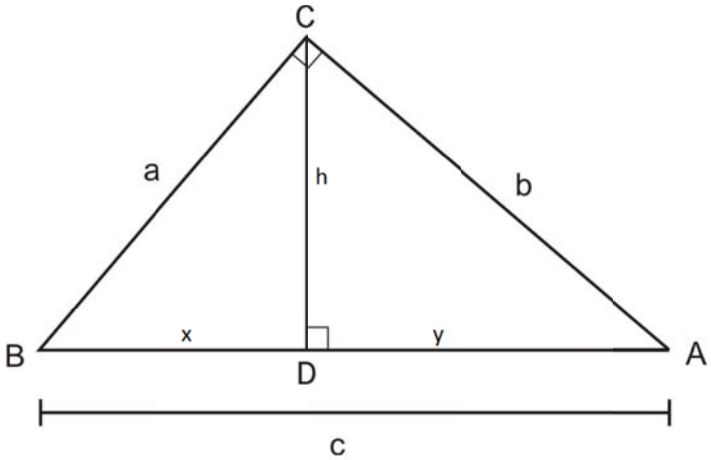
AMDG

NAME
Ms. Kresovic
Adv. Geo. -
Thurs 28 February 2013

9.3: Altitude Hypotenuse Theorems

Objective: After studying this section, you will be able to identify the relationships between the parts of a right triangle when an altitude is drawn to the hypotenuse.




Prior Knowledge: Pythogrean Theorem, as $leg^2 + leg^2 = hypotenuse^2$ where a & b are legs and c is the hypotenuse. In our worksheet, we used similar triangles to observe that the altitude is the geometric mean of the hypotenuse parts, that is $h^2 = xy$. Some of those exericses were leading us to observe two more theorems: $a^2 = xc$ and $b^2 = yc$.

Compare this diagram to the one in our book (below) and see how the formulas are similar. Can you come up with a more generalized (verbal) formula?

**Theorem 68** ***If an altitude is drawn to the hypotenuse of a right triangle, then***

**a** ***The two triangles formed are similar to the given right triangle and to each other***

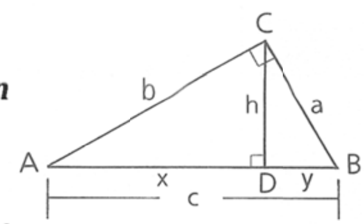
$$\triangle ADC \sim \triangle ACB \sim \triangle CDB$$

**b** ***The altitude to the hypotenuse is the mean proportional between the segments of the hypotenuse***

$$\frac{x}{h} = \frac{h}{y}, \text{ or } h^2 = xy$$

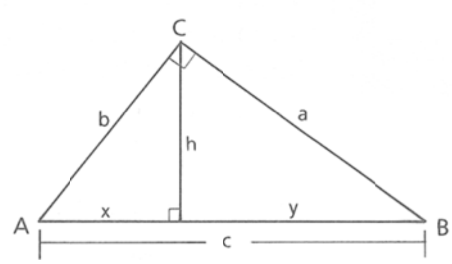
**c** ***Either leg of the given right triangle is the mean proportional between the hypotenuse of the given right triangle and the segment of the hypotenuse adjacent to that leg (i.e., the projection of that leg on the hypotenuse)***

$$\frac{y}{a} = \frac{a}{c}, \text{ or } a^2 = yc; \text{ and } \frac{x}{b} = \frac{b}{c}, \text{ or } b^2 = xc$$




Parts **b** and **c** of Theorem 68 can be summarized as follows.

$$h^2 = x \cdot y$$
$$b^2 = x \cdot c$$
$$a^2 = y \cdot c$$



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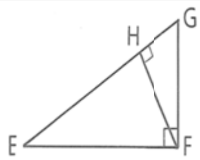
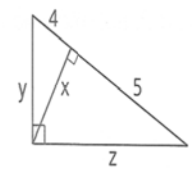
9.3: Altitude Hypotenuse Theorems

9.3: 377/ 1-8 all, 14, 16, 17, 21

**1** **a** If EH = 7 and HG = 3, find HF.
**b** If EH = 7 and HG = 4, find EF.
**c** If GF = 6 and EG = 9, find HG.




**2** **a** Find 2x. **b** Find $\frac{1}{2}$y. **c** Find z + 8.




1a

$alt^2 = part \cdot part$

$HF^2 = 7 \cdot 3$

$HF = \sqrt{21}$

2a

Find x → $alt^2 = part \cdot part$

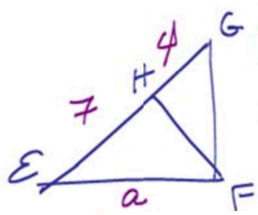
$x^2 = 4 \cdot 5$

$x = 2\sqrt{5}$

∴ $2 \cdot 2\sqrt{5} = 4\sqrt{5}$

1b

$leg^2$ = part connected · hypotenuse




$EF^2 = 7 \cdot 11$

$EF = \sqrt{77}$

2b

$leg^2$ = part connected · whole hypotenuse

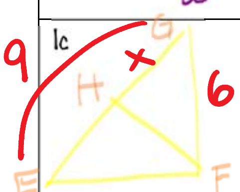
$y^2 = 4(9)$

$y^2 = 36$

$y = \sqrt{36}$

$y = 6$

1c




$GF^2 = GH \cdot GE$

$6^2 = x \cdot 9$

$\frac{36}{9} = x = 4$

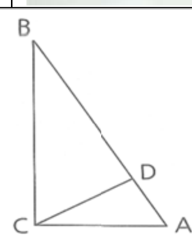
2c

$z^2 = 5(9)$

$z = 3\sqrt{5}$

$3\sqrt{5} + 8$

**3** Given: $\overline{AC} \perp \overline{CB}$, $\overline{CD} \perp \overline{AB}$
**a** If AD = 4 and BD = 9, find CD.
**b** If AD = 4 and AB = 16, find AC.
**c** If BD = 6 and AB = 8, find BC.
**d** If CD = 8 and BD = 16, find AD.
**e** If AD = 3 and BD = 24, find AC.
**f** If BC = 8 and BD = 20, find AB.




3a

$CD^2 = 36$

$CD = \sqrt{36}$

$CD = 6$

3b

$AC^2 = 4 \cdot 16$

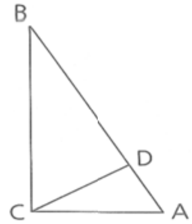
$AC^2 = 64$

$AC = \sqrt{64}$

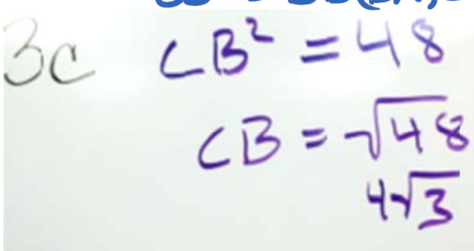
$AC = 8$

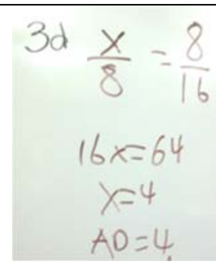
**3** Given: $\overline{AC} \perp \overline{CB}$, $\overline{CD} \perp \overline{AB}$

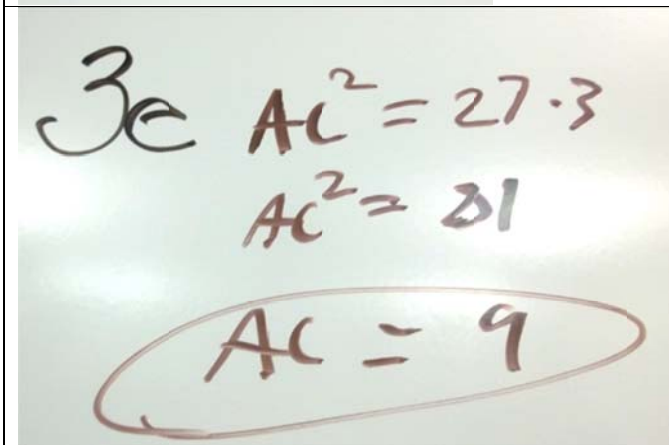
- **a** If AD = 4 and BD = 9, find CD.
- **b** If AD = 4 and AB = 16, find AC.
- **c** If BD = 6 and AB = 8, find BC.
- **d** If CD = 8 and BD = 16, find AD.
- **e** If AD = 3 and BD = 24, find AC.
- **f** If BC = 8 and BD = 20, find AB.

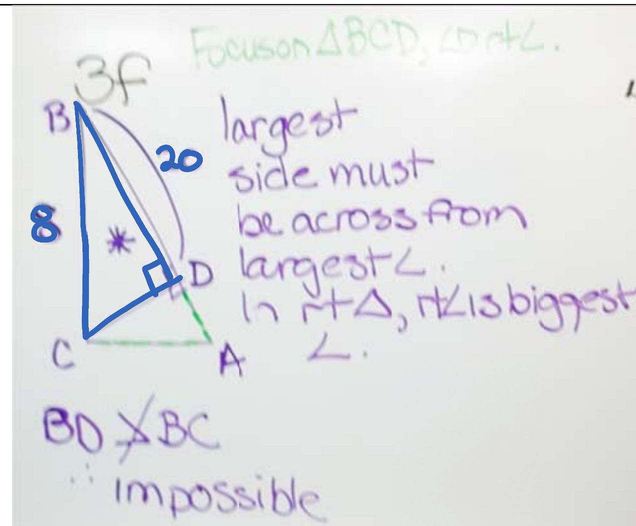



$CB^2 = BD(BA) = 6(8)$



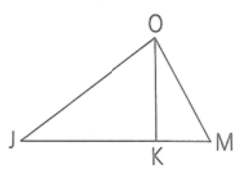



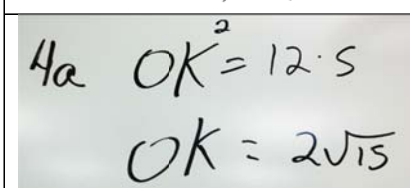



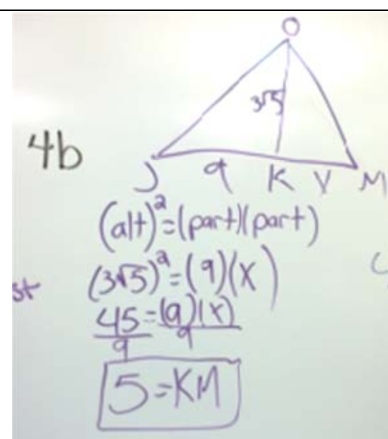




**4** Given: ∠JOM = 90°; $\overline{OK}$ is an altitude.

- **a** If JK = 12 and KM = 5, find OK.
- **b** If OK = $3\sqrt{5}$ and JK = 9, find KM.
- **c** If JO = $3\sqrt{2}$ and JK = 3, find JM.
- **d** If KM = 5 and JK = 6, find OM.



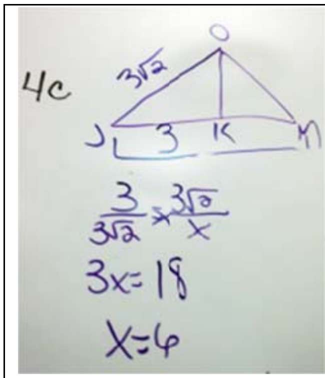

$(3\sqrt{5})(3\sqrt{5}) = \text{out}\cdot\text{out}\sqrt{\text{in}\cdot\text{in}}$

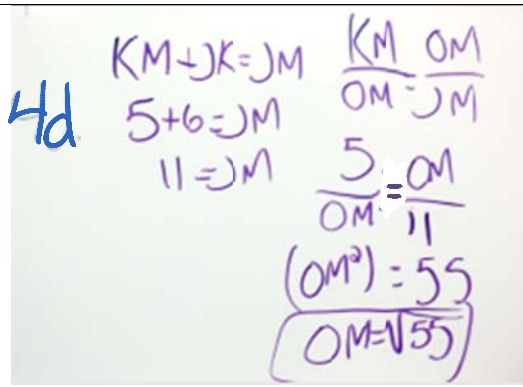
$3 \cdot 3\sqrt{5 \cdot 5}$

$9\sqrt{25}$

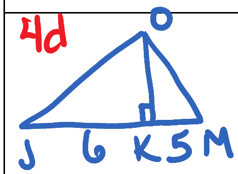
$9 \cdot 5$

$45$




4d




$OM^2 = KM \cdot JM$

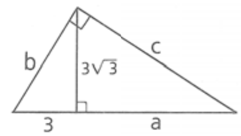
$OM^2 = 5 \cdot 11$

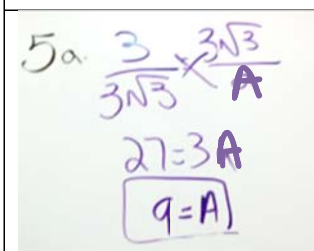
$OM = \sqrt{55}$

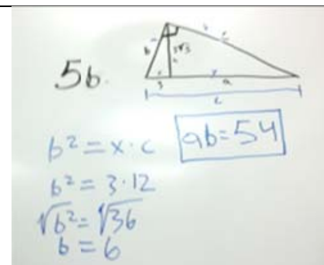
**5** **a** Find $a$.

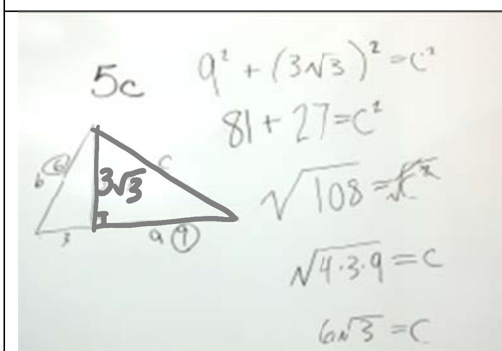
**b** Find $ab$.

**c** Find $a + b + c$.




…or 5c

$c^2 = 9 \cdot 12$

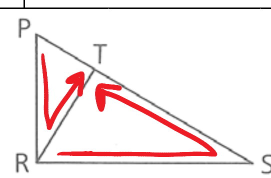
$c = \sqrt{9 \cdot 4 \cdot 3}$

$c = 3 \cdot 2\sqrt{3}$

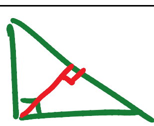
$c = 6\sqrt{3}$

**6** Given: $\overline{RT}$ is an altitude. ∠PRS is a right ∠.

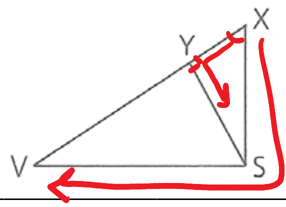
Conclusion: $\frac{PR}{RS} = \frac{RT}{ST}$




| Statements | Reasons |
|---|---|
| 1. $\overline{RT}$ alt, ∠PRS rt∠ | 1. given |
| 2. △PRS rt△ | 2. rt∠ ⇒ rt△ |
| 3. △PRT ~ △RST | 3. Alt - Hyp. |
| 4. $\frac{PR}{RS} = \frac{RT}{ST}$ | 4 ~△ ⇒ corr sds prop. |

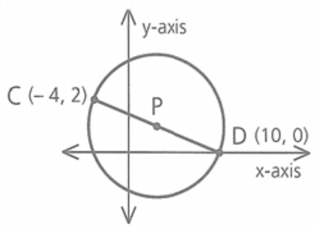


**7** Given: $\overline{SY}$ is an altitude. $\angle VSX$ is a right $\angle$.
Prove: $XY \cdot SV = XS \cdot YS$




| Statements | Reasons |
|---|---|
| 1. $\overline{SY}$ alt, $\angle VSX$ rt$\angle$ | 1. given |
| 2. $\Delta VSX$ rt$\Delta$ | 2. rt$\angle \Rightarrow$ rt$\Delta$ |
| 3. $\Delta XYS \sim \Delta XSV$ | 3 Alt-Hyp |
| 4. $\frac{XY}{XS} = \frac{YS}{SV}$ | 4 $\sim\Delta \Rightarrow$ corr sds prop |
| 5 $XY(SV) = YS(XS)$ | 5. Means-Extr Product |

**8** Find the coordinates of P, the center of the circle.

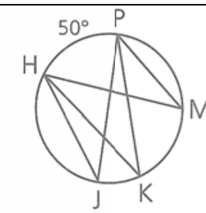



8. P mdpt CP

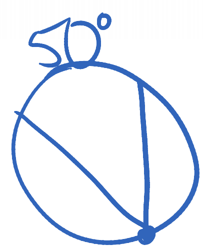
$P = \left(\frac{10-4}{2}, \frac{0+2}{2}\right)$

$P = (3, 1)$

**9** Given: Diagram as marked
Find: m∠HJP, m∠HKP, and m∠HMP



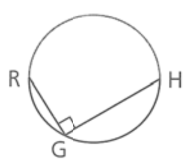

9.


Vertex ∠J is on ⊙
" ∠K "
" ∠M "
} all make same arc

$\therefore 50/2 = 25°$

**10** Find the measure of $\overset{\frown}{RH}$.

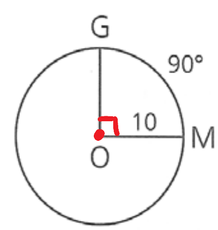



2(90)= 180 degrees

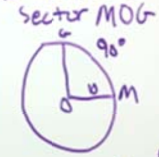
**11** Find the area of sector MOG.

$\frac{90}{360}$ (Area ⊙)

$1/4\ (\pi r^2)$




11. Find the area of sector MOG

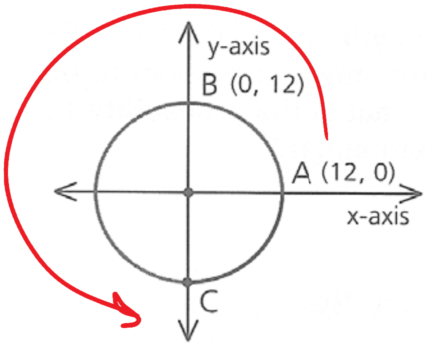



$\frac{90}{360} = \frac{1}{4}(100\pi) = \frac{100\pi}{4} =$ $25\pi$

**12** **a** Find the coordinates of point C.

**b** Find the measure of the arc from A to B to C ($m\widehat{ABC}$).

**c** Find the length of $\widehat{ABC}$.




12a

12a (0, −12)

12b

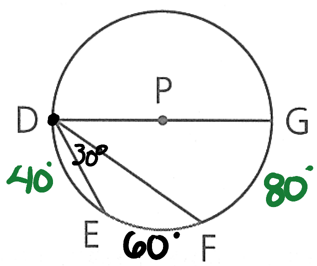
90 · 3

270°

12c

$\frac{270}{360}$ $C = \frac{3}{4}\pi d = \frac{3}{4}\cdot 24\pi = 18\pi$

**13** In ⊙P, $m\widehat{FG} = 80$ and $m\widehat{DE} = 40$. Find $m\widehat{EF}$ and $m\angle EDF$.

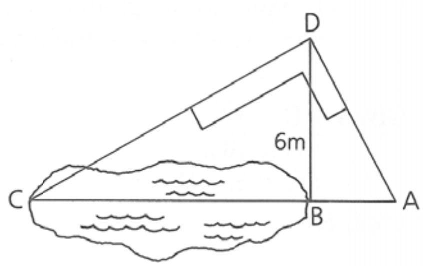


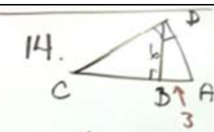

$m\widehat{DEG} = 180$

$m\widehat{EF} = 60°$

$m\angle EDF = 30°$

**14** As Slarpy stood at B, the foot of a 6-m pole, he asked Carpy how far it was across the pond from B to C. Carpy got his carpenter's square and climbed the pole. Using his lines of sight, he set up the figure shown. When Slarpy found that AB = 3 m, Carpy knew the answer. What was it?




$\frac{x}{6} = \frac{6}{3}$

$3x = 36$

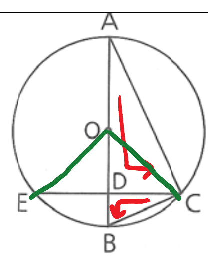
$x = 12$

BC = 12

**15** Given: ⊙O, $\overline{CD} \perp \overline{AB}$; ∠ACB is a right ∠.

Conclusions: **a** $\frac{AD}{CD} = \frac{CD}{BD}$

**b** $\frac{AD}{ED} = \frac{ED}{BD}$

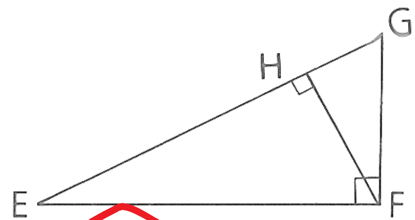


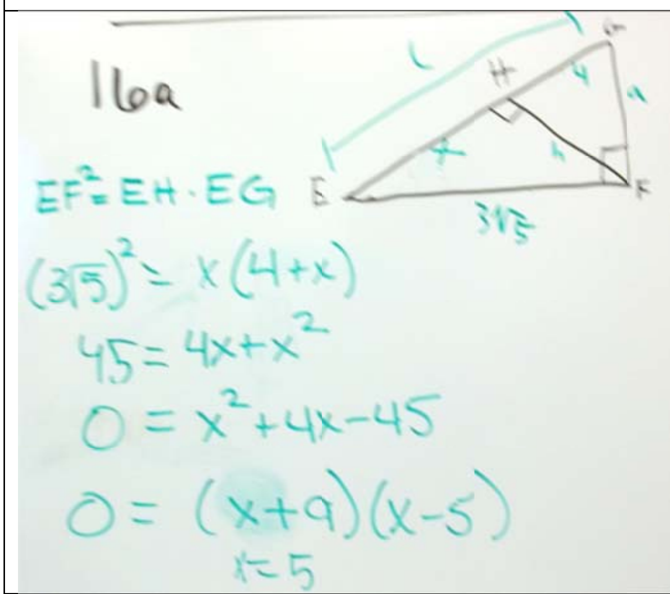

| STATEMENTS | REASONS |
|---|---|
| 1. ⊙O, ∠ACB rt∠ | 1. Given |
| 2. △ACB rt△ | 2. rt∠ ⇒ rt△ |
| 3. CD ⊥ AB | 3. Given |
| 4. △ADC ~ △CDB | 4. Alt Hyp |
| 5. $\frac{AD}{DC} = \frac{CD}{DB}$ | 5. ~△ ⇒ corr sds prop. |

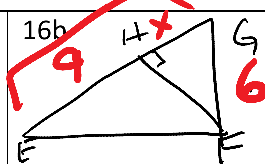
cont'd next pg.

| Statements | Reasons |
|---|---|
| 6. ∠ODE & ∠ODC rt∠s | 6. ⊥ ⇒ rt∠ (step 3) |
| 7. Draw $\overline{OE}$ & $\overline{OC}$ | 7. Aux |
| 8. $\overline{OE} \cong \overline{OC}$ | 8. ⊙ ⇒ ≅ radii |
| 9. $\overline{OD} \cong \overline{OD}$ | 9. Ref |
| 10. △ODE ≅ △ODC | 10. HL |
| 11. $\overline{DE} \cong \overline{DC}$ | 11. CPCTC |
| 12. $\frac{AD}{ED} = \frac{ED}{BD}$ | 12. Substitution |

**16** **a** If HG = 4 and EF = $3\sqrt{5}$, find EH.
**b** If GF = 6 and EH = 9, find EG.




$leg^2 = part \cdot whole$

$GF^2 = GH \cdot GE$

$6^2 = x(x+9)$

$36 = x^2 + 9x$

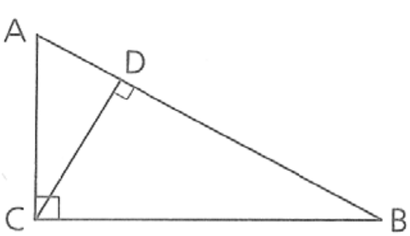
$0 = x^2 + 9x - 36$

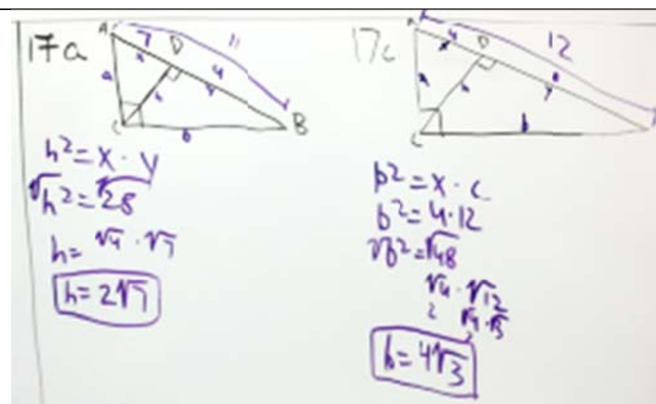
$0 = (x+12)(x-3)$

$x = 3$

$EG = 9 + 3 = 12$

**17** **a** If AD = 7 and AB = 11, find CD.
**b** If CD = 8 and AD = 6, find AB.
**c** If AB = 12 and AD = 4, find BC.
**d** If AC = 7 and AB = 12, find BD.



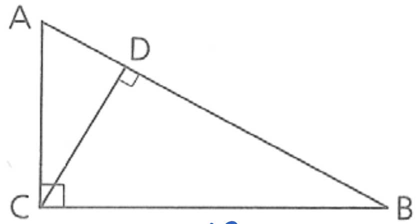



**17** **a** If AD = 7 and AB = 11, find CD.

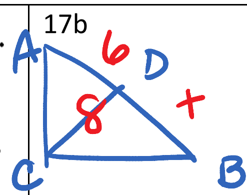
**b** If CD = 8 and AD = 6, find AB.

**c** If AB = 12 and AD = 4, find BC.

**d** If AC = 7 and AB = 12, find BD.




17b

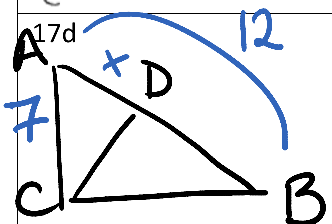



$\frac{6}{8} = \frac{8}{x}$, $x = \frac{64}{6} = \frac{32}{3}$

Then $AB = 6 + \frac{32}{3}$

$\frac{18}{3} + \frac{32}{3} = \frac{50}{3}$

17d




$AC^2 = AD \cdot AB$

$7^2 = x \cdot 12$

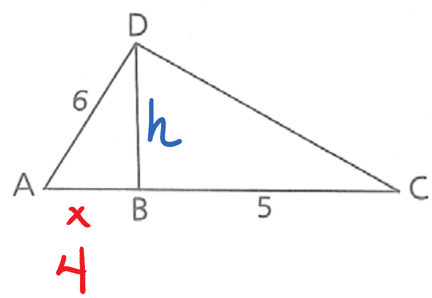
$49 = 12x$

$\frac{49}{12} = x = 4\frac{1}{12}$

$12 - 4\frac{1}{12} = 7\frac{11}{12}$

**21** Given: $\overline{AD} \perp \overline{CD}$,
$\overline{BD} \perp \overline{AC}$,
BC = 5, AD = 6

Find: BD




$AD^2 = AB\ AC$

$6^2 = x(x+5)$

$36 = x^2 + 5x$

$0 = x^2 + 5x - 36$

$0 = (x+9)(x-4)$

$x = 4$

$DB^2 = AB \cdot BC$

$DB^2 = 4 \cdot 5$

$DB = 2\sqrt{5}$