AMDG

Name KEY
Adv Geo -

3.8: HL

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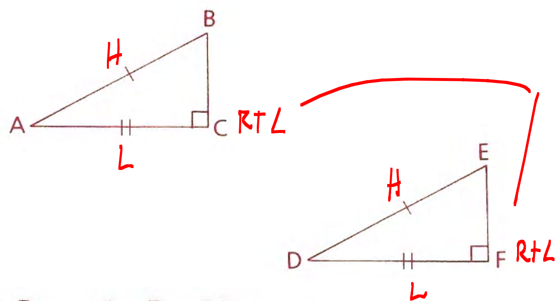
~~R 24 Oct 13~~
27 OCT 15

$\cong \triangle$ POSTULATES
1. SAS
2. ASA
3. SSS
4. HL

Objective

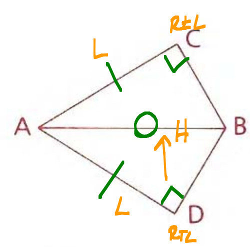
After studying this section, you will be able to
- Use the HL postulate to prove right triangles congruent

Postulate ***If there exists a correspondence between the vertices of two right triangles such that the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the other triangle, the two right triangles are congruent. (HL)***



Part Two: Sample Problems

Problem 1 Given: $\overline{BC} \perp \overline{AC}$,
$\overline{BD} \perp \overline{AD}$,
$\overline{AC} \cong \overline{AD}$
Prove: $\overrightarrow{AB}$ bisects $\angle CAD$.



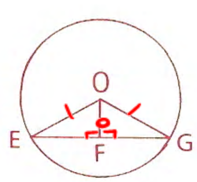
Proof

Statements	Reasons
1 $\overline{BC} \perp \overline{AC}$	1 GIVEN
2 $\angle ACB$ is a right $\angle$.	2 $\perp \Rightarrow$ RT∠
3 $\overline{BD} \perp \overline{AD}$	3 GIVEN
4 $\angle BDA$ is a right $\angle$.	4 $\perp \Rightarrow$ RT∠
5 $\overline{AC} \cong \overline{AD}$	5 GIVEN
6 $\overline{AB} \cong \overline{AB}$	6 Ref
7 $\triangle ACB \cong \triangle ADB$	7 HL
8 $\angle CAB \cong \angle DAB$	8 CPCTC
9 $\overrightarrow{AB}$ bisects $\angle CAD$.	9 $2 \cong \angle$s $\Rightarrow$ bis

2 rt∠s → don't need ≅ ∠s

Problem 2

Given: $\overline{OF}$ *is an altitude.*
$\odot O$
Conclusion: $\overline{EF} \cong \overline{FG}$



Statements	Reasons
1 $\overline{OF}$ is an altitude	1 Given
RT∠ 2 $\angle EFO$ and $\angle GFO$ are right $\angle$s	2 alt $\Rightarrow$ rt∠s (1)
L 3 $\overline{OF} \cong \overline{OF}$	3 Reflexive
4 $\odot O$	4 Given
H 5 $\overline{OE} \cong \overline{OG}$	5 $\odot \Rightarrow \cong$ radii (4)
6 $\triangle OEF \cong \triangle OGF$	6 HL (2 5 3)
7 $\overline{EF} \cong \overline{FG}$	7 CPCTC

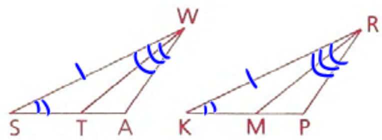
Problem 3

Prove: *Corresponding angle bisectors of congruent triangles are congruent.*

Once again we must set up the proof and draw the figure. (Although this may look like a simple two-step proof based on CPCTC, it isn't. *Corresponding parts of congruent triangles* refers only to corresponding sides and angles.)

Given: $\triangle KPR \cong \triangle SAW$;
$\overrightarrow{RM}$ bisects $\angle KRP$.
$\overrightarrow{WT}$ bisects $\angle SWA$.

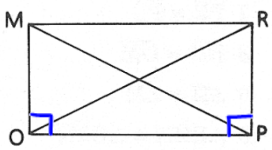
Prove: $\overline{RM} \cong \overline{WT}$

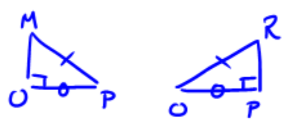


Statements	Reasons
1 $\triangle KPR \cong \triangle SAW$	1 GIVEN
2 $\overline{KR} \cong \overline{SW}$	2 CPCTC (1)
3 $\angle K \cong \angle S$	3 CPCTC (1)
4 $\angle KRP \cong \angle SWA$	4 CPCTC (1)
5 $\overrightarrow{RM}$ bisects $\angle KRP$.	5 Given
6 $\overrightarrow{WT}$ bisects $\angle SWA$.	6 Given
7 $\angle KRM \cong \angle SWT$	7 ÷
8 $\triangle KRM \cong \triangle SWT$	8 ASA
9 $\overline{RM} \cong \overline{WT}$	9 CPCTC

Problem 4

Given: $\overline{MO} \perp \overline{OP}$
$\overline{RP} \perp \overline{OP}$
$\overline{MP} \cong \overline{RO}$



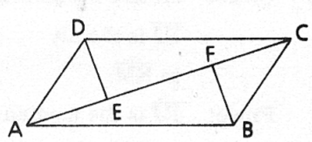


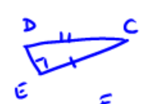
Prove: $\triangle MOP \cong \triangle RPO$

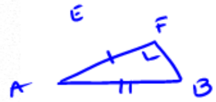
1 $\overline{MO} \perp \overline{OP}$	1 GIVEN
2 $\angle MOP$ is a rt $\angle$.	2 $\perp \Rightarrow$ rt $\angle$
3 $\overline{RP} \perp \overline{OP}$	3 Given
4 $\angle RPO$ is a rt $\angle$.	4 $\perp \Rightarrow$ rt $\angle$
5 $\overline{MP} \cong \overline{RO}$	5 Given
6 $\overline{OP} \cong \overline{OP}$	6 Ref
7 $\triangle MOP \cong \triangle RPO$	7 HL (2/4, 5, 6)

Problem 5

Given: $\overline{AE} \cong \overline{CF}$
$\overline{AB} \cong \overline{CD}$
$\angle BFA$ is a rt $\angle$.
$\angle DEC$ is a rt $\angle$.







Prove: $\angle CDE \cong \angle ABF$

1 $\overline{AE} \cong \overline{CF}$	1 GIVEN
2 $\overline{AF} \cong \overline{CE}$	2 ADD
3 $\overline{AB} \cong \overline{CD}$	3 Given
4 $\angle BFA$ is a rt $\angle$.	4 Given
5 $\angle DEC$ is a rt $\angle$.	5 Given
6 $\triangle DEC \cong \triangle BFA$	6 HL (4/5, 3, 2)
7 $\angle CDE \cong \angle ABF$	7 CPCTC (6)

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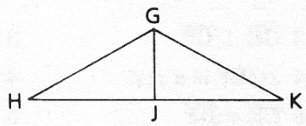
6 Given: $\overline{GH} \cong \overline{GK}$

$\overline{GJ}$ is an alt.

Prove: $\overrightarrow{GJ}$ bis $\angle HGK$.



1 $\overline{GH} \cong \overline{GK}$	1 Given
2 $\overline{GJ}$ is an alt.	2 Given
3 $\angle GJH$ is a rt $\angle$.	3 An alt of a $\triangle$ forms rt $\angle$s with one of the sides.
4 $\angle GJK$ is a rt $\angle$.	4 Same as 3
5 $\overline{GJ} \cong \overline{GJ}$	5 Reflexive prop
6 $\triangle GJH \cong \triangle GJK$	6 HL
7 $\angle HGJ \cong \angle KGJ$	7 CPCTC
8 $\overrightarrow{GJ}$ bis $\angle HGK$.	8 If a ray divides an $\angle$ into 2 $\cong$ $\angle$s, the ray bis the $\angle$.

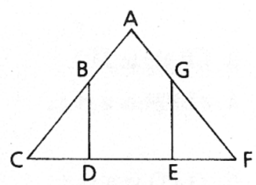
8 Given: $\overline{BD} \perp \overline{CF}$

$\overline{GE} \perp \overline{CF}$

$\overline{CE} \cong \overline{DF}$

$\overline{BC} \cong \overline{GF}$

Prove: $\triangle ACF$ isos.

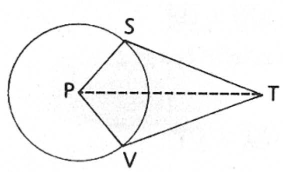


1 $\overline{BD} \perp \overline{CF}$	1
2 $\angle BDC$ is a rt $\angle$.	2
3 $\overline{GE} \perp \overline{CF}$	3
4 $\angle GEF$ is a rt $\angle$.	4
5 $\overline{CE} \cong \overline{DF}$	5
6 $\overline{DE} \cong \overline{DE}$	6
7 $\overline{CD} \cong \overline{EF}$	7
8 $\overline{BC} \cong \overline{GF}$	8
9 $\triangle BCD \cong \triangle GFE$	9
10 $\angle C \cong \angle F$	10
11 $\overline{AC} \cong \overline{AF}$	11
12 $\triangle ACF$ is isos.	12

10 Given: $\odot P$

$\overline{ST} \cong \overline{VT}$

Prove: $\angle PST \cong \angle PVT$



1 $\odot P$	1
2 $\overline{PS} \cong \overline{PV}$	2
3 $\overline{ST} \cong \overline{VT}$	3
4 Draw $\overline{PT}$	4
5 $\overline{PT} \cong \overline{PT}$	5
6 $\triangle PST \cong \triangle PVT$	6
7 $\angle PST \cong \angle PVT$	7

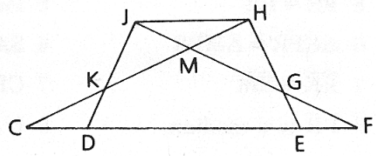
12 Given: $\overline{CD} \cong \overline{EF}$

$\overline{JF} \perp \overline{JD}$

$\overline{CH} \perp \overline{HE}$

$\overline{CH} \cong \overline{JF}$

Prove: $\overline{JD} \cong \overline{HE}$



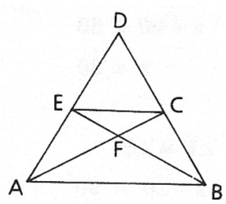
1 $\overline{CD} \cong \overline{EF}$	1
2 $\overline{DE} \cong \overline{DE}$	2
3 $\overline{CE} \cong \overline{DF}$	3
4 $\overline{JF} \perp \overline{JD}$	4
5 $\angle DJF$ is a rt $\angle$.	5
6 $\overline{CH} \perp \overline{HE}$	6
7 $\angle CHE$ is a rt $\angle$.	7
8 $\overline{CH} \cong \overline{JF}$	8
9 $\triangle JDF \cong \triangle HEC$	9
10 $\overline{JD} \cong \overline{HE}$	10

16 Given: $\overline{BE} \perp \overline{AD}$

$\overline{AC} \perp \overline{BD}$

$\overline{AC} \cong \overline{BE}$

$\overline{DE} \cong \overline{EC}$



Prove: $\triangle DEC$ is equilateral.

1 $\overline{BE} \perp \overline{AD}$.	1 Given
2 $\angle AEB$ is a rt $\angle$.	2 If 2 segs are $\perp$, they form rt $\angle$s.
3 $\overline{AC} \perp \overline{BD}$	3 Given
4 $\angle ACB$ is a rt $\angle$.	4 Same as 2
5 $\overline{AC} \cong \overline{BE}$	5 Given
6 $\overline{AB} \cong \overline{AB}$	6 Reflexive prop
7 $\triangle EAB \cong \triangle CBA$	7 HL
8 $\overline{DE} \cong \overline{EC}$	8 Given
9 $\overline{EA} \cong \overline{CB}$	9 CPCTC
10 $\angle EAB \cong \angle CBA$	10 CPCTC
11 $\overline{DA} \cong \overline{DB}$	11 If △ then △
12 $\overline{DE} \cong \overline{DC}$	12 Subtraction prop
13 $\overline{EC} \cong \overline{DC} \cong \overline{DE}$	13 Transitive prop
14 $\triangle DEC$ is equilateral.	14 If all sides of a $\triangle$ are $\cong$, it is equilateral.

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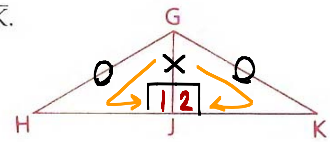
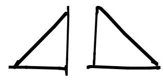
Homework

Problem Set A

1 Given: $\overline{GJ}$ is the altitude to $\overline{HK}$.
$\overline{HG} \cong \overline{KG}$

Prove: $\triangle HGJ \cong \triangle KGJ$

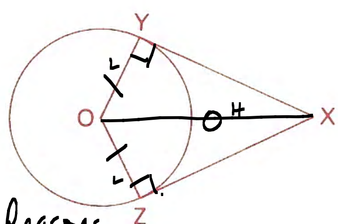




STATEMENTS	REASONS
1. $\overline{GJ}$ alt $\overline{HK}$	1. GIVEN
rt∠S 2. ∠1 & ∠2 rt∠S	2. alt → rt∠
H 3. $\overline{HG} \cong \overline{KG}$	3. GIVEN
4. $\overline{GJ} \cong \overline{GJ}$	4. Ref
5. $\triangle GHJ \cong \triangle GKJ$	5. HL

3 Given: $\odot O$,
$\overline{YO} \perp \overline{YX}$,
$\overline{ZO} \perp \overline{ZX}$

Conclusion: $\overline{YX} \cong \overline{ZX}$

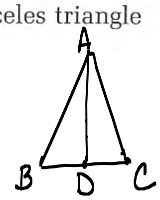


Statements | Reasons

5 Set up and prove: The altitude to the base of an isosceles triangle divides the triangle into two congruent triangles.

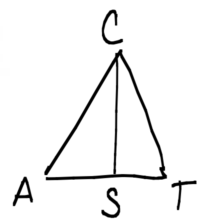
G: $\overline{AD}$ alt to $\overline{BC}$
$\overline{AB} \cong \overline{AC}$

P: $\triangle ABD \cong \triangle ACD$



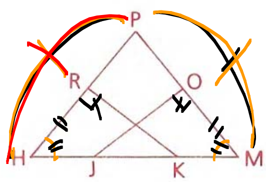
Problem Set B

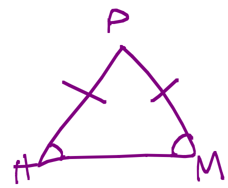
7 Prove: An altitude of an equilateral triangle is also a median of the triangle.



G: $\triangle CAT$ equilateral
$\overline{CS}$ alt $\overline{AT}$
P: $\overline{CS}$ median $\overline{AT}$

9 Given: $\overline{RK} \perp \overline{HR}$,
$\overline{JO} \perp \overline{PM}$,
$\overline{PH} \cong \overline{PM}$,
$\overline{PR} \cong \overline{PO}$
Conclusion: $\overline{RK} \cong \overline{JO}$

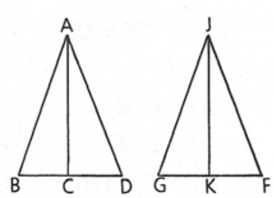




Statements	Reasons
1. $\overline{RK} \perp \overline{HR}$ & $\overline{JO} \perp \overline{PM}$	1. Given
2. $\angle HRK$ & $\angle MOJ$ rt $\angle$s	2. $\perp \Rightarrow$ rt $\angle$s
3. $\angle HRK \cong \angle MOJ$	3. rt $\angle$s $\Rightarrow \cong \angle$s
4. $\overline{PH} \cong \overline{PM}$, $\overline{PR} \cong \overline{PO}$	4. Given
5. $\overline{HR} \cong \overline{MO}$	5. Subtract
6. $\angle H \cong \angle M$	6. $\triangle \Rightarrow \triangle$
7. $\triangle HRK \cong \triangle MOJ$	7. ASA
8. $\overline{HK} \cong \overline{JM}$	8. CPCTC

11 Prove: Corresponding medians of congruent triangles are congruent.

Given: $\triangle ABD \cong \triangle JGF$
$\overline{AC}$ median to $\overline{BD}$
$\overline{JK}$ median to $\overline{GF}$
Prove: $\overline{AC} \cong \overline{JK}$



1 $\triangle ABD \cong \triangle JGF$	1 Given
2 $\overline{AB} \cong \overline{JG}$	2 CPCTC
3 $\angle B \cong \angle G$	3 CPCTC
4 $\overline{BD} \cong \overline{GF}$	4 CPCTC
5 $\overline{AC}$ median to $\overline{BD}$	5 Given
6 $\overline{JK}$ median to $\overline{GF}$	6 Given
7 $\overline{BC} \cong \overline{GK}$	7 Division prop
8 $\triangle ABC \cong \triangle JGK$	8 SAS
9 $\overline{AC} \cong \overline{JK}$	9 CPCTC

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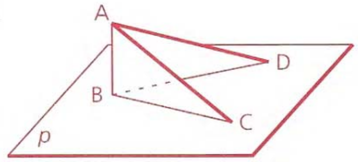
13 Given: $\triangle ABC$ and $\triangle ABD$ standing on plane p.
$\overline{AB} \perp \overline{BC}$, $\overline{AB} \perp \overline{BD}$,
$\overline{AC} \cong \overline{AD}$

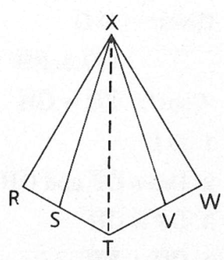
Prove: If $\overline{CD}$ is drawn, $\triangle BCD$ will be isosceles.



1 $\overline{AB} \perp \overline{BC}, \overline{AB} \perp \overline{BD}$	1 Given
2 $\angle$s ABC and ABD are rt $\angle$s.	2 If 2 segs are $\perp$, they form rt $\angle$s.
3 $\overline{AB} \cong \overline{AB}$	3 Reflexive prop
4 $\overline{AC} \cong \overline{AD}$	4 Given
5 $\triangle ABC \cong \triangle ABD$	5 HL
6 $\overline{BC} \cong \overline{BD}$	6 CPCTC
7 $\triangle BCD$ is isos.	7 If two sides $\cong$, then $\triangle$ is isos.

17 Given: $\angle R$ and $\angle W$ are rt $\angle$s.
$\overline{RX} \cong \overline{WX}$
S is $\frac{3}{7}$ of the way from R to T.
V is $\frac{4}{7}$ of the way from T to W.

Prove: $\overline{ST} \cong \overline{TV}$



1 $\angle R$ and $\angle W$ are rt $\angle$s.	1 Given
2 $\overline{RX} \cong \overline{WX}$	2 Given
3 Draw $\overline{XT}$	3 Two pts determine a line.
4 $\overline{XT} \cong \overline{XT}$	4 Reflexive prop
5 $\triangle RXT \cong \triangle WXT$	5 HL
6 S is $\frac{3}{7}$ of way from R to T.	6 Given
7 V is $\frac{4}{7}$ of way from T to W.	7 Given
8 $\overline{RT} \cong \overline{WT}$	8 CPCTC
9 S is $\frac{4}{7}$ of way from T to R.	9 Subtraction prop
10 $\overline{ST} \cong \overline{TV}$	10 Multiplication prop