AMDG

Name | Ms. Kresovic

Acc Geo - | 3.7: Angle-Side Theorems

Objective

After studying this section, you will be able to

- Apply theorems relating the angle measures and side lengths of triangles

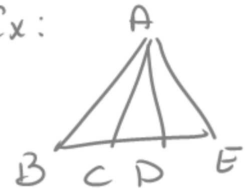
Part One: Introduction

It can be shown that the base angles of any isosceles triangle are congruent.

1 triangle

Theorem 20 ***If two sides of a triangle are congruent, the angles opposite the sides are congruent. (If △, then △.)***

Ex:



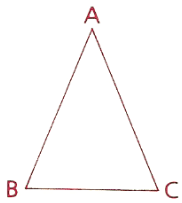
G: $\overline{AB} \cong \overline{AE}$

$\angle B \cong \angle E$

(△ ⇒ △)

Given: $\overline{AB} \cong \overline{AC}$

Prove: $\angle B \cong \angle C$

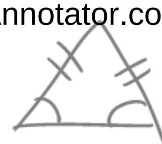


see also #7 in 3.5

Proof:

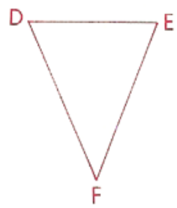
Statements	Reasons
1 $\overline{AB} \cong \overline{AC}$	1 Given
2 $\overline{BC} \cong \overline{BC}$	2 Reflexive Property
3 $\triangle ABC \cong \triangle ACB$	3 SSS (1, 2, 1)
4 $\angle B \cong \angle C$	4 CPCTC

You should be accustomed to proving that one triangle is congruent to another triangle. But notice that to prove the preceding theorem, we proved that a triangle is congruent to itself (its mirror image). We shall use the same type of proof to show that the converse of Theorem 20 is also true.



Theorem 21 ***If two angles of a triangle are congruent, the sides opposite the angles are congruent. (If △, then △.)***

Given: $\angle D \cong \angle E$
Conclusion: $\overline{DF} \cong \overline{EF}$



Proof:

Statements	Reasons
1 $\angle D \cong \angle E$	1 Given
2 $\overline{DE} \cong \overline{DE}$	2 Reflexive Property
3 $\triangle DEF \cong \triangle EDF$	3 ASA (1, 2, 1)
4 $\overline{DF} \cong \overline{EF}$	4 CPCTC

Theorem 21 tells us that a triangle is isosceles if two or more of its angles are congruent. We now have two ways of proving that a triangle is isosceles.

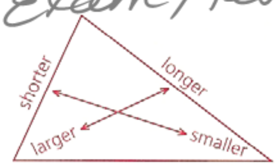
Ways to Prove That a Triangle Is Isosceles

1. If at least two sides of a triangle are congruent, the triangle is isosceles.
2. If at least two angles of a triangle are congruent, the triangle is isosceles.

Exam/Test

never use in proof
will use algebra

The inverses of Theorems 20 and 21 are also true. (Recall that the inverse of "If p, then q" is "If not p, then not q.") In fact, it can be proved that inequalities of sides and angles are related as shown in the diagram.



Theorem ***If two sides of a triangle are not congruent, then the angles opposite them are not congruent, and the larger angle is opposite the longer side. (If △, then △.)***

Theorem ***If two angles of a triangle are not congruent, then the sides opposite them are not congruent, and the longer side is opposite the larger angle. (If △, then △.)***

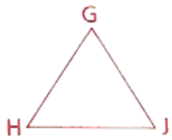
Ex

These theorems will be restated and proved in Chapter 15.

3.7: Angle-Side Theorems

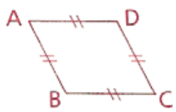
Let us now consider a question we raised in Section 3.6: Is an equilateral triangle also equiangular?

Given: $\overline{GH} \cong \overline{HJ} \cong \overline{GJ}$
Is $\angle H \cong \angle J \cong \angle G$?



If $\overline{GH} \cong \overline{HJ}$, which two angles must be congruent? If $\overline{HJ} \cong \overline{GJ}$, which two angles must be congruent? Do we therefore know that $\triangle GHJ$ is equiangular? Can we also prove that an equiangular triangle is equilateral?

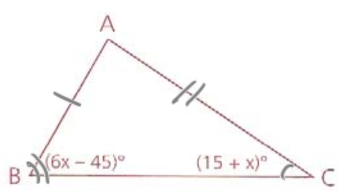
Because of their equivalence, the terms *equilateral triangle* and *equiangular triangle* will be used interchangeably throughout this book. We cannot, however, use the words *equilateral* and *equiangular* interchangeably when we apply them to other types of figures. For example, figure ABCD is equilateral but not equiangular. Figure EFGH, on the other hand, is equiangular but not equilateral.



Part Two: Sample Problems

Problem 1 Given: $AC > AB$,
$m\angle B + m\angle C < 180$,
$m\angle B = 6x - 45$,
$m\angle C = 15 + x$

What are the restrictions on the value of x?



Solution Since $AC > AB$, $m\angle B > m\angle C$.

$$6x - 45 > 15 + x$$
$$5x > 60$$
$$x > 12$$

We also know that $m\angle B + m\angle C < 180$.

$$6x - 45 + 15 + x < 180$$
$$7x < 210$$
$$x < 30$$

Therefore, x must be between 12 and 30.

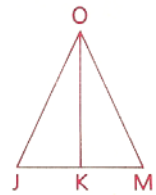
$\boxed{12 < x < 30}$

Problem 2 *Prove: The bisector of the vertex angle of an isosceles triangle is also the median to the base.*

Proof For a problem like this, we must set up the proof and supply the diagram.

Given: $\triangle JOM$ is isosceles, with $\angle JOM$ the vertex angle.
$\overrightarrow{OK}$ bisects $\angle JOM$.

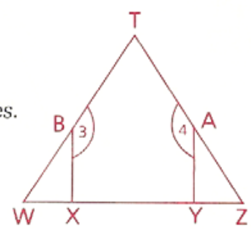
Conclusion: $\overline{OK}$ is the median to the base.



Statements	Reasons
1 $\triangle JOM$ is isosceles, with $\angle JOM$ the vertex angle.	1 Given
2 $\overline{OJ} \cong \overline{OM}$	2 The legs of an isosceles $\triangle$ are $\cong$.
3 $\overrightarrow{OK}$ bisects $\angle JOM$.	3 Given
4 $\angle JOK \cong \angle MOK$	4 If a ray bisects an $\angle$, it divides the $\angle$ into two $\cong$ $\angle$s.
5 $\overline{OK} \cong \overline{OK}$	5 Reflexive Property
6 $\triangle JOK \cong \triangle MOK$	6 SAS (2, 4, 5)
7 $\overline{JK} \cong \overline{MK}$	7 CPCTC
8 $\overline{OK}$ is the median to the base.	8 If a segment from a vertex of a $\triangle$ divides the opposite side into two $\cong$ segments, it is a median.

Problem 3 *Given:* $\angle 3 \cong \angle 4$,
$\overline{BX} \cong \overline{AY}$,
$\overline{BW} \cong \overline{AZ}$

Conclusion: $\triangle WTZ$ *is isosceles.*

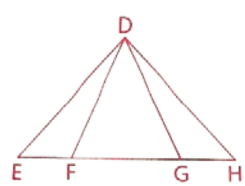


Proof

Statements	Reasons
1 $\angle 3 \cong \angle 4$	1 Given
2 $\angle 3$ is supp. to $\angle WBX$.	2 If two $\angle$s form a straight $\angle$, they are supplementary.
3 $\angle 4$ is supp. to $\angle YAZ$.	3 Same as 2
4 $\angle WBX \cong \angle YAZ$	4 $\angle$s supp. to $\cong$ $\angle$s, are $\cong$.
5 $\overline{BX} \cong \overline{AY}$	5 Given
6 $\overline{BW} \cong \overline{AZ}$	6 Given
7 $\triangle BWX \cong \triangle AZY$	7 SAS (5, 4, 6)
8 $\angle W \cong \angle Z$	8 CPCTC
9 $\triangle WTZ$ is isosceles.	9 If at least two $\angle$s of a $\triangle$ are $\cong$, the $\triangle$ is isosceles.

Problem 4 *Given:* $\angle E \cong \angle H$,
$\overline{EF} \cong \overline{GH}$

Conclusion: $\overline{DF} \cong \overline{DG}$



Proof

Statements	Reasons
1 $\angle E \cong \angle H$	1 Given
2 $\overline{DE} \cong \overline{DH}$	2 If $\triangle$, then $\triangle$.
3 $\overline{EF} \cong \overline{GH}$	3 Given
4 $\triangle DEF \cong \triangle DHG$	4 SAS (2, 1, 3)
5 $\overline{DF} \cong \overline{DG}$	5 CPCTC

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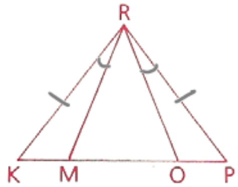
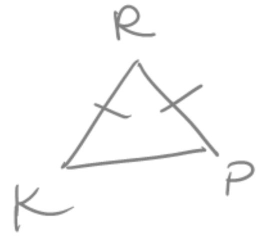
[redacted] Examples

2 Given: $\angle KRM \cong \angle PRO$,
$\overline{KR} \cong \overline{PR}$
Prove: $\overline{RM} \cong \overline{RO}$



~~SSS~~
~~SAS~~
ASA



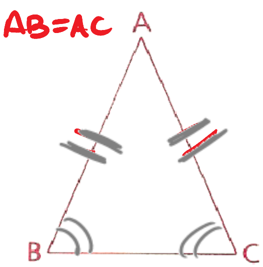
S	R
1. $\angle KRM \cong \angle PRO$	1. Given
2. $\overline{KR} \cong \overline{PR}$	2. Given
3. $\angle K \cong \angle P$	3. $\triangle \Rightarrow \triangle$
4. $\triangle RKM \cong \triangle RPO$	4. ASA (123)
5. $\overline{RM} \cong \overline{RO}$	5. CPCTC (4)

=

20 Given: $\angle A$ is the vertex of an isosceles $\triangle$. $\Rightarrow AB = AC$
- The number of degrees in $\angle B$ is twice the number of centimeters in $\overline{BC}$.
- The number of degrees in $\angle C$ is *three* times the number of centimeters in $\overline{AB}$.

$m\angle B = x + 6$,
$m\angle C = 2x - 54$

Find: The perimeter of $\triangle ABC$

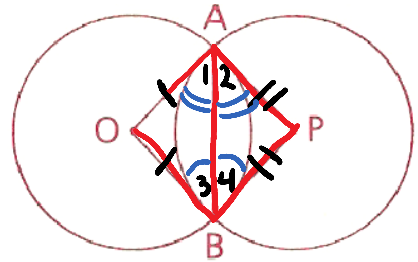


$\angle B = \angle C$
$x + 6 = 2x - 54$
$60 = x$

then $66 = 2(BC)$ & $66 = 3(AB)$
$33 = BC$ — $22 = AB$

P: $AB + AC + BC$
$22 + 22 + 33 = \boxed{77}$

23 Given: $\odot O$,
$\odot P$;
$\overleftrightarrow{AB}$ bisects $\angle$s OAP and OBP.
Prove: Figure AOBP is equilateral.



Statements	Reasons
1. $\odot O$	1. Given
2. $\overline{OA} \cong \overline{OB}$	2. $\odot \Rightarrow \cong$ rad
3. $\angle 1 \cong \angle 3$	3. $\triangle \Rightarrow \triangle$
4. $\odot P$	4. Given
5. $\overline{PA} \cong \overline{PB}$	5. $\odot \Rightarrow \cong$ rad
6. $\angle 2 \cong \angle 4$	6. $\triangle \Rightarrow \triangle$
7. $\overleftrightarrow{AB}$ bis $\angle$s OAP & OBP	7. Given
8. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	8. bis $\Rightarrow \cong \angle$s (7)
9. $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$	9. Trans. (3, 6, 8)
10. $\overline{AB} \cong \overline{BA}$	10. Ref
11. $\triangle OAB \cong \triangle PAB$	11. ASA (9, 10, 9)
12. $\overline{OA} \cong \overline{PA}$ & $\overline{OB} \cong \overline{PB}$	12. CPCTC (12)
13. $\overline{OA} \cong \overline{PA} \cong \overline{OB} \cong \overline{PB}$	13. Trans (12, 2, 5)
14. Quad OBPA = lat	14. $\cong$ sds $\Rightarrow$ = lat.