Pages 139-141 (Section 3.5)

1 Given: AB ≈ DC  $\overline{AC} \cong \overline{DB}$ 

Prove: △ABC ≅ △DCB

 $1 \overline{AB} \cong \overline{DC}, \overline{AC} \cong \overline{DB}$ 

 $2 \overline{BC} \cong \overline{BC}$ 

3 △ABC ≅ △DCB

1 Given

2 Reflexive prop

3 SSS

1 Given

2 Given

4 Given

6 SAS

3 Rt∠s are ≅.

5 Reflexive prop

2 Given: ∠FGH is a rt ∠.

∠JHG is a rt ∠. FG ≅ JH

Prove: △FGH ≅ △JHG

1 ∠FGH is a rt ∠.

2 ∠JHG is a rt ∠.

3 ∠FGH ≅ ∠JHG

4 FG ≅ JH

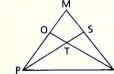
5 GH ≃ GH

6 △FGH ≅ △JHG

3 Given: PM ≈ RM

∠SPM ≃ ∠ORM

Prove: △PSM ≅ △ROM



 $1 \overline{PM} \cong \overline{RM}$ 

2 ∠SPM ≅ ∠ORM

 $3 \angle M \cong \angle M$ 

2 Given 3 Reflexive prop

1 Given

 $4 \triangle PSM \cong \triangle ROM$ 

4 ASA

4 Given: ∠1 ≅ ∠3

42 = 44

Concl:  $\overline{BC} \cong \overline{ED}$ 

 $1 \angle 1 \cong \angle 3, \angle 2 \cong \angle 4$ 

 $2 \overline{CD} \cong \overline{CD}$ 

3 ∠BCD ≃∠EDC

4 ΔBCD ≅ ΔEDC

 $5 \overline{BC} \cong \overline{ED}$ 

1 Given

2 Reflexive prop

3 Addition prop

4 ASA

5 CPCTC

Given: JH ≃ KH

 $\overline{HG} \cong \overline{HM}$ 

**∠5≅∠6** 

Concl: △JHG = △KHM

 $1 \overline{JH} \cong \overline{KH}, \overline{HG} \cong \overline{HM}$ 

2 ∠5 ≅ ∠6

3 ∠GHK ≅ ∠GHK

4 ∠JHG ≅ ∠KHM

3 Reflexive prop

4 Addition prop

5 △JHG ≅ △KHM

5 SAS

1 Given

2 Given

6 Given:  $\angle 1$  is comp to  $\angle 2$ . ∠3 is comp to ∠4.

 $\angle 1 \cong \angle 3$ 

Concl:  $\overline{AB} \cong \overline{CD}$ 

1  $\angle 1$  is comp to  $\angle 2$ .

2 ∠3 is comp to ∠4.

3 ∠1 ≅ ∠3

4 ∠2 ≅ ∠4

5 ∠ABC ≅ ∠DCB

 $6 \ \overline{BC} \cong \overline{BC}$ 

 $7 \triangle ABC \cong \triangle DCB$ 

 $8 \overline{AB} \cong \overline{CD}$ 

1 NOPRS is equilateral.

2 ∠OPR ≅ ∠PRS

 $3 \overline{OP} \cong \overline{SR}$ 

 $4 \overline{PR} \cong \overline{PR}$ 

1 Given

2 Given

3 Given

4 Comps of ≃ ∠s are ≅.

5 Addition prop

6 Reflexive prop

7 ASA

8 CPCTC

7 Given: Figure NOPRS is

equilateral.

∠OPR ≅ ∠PRS

 $\overline{PT} \cong \overline{TR}$ 

Prove:  $\overline{OT} \cong \overline{ST}$ 

5 △OPR ≅ △SRP

 $6 \overline{OR} \cong \overline{SP}$ 

 $7 \overline{PT} \cong \overline{TR}$ 

 $8 \overline{OT} \cong \overline{ST}$ 

1 Given

2 Given

3 All sides are ≅.

4 Reflexive prop

5 SAS

6 CPCTC

7 Given

8 Subtraction prop

Given: ∠9 ≅ ∠10

∠GFH ≅ ∠HJG

Concl:  $\overline{FG} \cong \overline{JH}$ 1 ∠9 ≅ ∠10

 $2 \angle 9 \cong \angle 11, \angle 10 \cong \angle 12$ 

3 ∠11 ≅ ∠12

4 ∠GFH ≅∠HJG

5 ∠GFJ ≅∠HJF

 $6 \overline{\text{FJ}} \cong \overline{\text{FJ}}$ 

7 △GFJ ≅ △HJF  $8 \overline{FG} \cong \overline{JH}$ 

G Given

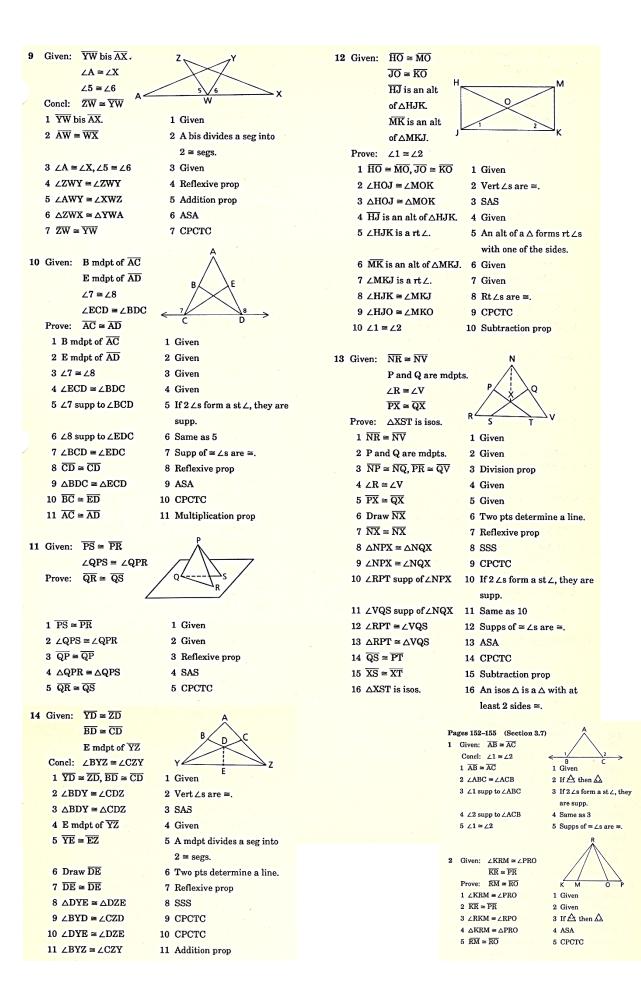
2 Vert∠s are ≅.

3 Transitive prop

4 Given

5 Addition prop 6 Reflexive prop

7 ASA



3 Given:  $\overline{SX} \cong \overline{TY}$ 

 $\overline{WX} \cong \overline{YZ}$ 

 $\overline{SW} \cong \overline{TZ}$ 

Prove:  $\overline{RW} \cong \overline{RZ}$ 

 $1 \overline{SX} \cong \overline{TY}$ 

 $2 \overline{WX} \cong \overline{YZ}$ 

 $3 \overline{SW} \cong \overline{TZ}$ 

 $4 \triangle SWX \cong \triangle TZY$ 

5 ∠RWX ≅ ∠RZY

 $6 \overline{RW} \cong \overline{RZ}$ 

4 Given: ∠3 ≅ ∠6

∠3 comp to ∠4

∠6 comp to ∠5

Prove: AEBC is isos.

1 ∠3 ≃ ∠6

2 ∠3 comp to ∠4

3 ∠6 comp to ∠5

4 ∠4 ≅ ∠5

 $5 \overline{EB} \cong \overline{EC}$ 

6 △EBC is isos.

6 If A then A

1 Given

1 Given

2 Given

3 Given

5 CPCTC

4 SSS

2 Given

3 Given

4 Comps of ≅ ∠s are ≅.

5 If A then A

6 If at least 2 sides of a △

are  $\cong$ , the  $\triangle$  is isos.

5 Given: FH ≈ GJ

△FKJ is isos

with  $\overline{FK} \cong \overline{JK}$ .

Prove: △FKH ≅ △JKG

1 FH ≅ GJ

2 △FKG is isos,

 $\overline{FK} \cong \overline{JK}$ 

3 ∠GFK ≅ ∠HJK

4 △FKH ≅ △JKG

6 Given:  $\angle 5 \cong \angle 6$ JG is alt to FH.

Prove:  $\triangle FJH$  is isos.

1 ∠5 ≅ ∠6

2 JG is alt to FH.

3 ∠JGF and ∠JGH are rt∠s.

4 ∠JGF ≅ ∠JGH

 $5 \overline{JG} \cong \overline{JG}$ 

6 △FJG ≅ △HJG

 $7 \overline{\text{FJ}} \cong \overline{\text{HJ}}$ 

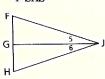
8 ΔFJH is isos.

1 Given

2 Given

3 If A then A

4 SAS



1 Given

2 Given

3 An alt of a △ forms rt ∠s with one of the sides.

4 Rt∠s are ≅.

5 Reflexive prop

6 ASA

7 CPCTC

8 If at least 2 sides of a A

are  $\cong$ , the  $\triangle$  is isos.

8  $m \angle R < m \angle P$  and  $m \angle R + m \angle P < 180$ 

4x < 7x - 18

4x + 7x - 18 < 180

18 < 3x

11x < 198

6 < x

x < 18

.6 < x < 18

9 Given: OP ≅ RS

 $\overline{KO} \cong \overline{KS}$ 

M mdpt of OK

T mdpt of KS

Prove:  $\overline{MP} \cong \overline{TR}$ 

 $1 \overline{OP} \cong \overline{RS}$ 

 $2 \overline{KO} \cong \overline{KS}$ 

3 M mdpt of OK

4 T mdpt of KS

5 MO ≅ TS

6 ∠0 ≅ ∠S

7 △MOP ≅ △TSR

 $8 \overline{MP} \cong \overline{TR}$ 

1 Given

2 Given

3 Given

4 Given

5 Division prop

6 If A then A

7 SAS

8 CPCTC

10 Given: ⊙ O

 $\overline{OX} \cong \overline{XW}$ 

Prove:  $\triangle XOW$  is

equilateral.

100

 $2 \overline{XO} \cong \overline{WO}$ 

 $3 \overline{OX} \cong \overline{XW}$ 

 $4 \overline{WO} \cong \overline{XW}$ 

1 Given

2 Radii of a ⊙ are ≅.

3 Given

4 Transitive prop

5 An equilateral △ is one in

which all sides are ≅.

11  $\angle$ ACB is 90° because  $\overline{AC} \perp \overline{BC}$ .

5 △XOW is equilateral.

3x = 90BC = x + 20

AC = 2x - 20,

AC = 2(30) - 20 = 40

x = 30BC = 30 + 20 = 50

No, △ABC is not isos because no sides are ≅.

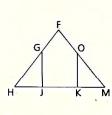
(AB > either leg)

12 a Since  $\overline{QS}$  and  $\overline{QP}$  are both radii of O Q,  $\overline{QS} \cong \overline{QP}$ .

 $m \angle PSQ = m \angle P = 36^{\circ}$ 

**b**  $\angle$ PSR is rt  $\angle$ , so m $\angle$ R = 180° - 90° - 36° = 54°

13 Given:  $\overline{BE} \cong \overline{BD}$  $\overline{\text{BE}} \perp \overline{\text{AE}}$ ∠BDC = 90° Prove: ∠AED ≅ ∠CDE  $1 \overline{BE} \cong \overline{BD}$ 1 Given  $2 \angle 1 \cong \angle 2$ 2 If A then △ 3 BE L AE 3 Given 4 ∠BEA is a rt ∠. 4 If 2 segs are ⊥, they form rt Ls. 5 ∠BDC = 90° 5 Given 6 ∠BDC is a rt∠. 6 Art∠is an∠of 90°. 7 ∠BEA ≃ ∠BDC 7 Rt∠s are ≅. 8 ∠AED ≡ ∠CDE 8 Addition prop 14 Given: △ABC is isos with  $\overline{AB} \cong \overline{AC}$ . AX is median to BC. Prove: AX bis ∠BAC. 1 △ABC is isos, 1 Given  $\overline{AB} \cong \overline{AC}$ . 2 If A then A  $2 \angle B \cong \angle C$ 3 AX is median to BC. 3 Given  $4 \ \overline{\text{BX}} \cong \overline{\text{XC}}$ 4 A median of a △ divides one side into  $2 \cong segs$ . 5 △ABX ≅ △ACX 5 SAS 6 ∠BAX ≃ ∠CAX 6 CPCTC 7 AX bis∠BAC. 7 If a ray divides an ∠ into  $2 \cong \angle s$ , it bis the  $\angle$ . 15 Given: HK ≅ JM  $\overline{GJ} \cong \overline{JK}$ 



to HM. Prove:  $\triangle FHM$  is isos.  $1 \overline{HK} \cong \overline{JM}$ 1 Given  $2 \overline{JK} \cong \overline{JK}$ 2 Reflexive prop 3 HJ ≅ KM 3 Subtraction prop  $4 \overline{GJ} \cong \overline{JK}$ 4 Given  $5 \ \overline{OK} \cong \overline{JK}$ 5 Given  $6 \overline{GJ} \cong \overline{OK}$ 6 Transitive prop 7 GJ and OK \(\pm\) HM 7 Given 8 ∠GJH and ∠OKM 8 If 2 segs are ⊥, they are rt∠s. form rt∠s.

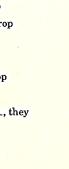
9 Rt∠s are ≅.

10 SAS

 $\overline{OK} \cong \overline{JK}$   $\overline{GJ}$  and  $\overline{OK}$  are  $\bot$ 

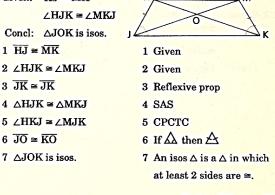
9 ∠GJH ≅ ∠OKM

10 △GJH ≃ △OMK



11 ∠H ≅ ∠M 11 CPCTC 12 HF ≅ MF 12 If A then A 13 △FHM is isos. 13 If  $a \triangle has 2 \cong sides$ , it is isos. 16 Given: PR = ST  $\overline{NP} \cong \overline{VT}$  $\angle P \cong \angle T$ Prove: \( \Delta WRS \) is isos. 1 PR ≈ ST 1 Given  $2 \overline{RS} \cong \overline{RS}$ 2 Reflexive prop  $3 \overline{PS} \cong \overline{RT}$ 3 Addition prop  $4 \overline{NP} \cong \overline{VT}$ 4 Given  $5 \angle P \cong \angle T$ 5 Given 6 △NPS ≅ △VTR 6 SAS  $7 \angle S \cong \angle R$ 7 CPCTC 8 RW ≃ SW 8 If A then A 9 AWRS is isos. 9 If  $a \triangle has 2 \cong sides$ , it is isos. 17 Given: YZ is base of an isos △. **∠2** ≃ **∠**Z  $\angle 1 \cong \angle Y$ Prove: XA bis ∠BXZ. 1  $\overline{YZ}$  base of isos  $\Delta$ . 1 Given  $2 \angle 2 \cong \angle Z, \angle 1 \cong \angle Y$ 2 Given  $3 \overline{XY} \cong \overline{XZ}$ 3 An isos  $\triangle$  is a  $\triangle$  in which at least 2 sides are ≅. 4 If A then A  $4 \angle Y \cong \angle Z$ 5 ∠1 ≅ ∠2 5 Transitive prop 6 XA bis∠BXZ. 6 If a ray divides an ∠ into

18 The base of the pyramid is a square, and since all 4 sides of a square are ≅, the 4 bases of the isos Δs are ≅. Each Δ is isos. Since each Δ shares a side with 2 other Δs, all 4 legs involved must be ≅. Then all 4 Δs are ≅ by SSS.
19 Given: HJ ≅ MK



 $2 \cong \angle s$ , it bis the  $\angle$ .

20 Since 
$$\overline{AB} \cong \overline{AC}$$
,  $\angle B \cong \angle C$  because if  $\triangle$  then  $\triangle$ .  
 $x + 6 = 2x - 54$ 

$$60 = x$$

$$\angle B = 60 + 6 = 66, \angle C = 2(60) - 54 = 66$$

BC = 
$$\frac{1}{2}(66) = 33$$

$$AB = \frac{1}{3}(66) = 22$$

 $\overline{AC} \cong \overline{AB}$  so AC = 22

Perimeter of △ABC = 77 cm

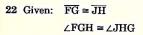
## 21 Given: CE ≅ CF

∠E supp to ∠5

Prove: \( \triangle CDG \) is isos.

- $1 \overline{CE} \cong \overline{CF}$
- $2 \angle E \cong \angle F$
- 3 ∠F≅∠3
- 4 ∠E ≅ ∠3
- 5 ∠E supp∠5
- 6 ∠4 supp∠5
- 7 ∠E ≅ ∠4
- 8 ∠3 ≅ ∠4
- $9 \ \overline{CD} \cong \overline{CG}$
- 10 ΔCDG is isos.

- 1 Given
- 2 If A then A
- 3 Given
- 4 Transitive prop
- 5 Given
- 6 If 2∠s form a st∠, they
- are supp. 7 Supp of same ∠ are ≅.
- 8 Transitive prop
- 9 If A then A
- 10 An isos △ is a △ in which at least 2 sides are ≅.



Concl: △FKJ is isos.

- $1 \overline{FG} \cong \overline{JH}$
- 2 ∠FGH ≅ ∠JHG
- 3 GH ≅ GH
- 4 △FGH ≅ △JHG
- 5 ∠JGH ≅ ∠FHG
- $6 \ \overline{GK} \cong \overline{KH}$
- 7 JG ≅ FH
- 8 FK ≅ KJ
- 9 △FKJ is isos.
- 7 CPCTC 8 Subtraction prop

6 If A then A

3 Reflexive prop

1 Given

2 Given

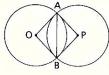
4 SAS

5 CPCTC

- 9 An isos △ is a △ in which at least 2 sides are ≅.
- 23 Given: OO, OP
  - AB bis ∠OAP
  - and∠OBP.

Prove: Figure AOBP is

equilateral.



- 1 0 0, 0 P
- $2 \overline{OA} \cong \overline{OB}$
- $3 \overline{AP} \cong \overline{PB}$
- 4 ∠OAB ≅ ∠OBA
- 5 ∠PAB ≅ ∠PBA
- 6 AB bis∠OAP and
- ∠OBP.
- $7 \angle OAB \cong \angle PAB$ ,
- ∠OBA ≅ ∠PBA
- 8 ∠OBA ≅ ∠PBA ≅
  - ∠OAB ≅ ∠PAB
- $9 \overline{AB} \cong \overline{AB}$
- 10 △OAB ≅ △PAB
- 11  $\overline{OA} \cong \overline{PA}, \overline{OB} \cong \overline{PB}$
- 12 OA ≅ PA ≅
  - $\overline{OB} \cong \overline{PB}$
- 13 Figure AOBP is
  - equilateral.

- 1 Given
- 2 Radii of a ⊙ are ≅.
- 3 Same as 2
- 4 If A then A
- 5 If A then A
- 6 Given
- 7 If a line bis an ∠, it divides the  $\angle$  into  $2 \cong \angle$ s.
- 8 Transitive prop
- 9 Reflexive prop
- 10 ASA
- 11 CPCTC
  - - 12 Transitive prop
    - - 13 If all sides of a figure are
      - ≅, the figure is equilateral.
- 24 Given: Figure XSTOW is equilateral and equiangular.
  - Prove: AYTO is isos.
  - 1 XSTOW is equilateral and equiangular.
  - $2 \overline{ST} \cong \overline{WO}$
- $3 \overline{TO} \cong \overline{TO}$ 
  - 4 ∠STO ≅ ∠WOT

  - $5 \triangle STO \cong \triangle WOT$
  - 6 ∠YOT ≅ ∠YTO
  - $7 \overline{\text{TY}} \cong \overline{\text{YO}}$
  - 8 ΔΥΤΟ is isos.
- 5 SAS

1 Given

- 6 CPCTC
- 7 If A then A
- 8 If a △ has at least 2 sides  $\cong$ , the  $\triangle$  is isos.

2 If a figure is equilateral,

4 If a figure is equiangular,

all sides are ≅.

3 Reflexive prop

all ∠s are ≅.

- 25 Since  $\angle D \cong \angle F \cong \angle DEF$  and  $\overline{GE} \perp \overline{DE}$ ,
  - $\angle FEG + \angle D = 90$  and  $\angle FEG + \angle F = 90$ .
    - 4x + y = 96

x + y + 3x - 6 = 90

x + 7y = 78

x + y + 6y + 12 = 90

- 4x = 96 y
- $x = 24 \frac{1}{4}y$  Substituting,
  - $(24 \frac{1}{4}y) + 7y = 78$
- Then  $x = 24 \frac{1}{4}y$ 
  - $x = 24 \frac{1}{4}(8) = 22$  and  $\angle F = 6(8) + 12$
  - $x = 22, y = 8, \angle F = 60^{\circ}$

22 Given:  $\overline{FG} \cong \overline{JH}$ 

∠FGH ≅ ∠JHG

Concl: AFKJ is isos.

 $1 \overline{FG} \cong \overline{JH}$ 

2 ∠FGH ≅ ∠JHG

 $3 \overline{GH} \cong \overline{GH}$ 

 $4 \triangle FGH \cong \triangle JHG$ 

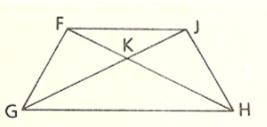
5 ∠JGH ≅ ∠FHG

 $6 \ \overline{GK} \cong \overline{KH}$ 

 $7 \overline{JG} \cong \overline{FH}$ 

 $8 \overline{FK} \cong \overline{KJ}$ 

9 △FKJ is isos.



1 Given

2 Given

3 Reflexive prop

4 SAS

5 CPCTC

6 If A then A

7 CPCTC

8 Subtraction prop

9 An isos △ is a △ in which

at least 2 sides are ≅.

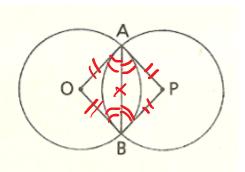
23 Given: ⊙ O, ⊙ P

AB bis ∠OAP

and ∠OBP.

Prove: Figure AOBP is

equilateral.



1 0 0, 0 P

 $2 \overline{OA} \cong \overline{OB}$ 

 $3 \overline{AP} \cong \overline{PB}$ 

4 ∠OAB ≅ ∠OBA

5 ∠PAB ≅ ∠PBA

6 AB bis∠OAP and

ZOBP.

 $7 \angle OAB \cong \angle PAB$ ,

∠OBA ≅ ∠PBA

8 ∠OBA ≅ ∠PBA ≅

∠OAB ≅ ∠PAB

 $9 \overline{AB} \cong \overline{AB}$ 

10 △OAB ≅ △PAB

11  $\overline{OA} \cong \overline{PA}, \overline{OB} \cong \overline{PB}$ 

 $12 \overline{OA} \cong \overline{PA} \cong$   $\overline{OB} \cong \overline{PB}$ 

13 Figure AOBP is equilateral.

1 Given

2 Radii of a ⊙ are ≅.

3 Same as 2

4 If A then A

5 If A then A

6 Given

7 If a line bis an ∠, it divides the ∠ into 2 ≅ ∠s.

8 Transitive prop

9 Reflexive prop

10 ASA

11 CPCTC

12 Transitive prop

13 If all sides of a figure are ≅, the figure is equilateral. 24 Given: Figure XSTOW is equilateral and equiangular.

Prove: AYTO is isos.

 XSTOW is equilateral and equiangular.

$$2 \overline{ST} \cong \overline{WO}$$



$$3 \overline{TO} \cong \overline{TO}$$

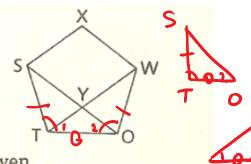
4 ∠STO ≅ ∠WOT

$$5 \triangle STO \cong \triangle WOT$$

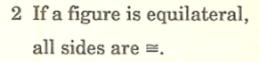
$$6 \angle YOT \cong \angle YTO$$

$$7 \overline{\text{TY}} \cong \overline{\text{YO}}$$

8 ΔΥΤΟ is isos.



1 Given



3 Reflexive prop

4 If a figure is equiangular, all ∠s are ≅.

- 5 SAS
- 6 CPCTC
- 7 If A then A
- 8 If  $a \triangle$  has at least 2 sides  $\cong$ , the  $\triangle$  is isos.

Since 
$$\angle D \cong \angle F \cong \angle DEF$$
 and  $\overline{GE} \perp \overline{DE}$ ,  
 $\angle FEG + \angle D = 90$  and  $\angle FEG + \angle F = 90$ .  
 $x + y + 3x - 6 = 90$   $x + y + 6y + 12 = 90$   
 $4x + y = 96$   $x + 7y = 78$   
 $4x = 96 - y$   
 $x = 24 - \frac{1}{4}y$  Substituting,

$$(24 - \frac{1}{4}y) + 7y = 78$$

$$\frac{27}{4} = 54$$

$$y = 8$$

Then x = 
$$24 - \frac{1}{4}y$$
  
x =  $24 - \frac{1}{4}(8) = 22$  and  $\angle F = 6(8) + 12$   
x =  $22$ , y =  $8$ ,  $\angle F = 60^{\circ}$ 

25 Given: 
$$\triangle FED$$
 is equilateral.  $\rightarrow a$ 11 LS&SdS  $\stackrel{\square}{=}$ 

$$\overline{GE} \perp \overline{DE}, \rightarrow 90$$

$$m \angle FEG = x + y,$$

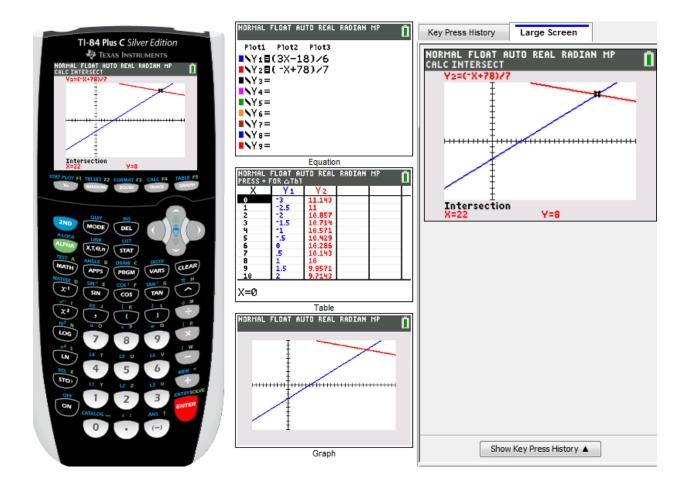
$$m \angle D = 3x - 6,$$

$$m \angle F = 6y + 12$$

Find: x, y, and  $\angle F$ 

$$2D=2F$$
 $3x-6=6y+12$ 
 $3x-6y=18$ 
 $3x-2y=6$ 

$$\angle DEF + \angle FEG = 90^{\circ}$$
  
 $6y + 12 + x + y = 90 *$   
 $\Rightarrow +7y = 78$   
 $x = -7y + 78$ 



$$3x-6 = 6y + 12$$
  
 $3x-18 = 6y$   
 $\frac{1}{6}(3x-18) = y$   
 $(3x-18)/6 = y$ 

$$6y+12+x+y=90$$

$$7y + x + 12 = 90$$

$$7y = -x + 78$$

$$4y= (-x+78)/7$$