AMDG

Name | Ms. Kresovic

Adv Geo - | 3.3: CPCTC & Circles | DATE

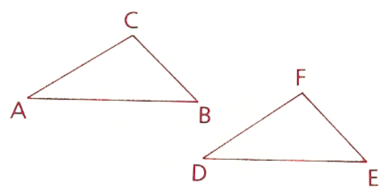
Objectives

After studying this section, you will be able to

- Apply the principle of CPCTC
- Recognize some basic properties of circles

CPCTC

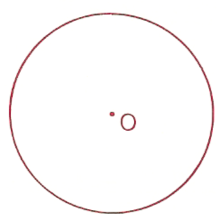
Suppose that in the figure $\triangle ABC \cong \triangle DEF$. Is it therefore true that $\angle B \cong \angle E$? If you refer to Section 3.1, you will find that we have already answered yes to this question in the definition of *congruent triangles*.



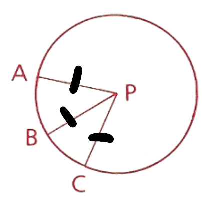
In the portions of the book that follow, we shall often draw such a conclusion *after* knowing that some triangles are congruent. We shall use CPCTC as the reason. *CPCTC* is short for *"Corresponding Parts of Congruent Triangles are Congruent."* By corresponding parts, we shall mean only the matching angles and sides of the respective triangles.

Introduction to Circles

Point O is the center of the circle shown at the right. By definition, every point of the circle is the same distance from the center. The center, however, is not an element of the circle; the circle consists only of the "rim." A circle is named by its center: this circle is called circle O (or $\odot$O).



Points A, B, and C lie on circle P ($\odot$P).
$\overline{PA}$ is called a *radius*.
$\overline{PA}$, $\overline{PB}$, and $\overline{PC}$ are called *radii*.



From previous math courses you may remember formulas for the area and the circumference of a circle:

$$A = \pi r^2$$
$$C = 2\pi r$$

By pressing the $\boxed{\pi}$ key on a scientific calculator, you can find that $\pi \approx 3.141592654$.

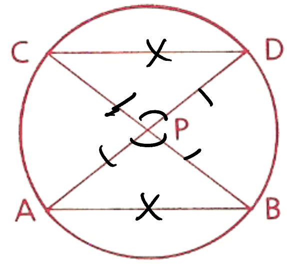
Theorem 19 ***All radii of a circle are congruent.***

$\odot \Rightarrow \cong$ rad

Problem 1

Given: ⊙P

Conclusion: $\overline{AB} \cong \overline{CD}$



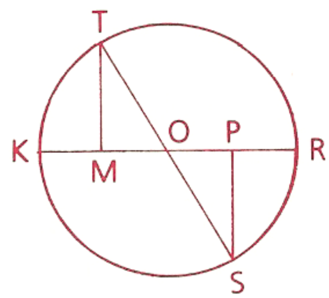
Proof

Statements	Reasons
1 ⊙P	1 GIVEN
2 $\overline{PA} \cong \overline{PB} \cong \overline{PC} \cong \overline{PD}$	2 ⊙ ⇒ ≅ rad
3 $\angle CPD \cong \angle APB$	3 vert∠ ⇒ ≅ ∠s
4 $\triangle CPD \cong \triangle APB$	4 SAS (2 3 2)
5 $\overline{AB} \cong \overline{CD}$	5 CPCTC (4)

Problem 2

Given: ⊙O;
∠T is comp. to ∠MOT.
∠S is comp. to ∠POS.

Prove: $\overline{MO} \cong \overline{PO}$



Proof

Statements	Reasons
1 ⊙O	1
2 $\overline{OT} \cong \overline{OS}$	2
3 ∠T is comp. to ∠MOT.	3
4 ∠S is comp. to ∠POS.	4
5 $\angle MOT \cong \angle POS$	5
6 $\angle T \cong \angle S$	6
7 $\triangle MOT \cong \triangle POS$ (Watch the correspondence.)	7
8 $\overline{MO} \cong \overline{PO}$	8

Problem 3

Find the exact area & circumference of a circle whose radius is 12.5 cm.

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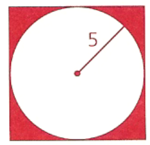
Problem 4

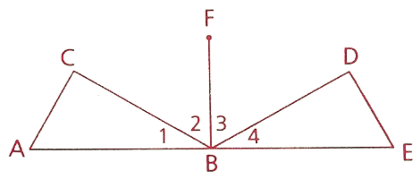
Explain why the area of the shaded region is $100 - 25\pi$.



Problem 5

Given: $\angle A \cong \angle E$,
$\overline{AB} \cong \overline{BE}$,
$\overline{FB} \perp \overline{AE}$,
$\angle 2 \cong \angle 3$

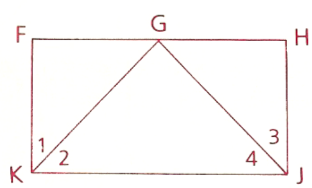
Prove: $\overline{CB} \cong \overline{DB}$



Problem 6

Given: $\overline{KG} \cong \overline{GJ}$,
$\angle 2 \cong \angle 4$,
$\angle 1$ is comp. to $\angle 2$.
$\angle 3$ is comp. to $\angle 4$.
$\angle FGJ \cong \angle HGK$

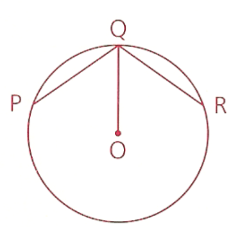
Conclusion: $\overline{FG} \cong \overline{HG}$



Problem 7

Given: $\odot O$,
$\overline{PQ} \cong \overline{QR}$

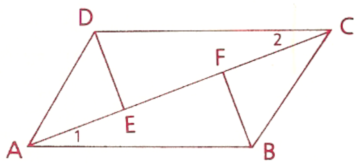
Prove: $\overrightarrow{QO}$ bisects $\angle PQR$.



Problem 8

21 Given: $\overline{AE} \cong \overline{FC}$,
$\overline{FB} \cong \overline{DE}$,
$\angle CFB \cong \angle AED$

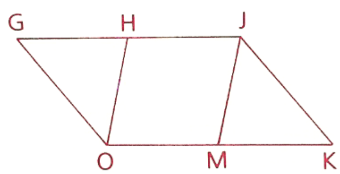
Prove: $\angle 1 \cong \angle 2$



Problem 9

Given: H is the midpt. of $\overline{GJ}$.
M is the midpt. of $\overline{OK}$.
$\overline{GO} \cong \overline{JK}$,
$\overline{GJ} \cong \overline{OK}$,
$\angle G \cong \angle K$,
$OK = 27$,
$m\angle GOH = x + 24$, $m\angle GHO = 2y - 7$,
$m\angle JMK = 3y - 23$, $m\angle MJK = 4x - 105$

Find: $m\angle GOH$, $m\angle GHO$, and GH



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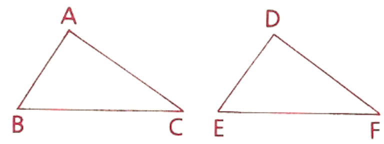
Homework

Problem Set A

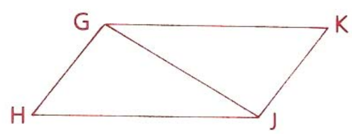
1 Given: $\overline{AB} \cong \overline{DE}$,
$\overline{BC} \cong \overline{EF}$,
$\overline{AC} \cong \overline{DF}$

Prove: $\angle A \cong \angle D$



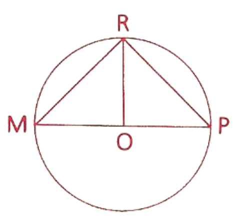
2 Given: $\angle HGJ \cong \angle KJG$,
$\angle KGJ \cong \angle HJG$

Conclusion: $\overline{HG} \cong \overline{KJ}$



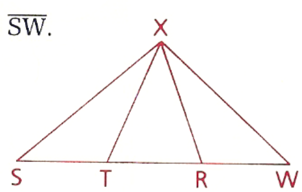
3 Given: $\odot O$,
$\overline{RO} \perp \overline{MP}$

Prove: $\overline{MR} \cong \overline{PR}$

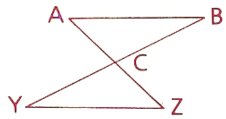


4 Given: T and R trisect $\overline{SW}$.
$\overline{XS} \cong \overline{XW}$,
$\angle S \cong \angle W$

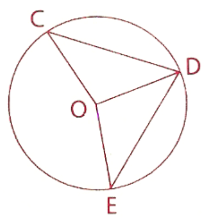
Prove: $\overline{XT} \cong \overline{XR}$



5 Given: $\angle B \cong \angle Y$;
C is the midpt. of $\overline{BY}$.
Conclusion: $\overline{AB} \cong \overline{YZ}$

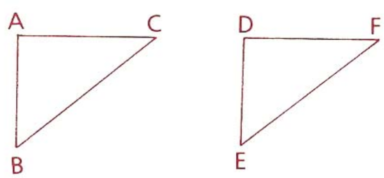


6 Given: $\odot O$,
$\overline{CD} \cong \overline{DE}$
Prove: $\angle COD \cong \angle DOE$



7) Find, to the nearest tenth, the area & circumference of a circle whose radius is 12.5 cm.

8 $\triangle ABC \cong \triangle DEF$,
$\angle A = 90°$, $\angle B = 50°$, $\angle C = 40°$,
$m\angle E = 12x + 30$, $m\angle F = \frac{y}{2} - 10$,
$m\angle D = \sqrt{z}$
Solve for x, y, and z.



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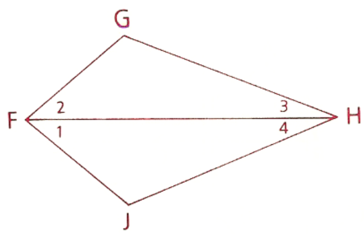
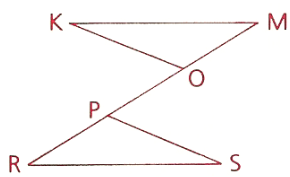
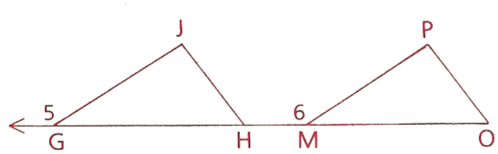
9 Given: $\overleftrightarrow{FH}$ bisects $\angle GFJ$ and $\angle GHJ$.
Conclusion: $\overline{FG} \cong \overline{FJ}$



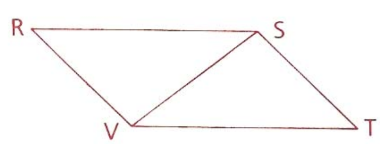
10 Given: $\angle M \cong \angle R$, $\angle RPS \cong \angle MOK$, $\overline{MP} \cong \overline{RO}$
Conclusion: $\overline{KM} \cong \overline{RS}$



14 Given: $\angle 5 \cong \angle 6$, $\angle JHG \cong \angle O$, $\overline{GH} \cong \overline{MO}$
Conclusion: $\angle J \cong \angle P$

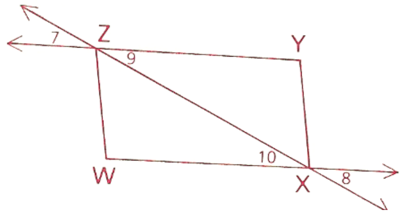


15 Given: $\angle RST \cong \angle RVT$, $\angle RVS \cong \angle TSV$
Conclusion: $\overline{RS} \cong \overline{VT}$



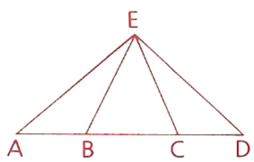
16 Given: $\angle 7 \cong \angle 8$,
$\overline{ZY} \cong \overline{WX}$

Prove: $\angle W \cong \angle Y$



17 Given: $\angle AEC \cong \angle DEB$,
$\overline{BE} \cong \overline{CE}$,
$\angle ABE \cong \angle DCE$

Prove: $\overline{AB} \cong \overline{CD}$



19 **a** Find the coordinates of point P.
b Find the area of the circle.