Name Stu Dent | | Ms. Kreosvic
Adv Geo 8 | 12.3: Surface Area of Circular Solids & 12.4: Volume of Prisms and Cylinders | M 12 May 14

SURFACE AREAS OF CIRCULAR SOLIDS

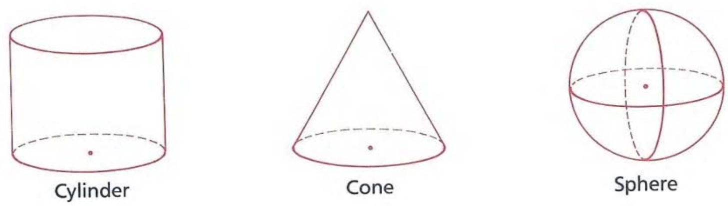
Objective

After studying this section, you will be able to

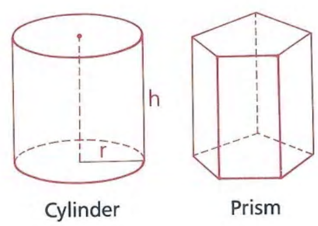
- Find the surface areas of circular solids

Part One: Introduction

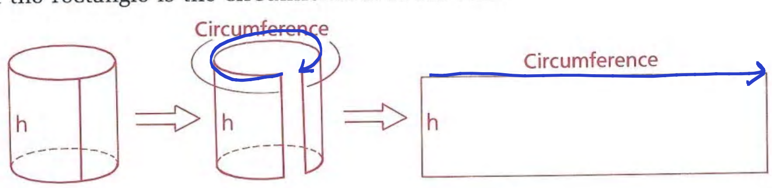
Consider the following three solids that are based on the circle.



A ***cylinder*** resembles a prism in having two congruent parallel bases. The bases of a cylinder, however, are circles. In this text, *cylinder* will mean a right circular cylinder—that is, one in which the line containing the centers of the bases is perpendicular to each base.



The lateral area of a cylinder can be visualized by thinking of a cylinder as a can, and the lateral area as the label on the can. If we cut the label and spread it out, we see that it is a rectangle. The height of the rectangle is the same as the height of the can. The base of the rectangle is the circumference of the can.

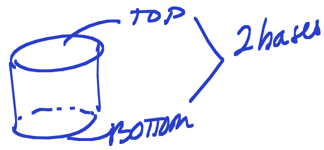


$d \cdot \pi \cdot h$

Theorem 113 ***The lateral area of a cylinder is equal to the product of the height and the circumference of the base.***

$$L.A._{cyl} = Ch = 2\pi rh = h \cdot d\pi$$

where C is the circumference of the base, h is the height of the cylinder, and r is the radius of the base.

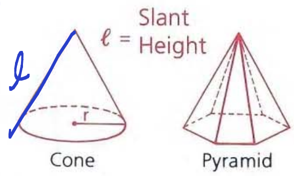


Definition The total area of a cylinder is the sum of the cylinder's lateral area and the areas of the two bases.

$$T.A._{cyl} = L.A. + 2A_{base}$$

$$h \cdot d\pi + 2[\pi r^2]$$

A ***cone*** resembles a pyramid, but its base is a circle. In a pyramid the slant height and the lateral edge are different; in a cone they are the same.



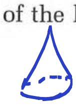
In this book, the word *cone* will mean a right circular cone—one in which the altitude passes through the center of the circular base.

Theorem 114 ***The lateral area of a cone is equal to one-half the product of the slant height and the circumference of the base.***

$$L.A._{cone} = \frac{1}{2}C\ell = \pi r\ell$$

where C is the circumference of the base, ℓ is the slant height, and r is the radius of the base.

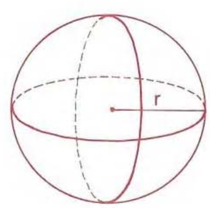
Definition The total area of a cone is the sum of the lateral area and the area of the base.



$$T.A._{cone} = L.A. + A_{base}$$

$$\pi(r \cdot \ell) + \pi(r^2)$$

A ***sphere*** is a special figure with a special surface-area formula. (A sphere has no lateral edges and no lateral area.) The proof of the formula requires the concept of limits and will not be given here.



Postulate $T.A._{sphere} = \boxed{4\pi r^2}$

where r is the sphere's radius.

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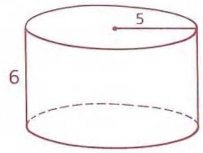
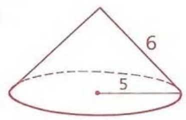
12. 3 Class Examples

Problem *Find the total area of each figure.*

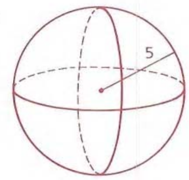
a



b



c



Solution

a $T.A._{cyl} = L.A. + 2A_{base}$
$= 2\pi rh + 2\pi r^2$
$= 2\pi(5)(6) + 2\pi(5^2)$
$= 110\pi$

b $T.A._{cone} = L.A. + A_{base}$
$= \pi r\ell + \pi r^2$
$= \pi(5)(6) + \pi(5^2)$
$= 55\pi$

c $T.A._{sphere} = 4\pi r^2$
$= 4\pi(5^2)$
$= 100\pi$

Part Three: Problem Sets

Problem Set A

1 What is the total area of a sphere having

a A radius of 7?

$A_{SPHERE} = 4\pi r^2, \; r = 7$

$4(49)\pi = 196\pi$

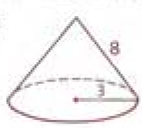
c A diameter of 6?

$r = 3, \; A_{SPH} = 4\pi r^2$

$4 \cdot 3 \cdot 3\pi = 36\pi$

2 Find the lateral area and the total area of each solid.

a

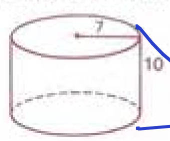


$LA_{CONE}: \pi \cdot r \cdot \ell = 24\pi$

$A_{CIRCLE}: \pi r^2 = 9\pi$

$TSA = 33\pi$

b



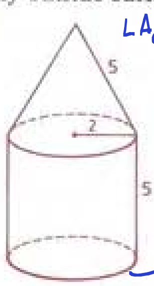
$LA_{CYL} = d \cdot h \cdot \pi = 140\pi$

$2A_{\odot} : 2\pi r^2 = 2(49\pi) = 98\pi$

238π

4 Find the total area of each solid. (Hint: Be sure that you include only outside surfaces and that you do not miss any.)

a



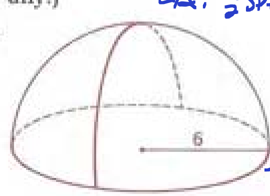
$LA_{CONE}: \pi r \cdot \ell = 10\pi$

$LA_{CYL}: d \cdot \pi \cdot h = 20\pi$

$-1\,Base = \pi r^2 = 4\pi$

$TSA = 34\pi$

b



This is a *hemisphere* ("half sphere"). The T.A. includes the area of the circular base.

$LA: \frac{1}{2}\,SPHERE = \frac{1}{2}(4\pi r^2)$

$\frac{4 \cdot 6 \cdot 6\pi}{2} = 72\pi$

$-Base: \pi r^2 = 36\pi$

$TSA = 108\pi$

5 ABCD is a parallelogram, with A =(-4, 3), B = (6, 3), C =(-3, 12), and D = (5, 12).

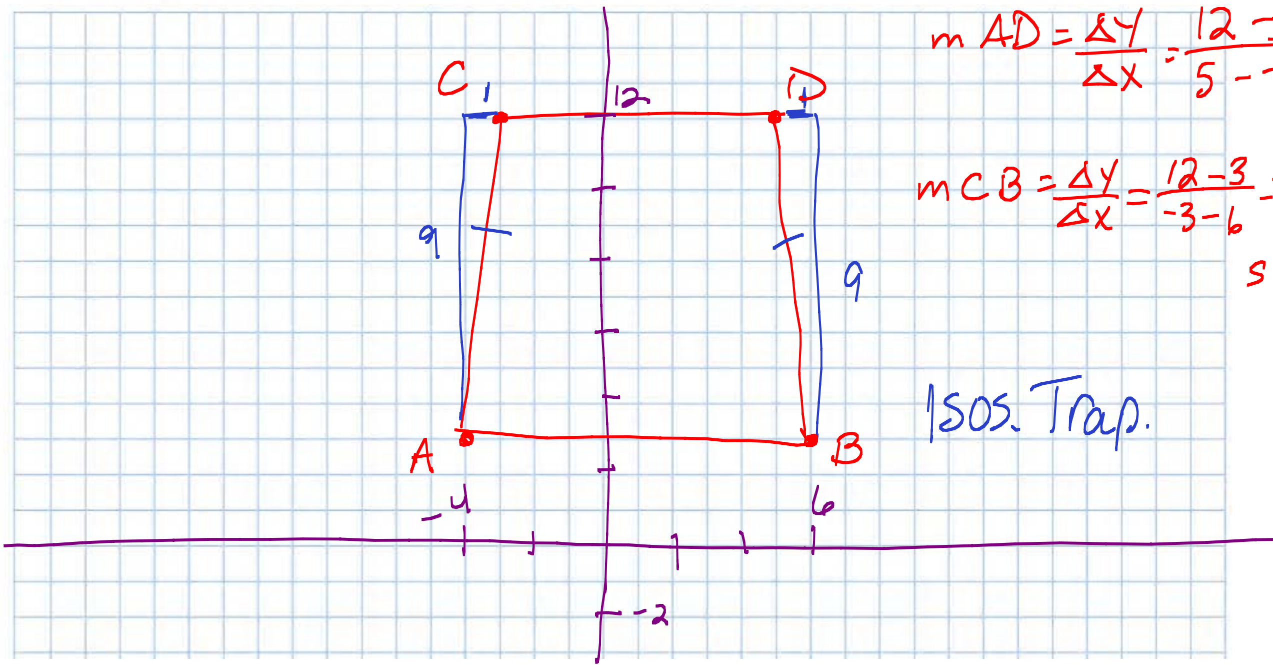
a Find the slopes of the diagonals, $\overline{AC}$ and $\overline{BD}$.

b Use your answers to part a to identify ▱ABCD by its most specific name.

−4 12 6

3

You could use your ID as a straight edge!



$m\,AD = \frac{\Delta y}{\Delta x} = \frac{12-3}{5--4} = \frac{6}{9} = \frac{2}{3}$

$m\,CB = \frac{\Delta y}{\Delta x} = \frac{12-3}{-3-6} = \frac{9}{-9} = 1$

slopes are not ⊥

Isos. Trap.

6 Find the total (including the rectangular face) surface area of a half cylinder with a radius of 5 and a height of 2.

6 $A \text{ of } \frac{1}{2}\text{cyl} = LA + A_{\text{base }1} + A_{\text{base }2} + A_{\text{cut surface}}$

$LA = \frac{1}{2}(C \cdot h)$ $A_{\text{base}} = \frac{1}{2}\pi r^2$

$LA = \frac{1}{2}(2\pi rh)$ $A_{\text{base}} = \frac{1}{2}\pi(5)^2$

$LA = \frac{1}{2}(2)\pi(5)(2)$ $A_{\text{base}} = \frac{25}{2}\pi$

$LA = 10\pi$

$A_{\text{cut surface}} = \ell \cdot w$

$A_{\text{cut surface}} = 2 \cdot 10$

$A_{\text{cut surface}} = 20$

$A_{\frac{1}{2}\text{cyl}} = 10\pi + 2(\frac{25}{2})\pi = 20$

$A_{\frac{1}{2}\text{cyl}} = 35\pi + 20$

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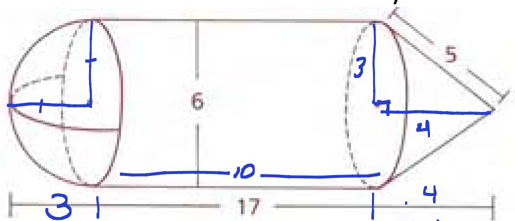
10 Find the total area of the solid.



TSA: $\frac{1}{2}$ SPHERE + LA_{CYL} + LA_{CONE}

$= \frac{4\pi r^2}{2} + d\pi \cdot h + \pi r \cdot l$

$\frac{4 \cdot 3 \cdot 3 \cdot \pi}{2} + 6 \cdot \pi \cdot 10 + \pi\, 3 \cdot 5$

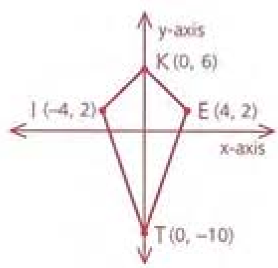
$18\pi + 60\pi + 15\pi$

93π

11 KITE is a kite.

a Find the area of KITE.

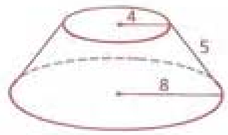
b Find the area of the rectangle formed when consecutive midpoints of the sides of KITE are connected.



a $A_{kite} = \frac{1}{2} d_1 \cdot d_2 = \frac{1}{2}(16)(8) = 64$

b From mdpt form, mid $\overline{KE} = (2, 4)$, mid $\overline{KI} = (-2, 4)$, and mid $\overline{ET} = (2, -4)$. Area $\square = 4 \cdot 8 = 32$

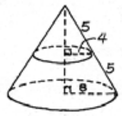
13 The solid at the right is called a *frustum* of a cone. Find its total area if the radii of the top and bottom bases are 4 and 8 respectively, and the slant height is 5.



Draw in the rest of the cone and find the area.

$LA = \pi(8)(10) = 80\pi$

Subtract area of top cone.



$LA = \pi(4)(5) = 20\pi$

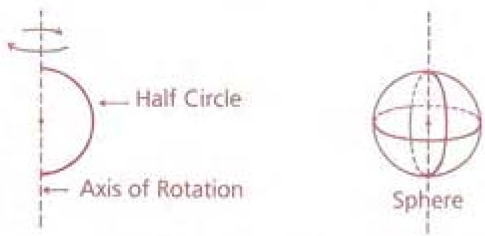
$80\pi - 20\pi = 60\pi$

radius of large $\odot = 8$, small $\odot = 4$

slant ht large = 10, small = 5, (converse of Midline Th)

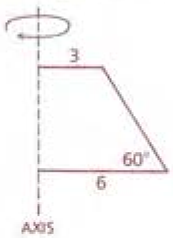
$TA = 16\pi + 64\pi + (\pi \cdot 8 \cdot 10 - \pi \cdot 4 \cdot 5) = 140\pi$

14 A *surface of rotation* is generated by revolving a shape about a fixed line, called the *axis of rotation*. For example, revolving a half circle about the line containing its endpoints produces a sphere.

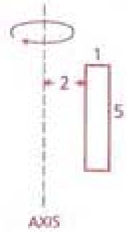


Identify the surface of rotation generated in each diagram below and compute the total area of each of these surfaces.

a



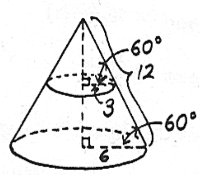
b



c

a Forms the frustrum of a cone.

Extend the frustrum to form a cone. There are two 30°60°90° Δs formed with $\parallel$ bases.



In the large Δ, hypotenuse = ℓ of cone = 12

In small Δ, hypotenuse = 2(3) = 6

$TA = LA_{\text{large cone}} - LA_{\text{small cone}} + A_{\text{base 1}} + A_{\text{base 2}}$

$TA = \pi r \cdot \ell - \pi r \cdot \ell + \pi r^2 + \pi r^2$

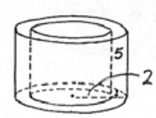
$TA = \pi(6)(12) - \pi(3)(6) + \pi(6)^2 + \pi(3)^2$

$TA = 72\pi - 18\pi + 36\pi + 9\pi = 99\pi$

b Forms cylindrical shell.

radius sm $\odot = 2$

radius lg $\odot = 2 + 1 = 3$



height = 5

$TA = LA_{\text{large cyl}} + LA_{\text{small cyl}} + 2(A_{\text{large base}} - A_{\text{small base}})$

$TA = 2\pi rh + 2\pi rh + 2(\pi r^2 - \pi r^2)$

$TA = 2\pi(3)(5) + 2\pi(2)(5) + 2[\pi(3)^2 - \pi(2)^2]$

$TA = 2\pi(15) + 20\pi + 2(9\pi - 4\pi)$

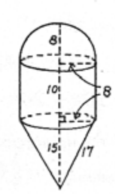
$TA = 30\pi + 20\pi + 10\pi = 60\pi$

c Forms same solid as problem 9.

radius = 8

height cyl = 10

slant height cone = 17



$TA = \frac{1}{2}(A_{\text{sphere}}) + LA_{\text{cylinder}} + LA_{\text{cone}}$

$TA = \frac{1}{2}(4\pi r^2) + 2\pi rh + \pi r\ell$

$TA = \frac{1}{2}[4\pi(8)^2] + 2\pi(8)(10) + \pi(8)(17)$

$TA = 128\pi + 160\pi + 136\pi = 424\pi$

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VOLUMES OF PRISMS AND CYLINDERS

Objectives

After studying this section, you will be able to

- Find the volumes of right rectangular prisms
- Find the volumes of other prisms
- Find the volumes of cylinders
- Use the area of a prism's or a cylinder's cross section to find the solid's volume

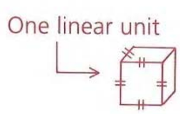
Part One: Introduction

Volume of a Right Rectangular Prism

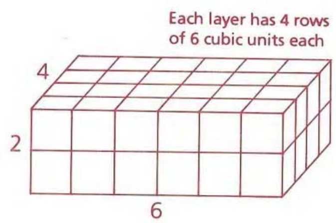
The measure of the space enclosed by a solid is called the solid's ***volume***. In a way, volume is to solids what area is to plane figures.

Definition The ***volume*** of a solid is the number of cubic units of space contained by the solid.

A cubic unit is the volume of a cube with edges one unit long. A cube is a right rectangular prism with congruent edges, so all its faces are squares. In Section 12.1 we used the word *box* for a right prism. Thus, a right rectangular prism can also be called a rectangular box.



One Cubic Unit



Rectangular Box

The rectangular box above contains 48 cubic units. The formula that follows is not only a way of counting cubic units rapidly but also works with fractional dimensions.

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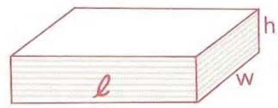
Postulate ***The volume of a right rectangular prism is equal to the product of its length, its width, and its height.***

$$V_{rect.\ box} = \ell wh$$

where ℓ is the length, w is the width, and h is the height.

Another way to think of the volume of a rectangular prism is to imagine the prism to be a stack of congruent rectangular sheets of paper. The area of each sheet is $\ell \cdot w$, and the height of the stack is h. Since the base of the prism is one of the congruent sheets, there is a second formula for the volume of a rectangular box.



$$\begin{aligned} V &= \ell wh \\ &= (\ell w)h \\ &= (\text{area of sheet}) \cdot h \end{aligned}$$

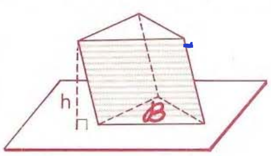
Theorem 115 ***The volume of a right rectangular prism is equal to the product of the height and the area of the base.***

$$V_{rect.\ box} = \mathcal{B}h$$

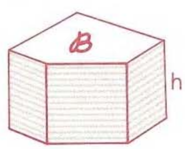
where $\mathcal{B}$ is the area of the base and h is the height.

Volumes of Other Prisms

The formula in Theorem 115 can be used to compute the volume of any prism, since any prism can be viewed as a stack of sheets the same shape and size as the base.

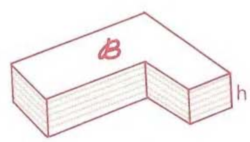


$V = \mathcal{B}h$



$V = \mathcal{B}h$

area base



$V = \mathcal{B}h$

Theorem 116 ***The volume of any prism is equal to the product of the height and the area of the base.***

$$V_{prism} = \mathcal{B}h$$

where $\mathcal{B}$ is the area of the base and h is the height.

Notice that the height of a *right* prism is equivalent to the measure of a lateral edge.

Volume of a Cylinder

The stacking property applies to a cylinder as well as to a prism, so the formula for a prism's volume can also be used to find a cylinder's. Furthermore, since the base of a cylinder is a circle, there is a second, more popular formula.

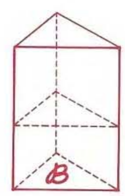
Theorem 117 ***The volume of a cylinder is equal to the product of the height and the area of the base.***

$$V_{cyl} = \mathcal{B}h = \pi r^2 h$$

where $\mathcal{B}$ is the area of the base, h is the height, and r is the radius of the base.

Cross Section of a Prism or a Cylinder

When we visualize a prism or a cylinder as a stack of sheets, all the sheets are congruent, so the area of any one of them can be substituted for $\mathcal{B}$. Each of the sheets between the bases is an example of a ***cross section***.



Definition A ***cross section*** is the intersection of a solid with a plane.

In this book, unless otherwise noted, all references to cross sections will be to cross sections *parallel to the base*. We can now combine Theorems 116 and 117, using the symbol $\not{C}$ to represent the area of a cross section parallel to the base.

Theorem 118 ***The volume of a prism or a cylinder is equal to the product of the figure's cross-sectional area and its height.***

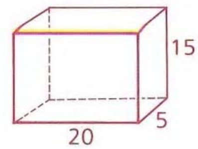
$$V_{prism\ or\ cyl} = \not{C}h$$

where $\not{C}$ is the area of a cross section and h is the height.

12.4: Class Examples

Part Two: Sample Problems

Problem 1 *Find the volume of the rectangular prism.*



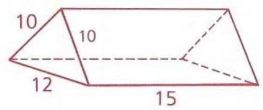
Solution

$V = \ell wh$ or $V = \mathcal{B}h$

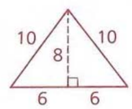
$= 20(5)(15)$ $= (5 \cdot 20)(15)$ (Using a 5 × 20 face as base)

$= 1500$ $= 1500$

Problem 2 *Find the volume of the triangular prism.*



Solution Notice that the base of the prism is the triangle at the right.



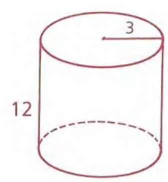
$A_\triangle = \frac{1}{2}(12)(8) = 48$

$V = \mathcal{B}h$

$= 48(15)$

$= 720$

Problem 3 *Find the volume of a cylinder with a radius of 3 and a height of 12.*



Solution

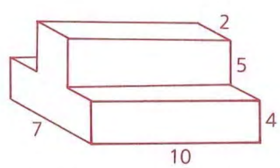
$V = \pi r^2 h$

$= \pi(3^2)(12)$

$= \pi(9)(12)$

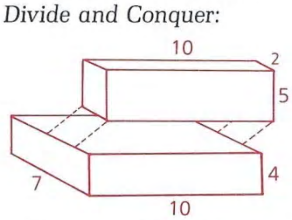
$= 108\pi$

Problem 4 *Find the volume of the right prism shown. (Take the left face as a representative cross section.)*



Solution Use either of two methods.

a *Divide and Conquer:*

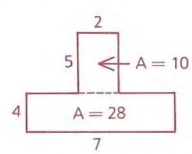


$V_{\text{top box}} = 2(5)(10) = 100$

$V_{\text{bottom box}} = 7(10)(4) = 280$

$V_{\text{solid}} = 280 + 100 = 380$

b *Cross Section Times Height:*



$\not{C} = 10 + 28 = 38$

$V = \not{C}h$

$= 38(10)$

$= 380$

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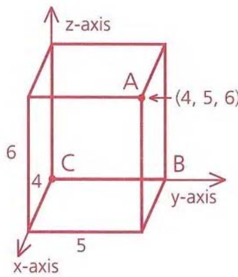
12.3: Surface Area of Circular Solids & 12.4: Volume of Prisms and Cylinders

Problem 5 *A box (rectangular prism) is sitting in a corner of a room as shown.*

a *Find the volume of the prism.*

b *If the coordinates of point A in a three-dimensional coordinate system are (4, 5, 6), what are the coordinates of B?*



Q: What are the coordinates of the other "corners" of the box?

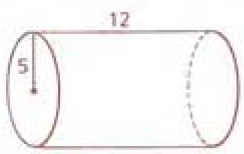
Solution

a $V = \mathscr{B}h$
$= (4 \cdot 5)6$
$= 120$

b Point C = (0, 0, 0) is the corner. To get from C to B, you would travel 0 units in the x direction, 5 units in the y direction, and 0 units in the z direction. So the coordinates of B are (0, 5, 0).

1 Find the volume of each solid.

a

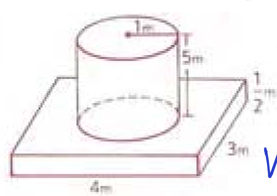


$V = B \cdot h$

πr^2

$25\pi \cdot 12 = 300\pi$

2 Find the volume of cement needed to form the concrete pedestal shown. (Leave your answer in π form.)



$V_{CYL} : B \cdot h$

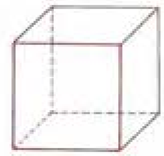
$\pi \cdot 5 = 5\pi$

V_{RECT} $4(3)\left(\frac{1}{2}\right) = 6$

$(5\pi + 6)\, m^3$

AMDG

5 a Find the volume of a cube with an edge of 7.
b Find the volume of a cube with an edge of e.
c Find the edge of a cube with a volume of 125.



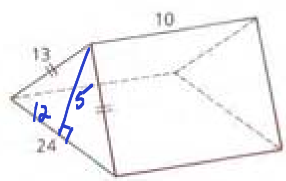
$\boxed{5a}$ $7^3 = 343$

$\boxed{5b}$ e^3

$\boxed{5c}$ $125 = x^3$

$\sqrt[3]{125} = x$ → $125 \wedge (1/3) = 5$

9 Find the volume and the total area of the prism.



LA = 13 × 10 + 13 × 10 + 24 × 10

130 + 130 + 240 = 500

2 Bases = $2\left(\frac{1}{2}\right) b \cdot h = 24 \cdot 5 =$ 120

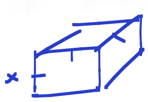
$620 u^2$

$B = \frac{1}{2} 24 \cdot 5 = 60$

$h = 10$

$V = 60(10) = 600 u^3$

11 When Hilda computed the volume and the surface area of a cube, both answers had the same numerical value. Find the length of one side of the cube.



$V_{CUBE} = x^3$

6 SQ FACES, EACH: x^2 → TSA: $6x^2$

$V_{CUBE} = TSA_{CUBE}$

$x^3 = 6x^2$

$x^3 - 6x^2 = 0$

$x^2(x-6) = 0$

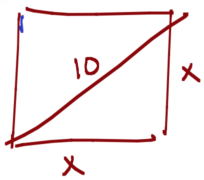
~~x = 0 or~~ 6 = x

Name
Adv Geo

12.3: Surface Area of Circular Solids &
12.4: Volume of Prisms and Cylinders

Ms. Kreosvic
M 12 May 14

13 Find the volume of a cube in which a face diagonal is 10.

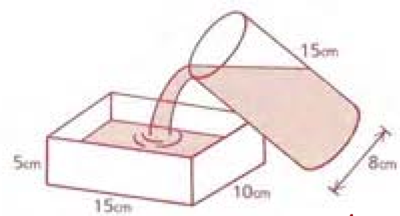


$x^2 + x^2 = 10^2$
$2x^2 = 100$
$x^2 = 50$
$x = \sqrt{25}\cdot\sqrt{2}$
$x = 5\sqrt{2}$

$x^3 \to 5^3\cdot(\sqrt{2})^3$
$125 \cdot 2\sqrt{2}$
$250\sqrt{2}$

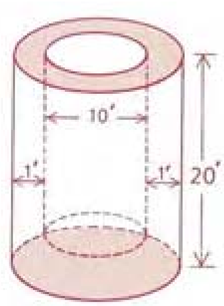
16 The cylindrical glass is full of water, which is poured into the rectangular pan. Will the pan overflow?

yes
by ≈3.6 cm^3



$V_{PAN}: 5(15)(10) = 750$, $V_{CYL}: 4^2\pi\cdot 15$
$750 cm^3$ $: 16\cdot 15\pi$
$753.6\ cm^3$

18 A cistern is to be built of cement. The walls and bottom will be 1 ft thick. The outer height will be 20 ft. The inner diameter will be 10 ft. To the nearest cubic foot, how much cement will be needed for the job?



Diameter of outer cylinder = 10 + 1 + 1 = 12
Radius of outer cylinder = 6

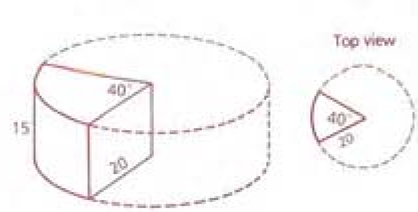
$$\begin{aligned} V_{\text{outer cyl}} &= Bh \\ &= \pi r^2 h \\ &= \pi(6)^2(20) \\ &= 720\pi \text{ cu ft} \end{aligned}$$

Because the bottom will be 1 ft thick, height inner cyl = height outer cyl – 1 ft = 19 ft and the radius inner cyl = $\frac{1}{2}(10) = 5$

$$\begin{aligned} V_{\text{inner cyl}} &= Bh \\ &= \pi r^2 h \\ &= \pi(5)^2(19) \\ &= 475\pi \text{ cu ft} \end{aligned}$$

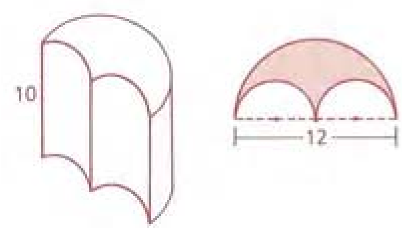
$$\begin{aligned} \text{Amt cement} &= V_{\text{outer cyl}} - V_{\text{inner cyl}} \\ &= 720\pi \text{ cu ft} - 475\pi \text{ cu ft} \\ \text{Amt cement} &= 245\pi \approx 770 \text{ cu ft} \end{aligned}$$

19 A wedge of cheese is cut from a cylindrical block. Find the volume and the total surface area of this wedge.



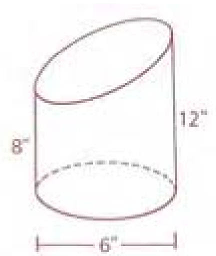
$$\begin{aligned}
V_{prism} &= Bh \\
&= A \text{ sector } h \\
&= \frac{40}{360}\pi(20)^2(15) \\
V_{prism} &= \frac{1}{9}\pi(400)(15) = \frac{2000}{3}\pi \\
TA &= \frac{40}{360}(2\pi \cdot 20 \cdot 15) + 2[\frac{40}{360}\pi(20)^2 + 15 \cdot 20] \\
&= \frac{1}{9}(40\pi)(15) + 2(\frac{1}{9})(400\pi) + 2(15)(20) \\
&= \frac{600}{9}\pi + \frac{800}{9}\pi + 600 = \frac{1400}{9}\pi + 600 \\
TA &= \frac{1400}{9}\pi + 600
\end{aligned}$$

21 Find the volume of the solid at the right. (A representative cross section is shown.)



$$\begin{aligned}
V_{prism} &= Bh \\
V &= [\frac{1}{2}\pi(6)^2 - 2(\frac{1}{2})\pi(3^2)](10) \\
V &= (18\pi - 9\pi)10 = 90\pi
\end{aligned}$$

22 A cylinder is cut on a slant as shown. Find the solid's volume.



Separate the cylinder into a cylinder and a half cylinder.

The height of the top cylinder is $12 - 8 = 4$.

$$\begin{aligned}
\text{Total Volume} &= V_{cyl\,1} + V_{cyl\,2} \\
&= Bh_1 + \frac{1}{2}Bh_2 \\
&= \pi r^2 h_1 + \frac{1}{2}(\pi r^2 h_2) \\
&= \pi(3)^2(8) + \frac{1}{2}\pi(3)^2(4) \\
&= 72\pi + 18\pi \\
\text{Total Volume} &= 90\pi
\end{aligned}$$

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12.3: Surface Area of Circular Solids &
12.4: Volume of Prisms and Cylinders

12.3 Homework

1 What is the total area of a sphere having $A = 4\pi r^2$

b A radius of 3?

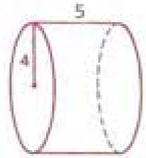
$4 \cdot 3 \cdot 3\pi = 36\pi$

d A diameter of 5?

$r = \frac{5}{2}$ $A = \cancel{4} \cdot \frac{5}{\cancel{2}} \cdot \frac{5}{\cancel{2}}\pi = 25\pi$

2 Find the lateral area and the total area of each solid.

c



d

2c LA: $C = 8\pi$, 5; 40π $+ 2$ Bases $= \pi r^2$, $2 \cdot 16\pi$ TSA $\cancel{72\pi}$

2d. $\frac{1}{2}C\ell = \frac{1}{2}\pi 2 \cdot 3 = 3\pi$ $+ 1$ base πr^2, 1π 4π

3 Find the radius of a sphere whose surface area is 144π.

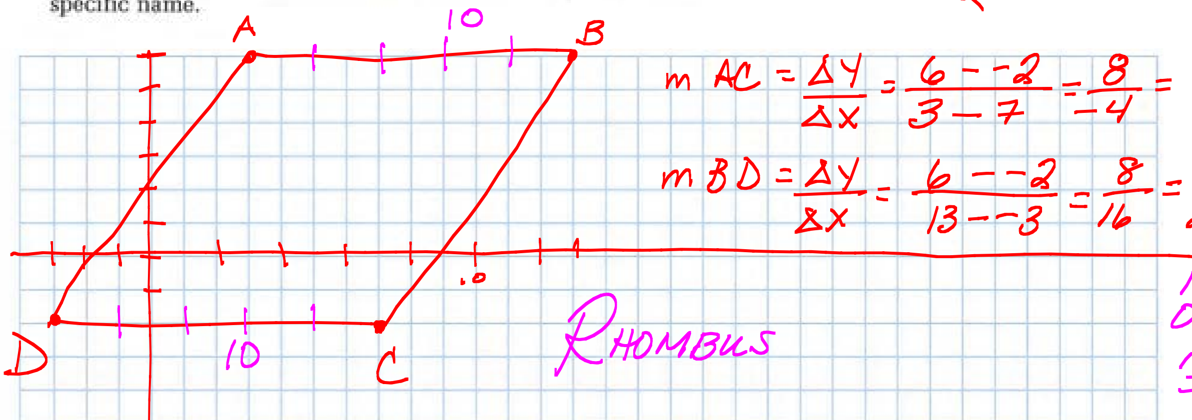
$SA = 4\pi r^2$
$144\pi = 4\pi r^2$
$36 = r^2$
$6 = r$

5 ABCD is a parallelogram, with A = (3, 6), B = (13, 6), C = (7, −2), and D = (−3, −2).

a Find the slopes of the diagonals, $\overline{AC}$ and $\overline{BD}$.

b Use your answers to part **a** to identify ▱ABCD by its most specific name.

6
−3 13
−2



$m\,AC = \frac{\Delta y}{\Delta x} = \frac{6 - -2}{3 - 7} = \frac{8}{-4} = -2$

$m\,BD = \frac{\Delta y}{\Delta x} = \frac{6 - -2}{13 - -3} = \frac{8}{16} = \frac{1}{2}$

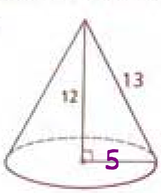
OPP & RECIP ⇒ ⊥

RHOMBUS

1 PR OPP SDS BOTH ≅ & ∥

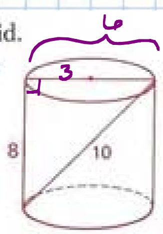
7 Find the total area of each solid.

a



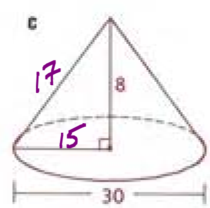
$LA_{CONE} = (\frac{1}{2}\pi d)\ell$
$= \pi r \ell$
$= 5 \cdot 13\pi$
$= 65\pi$
$+ BASE = 25\pi$
$TSA = 90\pi$

b



$LA_{CYL} = (\pi d)h$
$= 6 \cdot 8\pi$
$= 48\pi$
$+ Base = \pi r^2$
$= 9\pi$ but 2 bases so
$TSA = 48\pi + 18\pi = 66\pi$

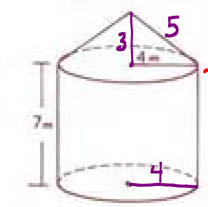
c



$LA_{CONE} = \frac{1}{2}C\ell$
$= \frac{1}{2}(\pi d)\ell$
$= r\ell\pi$
$= 15 \cdot 17\pi$
$= 255\pi$
$A_{BASE} = 225\pi$
$TSA = 480\pi$

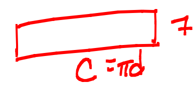
AMDG

8 The total height of the tower shown is 10 m. If one liter of paint will cover an area of 10 sq m, how many 1-L cans of paint are needed to paint the entire tower? (Hint: First find the total area to be painted, using 3.14 for π.)



$LA_{CONE} = \frac{1}{2}C\ell = \frac{1}{2}\pi d \cdot \ell = r\ell\pi = 4 \cdot 5\pi = 20\pi$

LA_{CYL}



$= 7(8)\pi$
$= 56\pi$

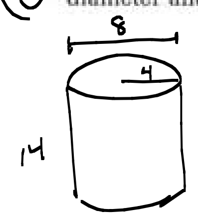
Hint: Think Stoichiometry. Use proportions with units.

$TSA = 20\pi + 56\pi = 76\pi \approx 238.64\,m^2$

$238.64\,\cancel{m^2} \cdot \frac{1\text{-L can}}{10\ \cancel{m^2}} = 23.864$ cans

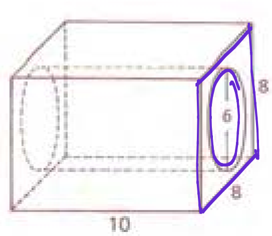
24 cans

9 What size label (length and width) will just fit on a can 8 cm in diameter and 14 cm high?



$LA_{CYL} = C \cdot h$
$= \pi d \cdot h \longrightarrow (8\pi \text{ by } 14)\text{cm}$

12 Find the total surface area of the solid shown, including the surface inside the hole.



$TSA = LA_{PRISM} + 2\boxed{O} + LA_{CYL}$

$4 \cdot [\,10 \times 8\,] + 2[\,8 \times 8 - (6)\,] + [\,C = 6\pi \times 10\,]$

$320 + 2[64 - 9\pi] + 60\pi$

$320 + 128 - 18\pi + 60\pi$

$448 + 42\pi$

Nice Exam Question :)

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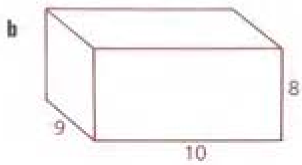
12.4 Homework

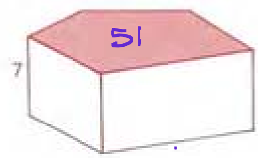
1 Find the volume of each solid.

b



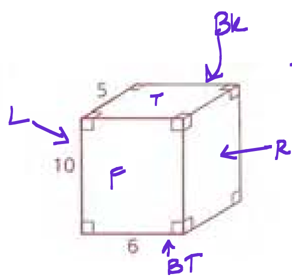
$V = l \cdot w \cdot h$
$= 9 \cdot 8 \cdot 10$
$= 720$

3 The area of the shaded face of the right pentagonal prism is 51. Find the prism's volume.



$V = B \cdot 7$
$= 51 \cdot 7$
$= 357$

4 Find the volume and the total surface area of the rectangular box shown.

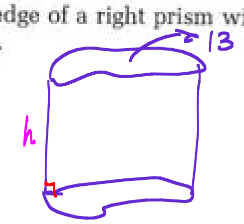


$V = 10 \cdot 5 \cdot 6$
$= 300$

TSA: F+Bk: 10, 6 + L+R: 10, 5 + T+BT: 5, 6

$2(60) + 2(50) + 2(30)$
$2(60 + 50 + 30)$
$2(140)$
280

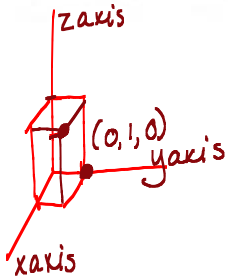
6 Find the length of a lateral edge of a right prism with a volume of 286 and a base area of 13.



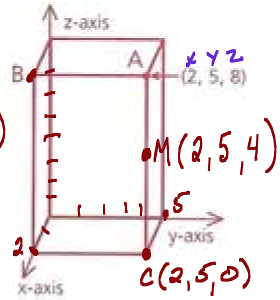
$V = Bh$
$286 = 13 \cdot h$
$22 = h$

(1, 1, 2)

8 If point A's coordinates in the three-dimensional coordinate system are (2, 5, 8), what are the coordinates of B?

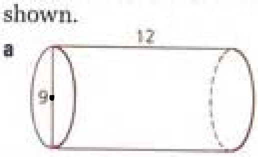


(2, 0, 8)

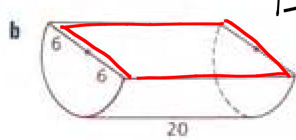


10 Find the volume and the total area of each right cylindrical solid shown.

a



b



$\boxed{10a}$

$V = B \cdot h$

$= \left(\frac{9}{2}\right)^2 \pi \cdot 12$

$= \frac{81}{4} \cdot 12\ \pi$

$= 81\,(3)\pi$

$= 243\pi$

$TSA = 2 \text{ bases} + LA_{CYL}$

$2(\pi r^2) + \boxed{}\ 12$, C

$2\left(\pi\left(\frac{9}{2}\right)^2\right) + 9\pi \cdot 12$

$\frac{81 \cdot \cancel{2}^{1}}{\cancel{4}_{2}}\pi + 108\pi$

$\frac{81}{2}\pi +$

$40.5\pi + 108\pi$

148.5π

$\boxed{10b}$ $V = B \cdot h$

$\frac{1}{2}\pi r^2 \cdot h$

$\frac{36}{2}\pi \cdot 20$

$18 \cdot 20 \cdot \pi$

360π

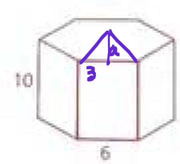
$TSA = [2\tfrac{1}{2}\bigcirc s] + \boxed{}\ 20 + \boxed{}\ 12$, $\frac{1}{2}C$, 20

$\pi 6^2 + \frac{1}{2}\pi 12 \cdot 20$

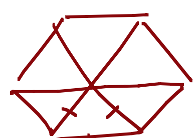
$36\pi + 120\pi$

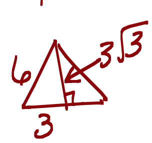
$156\pi + 240$

12 Find the volume and the surface area of the regular hexagonal right prism.



$a = 3\sqrt{3}$





$V = Bh$

$= \left(\frac{1}{2}ap\right)h$

$= \frac{1}{2}\, 3\sqrt{3}\ 36 \cdot 10$

$= 36 \cdot 5 \cdot 3\sqrt{3}$

$\boxed{V = 540\sqrt{3}}$

$TSA = 2 \text{ hex bases} + LA_{PRISM}$

$2\left[\frac{1}{2}ap\right] + 6\left[10 \cdot 6\right]$

$3\sqrt{3} \cdot 36 + 360$

$\boxed{108\sqrt{3} + 360}$

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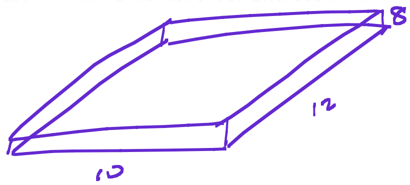
14 A rectangular cake pan has a base 10 cm by 12 cm and a height of 8 cm. If 810 cu cm of batter is poured into the pan, how far up the side will the batter come?



$V = 960\ cm^3$

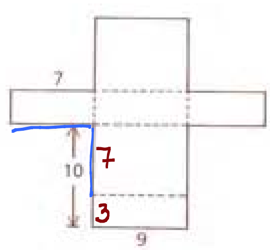
$\approx 84\%$

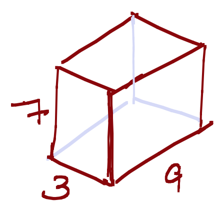
$\frac{BATTER}{PAN} : \frac{810}{960} = \frac{27}{32}$ up the side

side is 8 : proportion $\frac{27}{\cancel{32}_4} = \frac{27}{4}$ or 6.75 cm is covered w/ batter

$8cm(84\%) = 6.75\ cm$

15 A rectangular container is to be formed by folding the cardboard along the dotted lines. Find the volume of this container.





$7 \cdot 3 \cdot 9$

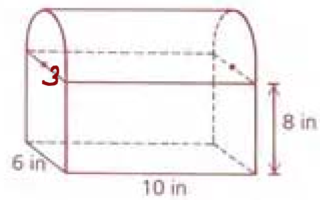
$7 \cdot 27$

$140 + 49 = 189$

17 Jim's lunch box is in the shape of a half cylinder on a rectangular box. To the nearest whole unit, what is

a The total volume it contains?

b The total area of the sheet metal needed to manufacture it?



$\boxed{17a}$ $V_{LUNCHBOX} = V_{PRISM} + V_{\frac{1}{2}CYLINDER}$

$= 6 \cdot 8 \cdot 10 + \frac{1}{2}\pi 3^2 \cdot 10$

$= 480 + 45\pi$

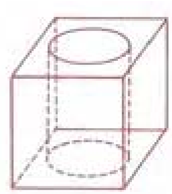
$\boxed{17b}$ $TSA = \text{FRONT \& BACK} + \text{L\&R} + \text{BOTTOM} + 2\tfrac{1}{2}\text{Os} + LA_{CYLIN}$

$2 \cdot 8 \cdot 10 + 2 \cdot 6 \cdot 8 + 6 \cdot 10 + 9\pi +$ 10 $\frac{1}{2}C = \frac{1}{2}\pi 6$

$160 + 96 + 60 + 9\pi + 30\pi$

$316 + 39\pi$

20 An ice-cube manufacturer makes ice cubes with holes in them. Each cube is 4 cm on a side and the hole is 2 cm in diameter.



20a. $V_{CUBE} = s^3 = 4^3 = 64\ cm^3$

$V_{HOLE} = Bh = \pi r^2 \cdot h = 1\pi \cdot 4 = 4\pi$

$64 - 4\pi \approx 51.4\ cm^3$

a To the nearest tenth, what is the volume of ice in each cube?

b To the nearest tenth, what will be the volume of the water left when ten cubes melt? (Water's volume decreases by 11% when it changes from a solid to a liquid.)

c To the nearest tenth, what is the total surface area (including the inside of the hole) of a single cube?

d The manufacturer claims that these cubes cool a drink twice as fast as regular cubes of the same size. Verify whether this claim is true by a comparison of surface areas. (Hint: The ratio of areas is equal to the ratio of cooling speeds.)

20b. $10(51.4) = 514.4\ cm^3$

$- 11\%(514.4)$

```
64-4π
                      51.43362939
Ans*10
                      514.3362939
514.3365939-.11*514.336
                      457.7596339
```

457.8

c TA = 6(A face of the cube) – 2(A base of cyl) + LA_{cyl}

TA = $6(s^2) - 2(\pi r^2) + Ch$

$= 6(4)^2 - 2(\pi(1)^2) + 2\pi(1)(4)$

$= 6(16) - 2\pi + 8\pi$

$\approx 96 - 2(3.14) + 8(3.14)$

TA $\approx$ 114.8 sq cm

d TA of a cube without hole = $6(s^2)$

TA = $6(4^2)$

TA = 96 sq cm

$\frac{114.8}{96} = 1.20$

His claim is not true, 114.8 sq cm vs. 96 sq cm.