AMDG
Name
Adv Geo
10-9: Circumference and Arc Length

Objectives

After studying this section, you will be able to
▪ Determine the circumference of a circle
▪ Determine the length of an arc

Definition   The **circumference** of a circle is its perimeter.

Postulate $C = \pi d$

Theorem 98   **The length of an arc is equal to the circumference of its circle times the fractional part of the circle determined by the arc.**

$$\text{Length of } \widehat{PQ} = \left(\frac{m\widehat{PQ}}{360}\right)\pi d$$

where d is the diameter and $\widehat{PQ}$ is measured in degrees.

Problem 1   Find the radius of a circle whose circumference is 50π.

$$50\pi = d\pi$$
$$50 = d$$
$$25 = r$$

Problem 2   Find the length of each arc of a circle with a 12-cm radius.

$\to C = 24\pi$ cm

a A 30° arc

$$\frac{30}{360} \cdot 24\pi$$
$$\frac{1}{12} \cdot 24\pi = 2\pi$$

b A 105° arc

$$\frac{105}{360} \cdot 24\pi$$
$$\frac{7}{24} \cdot 24\pi = 7\pi$$

$\to C = 70\pi$ cm

Problem 3   The diameter of a bicycle wheel (including the tire) is 70 cm.

a How far will the bicycle travel if the wheel rotates 1000 times? (Approximate the answer in meters.) $.7\pi$ m $(1000) = 700\pi$ m ≈ 2199 m

b How many revolutions will the wheel make if the bicycle travels 15 m? (Approximate to the nearest tenth of a revolution.)

$$15 = \frac{7}{10}\pi (\text{rev})$$
$$150 = 7\pi (\text{rev})$$
$$\frac{150}{7\pi} = \text{rev}$$

1500/(70π)
6.820926133

Problem Set A

1 Find the circumference of the circle. Then approximate the circumference to the nearest ~~hundredth~~

   a A circle whose diameter is 21 mm $21\pi \approx$ ~~65.97344573~~

   b A circle whose radius is 6 mm

$$C = 12\pi \, mm$$
$$\approx ~~37.69911184~~$$
$$37.70$$

2 Find, to the nearest hundredth, the radius of a circle whose circumference is

  a 56π **b** 314   **c** 17π **d** 88

$56\pi = d\pi$ $314 = d\pi$ $17\pi = d\pi$ $88 = d\pi$

$56 = d$ $99.95 = d$ $17 = d$ $88/\pi = d \approx 28.01$

$28 = r$ $49.97 = r$ $\frac{17}{2} = r$ $14.01 \approx r$

                              8.50

3 Find the length of each arc of a circle with a radius of 10. $C = 20\pi$

  a A 72° arc   **b** A 90° arc   **c** A 60° arc   **d** A semicircle

$\dfrac{72}{360} \cdot 20\pi$ $\dfrac{90}{360} \cdot 20\pi$ $\dfrac{60}{360} \cdot 20\pi$ $\dfrac{180}{360} \cdot 20\pi$

$\dfrac{1}{5} \cdot 20\pi$ $\dfrac{1}{4} \cdot 20\pi$ $\dfrac{1}{6} \cdot 20\pi$ $\dfrac{1}{2} \cdot 20\pi$

4π 5π $\dfrac{10}{3}\pi$ 10π

$\to C = 30\pi$

4 A bicycle has wheels 30 cm in diameter. Find, to the nearest tenth of a centimeter, the distance that the bicycle moves forward during

  a) 1 revolution

$$30\pi \approx 94.2 \, cm$$
Exact   Estimate

  b) 10 revolutions
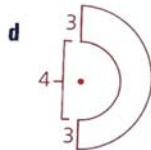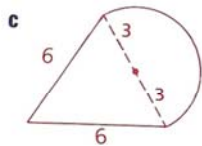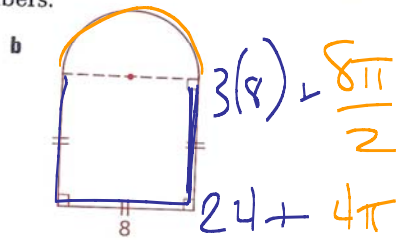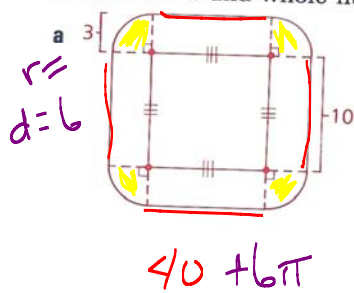
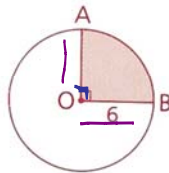$$300\pi \approx 942.5 \, cm$$
Exact

  c) 1000 revolutions

$$30000\pi \approx 94,247.8 \, cm$$

5 Find the complete perimeter of each figure. Leave your answers in terms of π and whole numbers.

a 3

$r =$
$d = 6$

10

$40 + 6\pi$

b

$3(8) \angle \dfrac{8\pi}{2}$
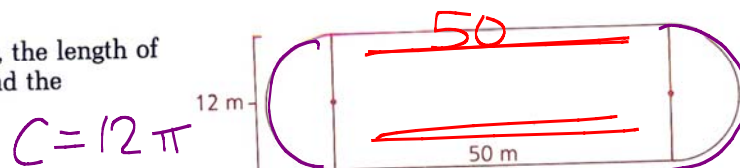
$24 + 4\pi$

8

c

3
6
3
6

d

3
4
3

6 a Find the length of $\widehat{AB}$. $\leftarrow \dfrac{deg}{360} \cdot C$

b Find the perimeter of sector AOB. (The shaded region is a sector.)

A
O
6
B

6a: $\dfrac{1}{4} \cdot 12\pi = 3\pi$

6b: $6 + 6 + 3\pi = 12 + 3\pi$

7 Find, to the nearest meter, the length of fencing needed to surround the racetrack.

$C = 12\pi$

12 m

50

50 m

$100 + C$

$(100 + 12\pi)\text{m} \approx 138\text{m}$