

10-6: More Angle-Arc Theorems

Review

If the vertex of the angle is ____ the circle	Then use this formula to find the angle's measure:
IN	
ON	
OUT	

Objectives

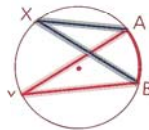
After studying this section, you will be able to

- Recognize congruent inscribed and tangent-chord angles
- Determine the measure of an angle inscribed in a semicircle
- Apply the relationship between the measures of a tangent-tangent angle and its minor arc

Theorem 89 *If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.*

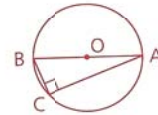
Given: X and Y are inscribed angles intercepting arc AB.

Conclusion: $\angle X \cong \angle Y$



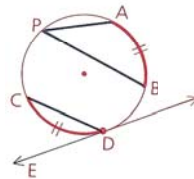
Theorem 91 *An angle inscribed in a semicircle is a right angle.*

Given: \overline{AB} is a diameter of $\odot O$.
Prove: $\angle C$ is a right angle.



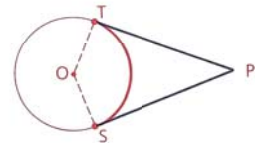
Theorem 90 *If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.*

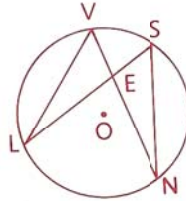
If \overleftrightarrow{ED} is the tangent at D and $\widehat{AB} \cong \widehat{CD}$, we may conclude that $\angle P \cong \angle CDE$.



Theorem 92 *The sum of the measures of a tangent-tangent angle and its minor arc is 180.*

Given: \overline{PT} and \overline{PS} are tangent to circle O.
Prove: $m\angle P + m\widehat{TS} = 180$

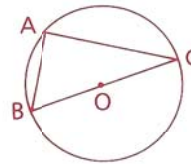


Problem 1Given: $\odot O$ Conclusion: $\triangle LVE \sim \triangle NSE$,
 $EV \cdot EN = EL \cdot SE$ **Proof**

1 $\odot O$	1
2 $\angle V \cong \angle S$	2
3 $\angle L \cong \angle N$	3
4 $\triangle LVE \sim \triangle NSE$	4
5 $\frac{EV}{SE} = \frac{EL}{EN}$	5
6 $EV \cdot EN = EL \cdot SE$	6

Problem 2

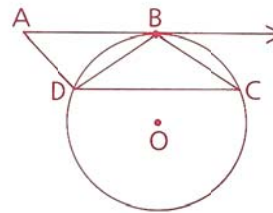
In circle O, \overline{BC} is a diameter and the radius of the circle is 20.5 mm.
Chord \overline{AC} has a length of 40 mm. Find AB.

**Solution**

Since $\angle A$ is inscribed in a semicircle, it is a right angle. By the Pythagorean Theorem,

Problem 3

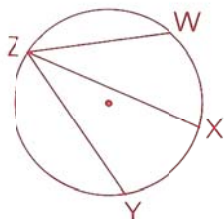
Given: $\odot O$ with \overleftrightarrow{AB} tangent at B, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
Prove: $\angle C \cong \angle BDC$

**Proof**

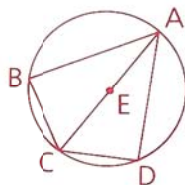
1 \overleftrightarrow{AB} is tangent to $\odot O$.	1 Given
2 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	2 Given
3 $\angle ABD \cong \angle BDC$	3
4 $\angle C \cong \angle ABD$	4
5 $\angle C \cong \angle BDC$	5

10-6: More Angle-Arc Theorems

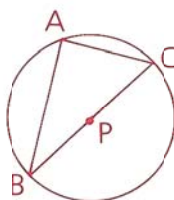
- 1** Given: X is the midpt. of \widehat{WY} .
Prove: \overrightarrow{ZX} bisects $\angle WZY$.



- 2** Given: $\odot E$ with diameter \overline{AC} , $\overline{BC} \cong \overline{CD}$
Conclusion: $\triangle ABC \cong \triangle ADC$

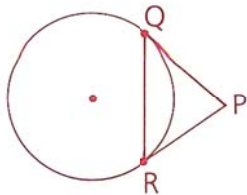


- 3** In $\odot P$, \overline{BC} is a diameter, $AC = 12$ mm,
and $BA = 16$ mm. Find the radius of the
circle.



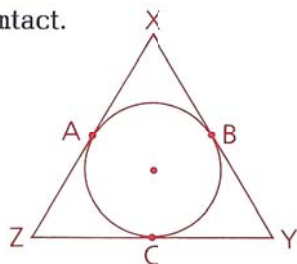
- 4 Given: \overline{PQ} and \overline{PR} are tangent segments.
 $\widehat{QR} = 163^\circ$

Find: **a** $\angle P$
b $\angle PQR$

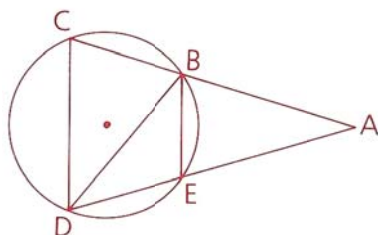


- 5 Given: A, B, and C are points of contact.
 $\widehat{AB} = 145^\circ$, $\angle Y = 48^\circ$

Find: $\angle Z$

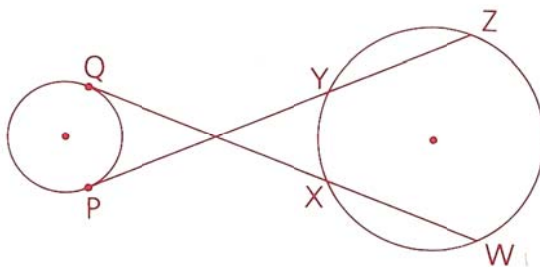


- 6 Given: $\widehat{BC} \cong \widehat{ED}$, $AB = 8$,
 $BC = 4$, $CD = 9$
a Are \overline{BE} and \overline{CD} parallel?
b Find BE.
c Is $\triangle ACD$ scalene?

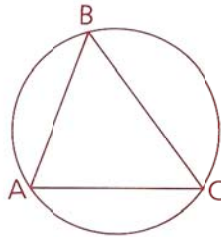


- 7 Given: \overleftrightarrow{PY} and \overleftrightarrow{QW} are tangents.
 $\widehat{WZ} = 126^\circ$, $\widehat{XY} = 40^\circ$

Find: \widehat{PQ}

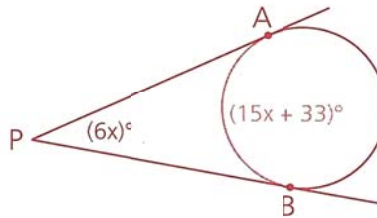


- 8 If $\triangle ABC$ is inscribed in a circle and $\widehat{AC} \cong \widehat{AB}$, tell whether each of the following must be true, could be true, or cannot be true.

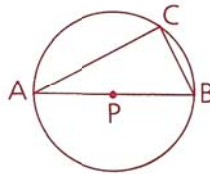


- a $\overline{AB} \cong \overline{AC}$
b $\overline{AC} \cong \overline{BC}$
c \overline{AB} and \overline{AC} are equidistant from the center of the circle.
d $\angle B \cong \angle C$
e $\angle BAC$ is a right angle.
f $\angle ABC$ is a right angle.

- 9 In the figure shown, find $m\angle P$.

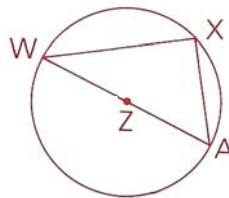


- 10 If \overline{AB} is a diameter of $\odot P$, $CB = 1.5$ m, and $CA = 2$ m, find the radius of $\odot P$.

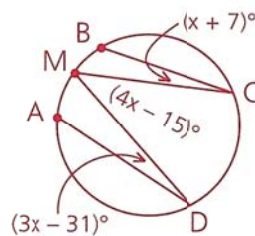


- 11 The radius of $\odot Z$ is 6 cm and $\widehat{WX} = 120^\circ$.

- Find: a AX
b The perimeter of $\triangle WAX$

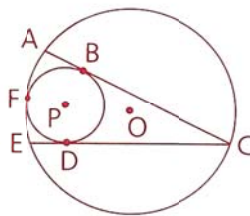


- 12 M is the midpoint of \widehat{AB} . Find $m\widehat{CD}$.



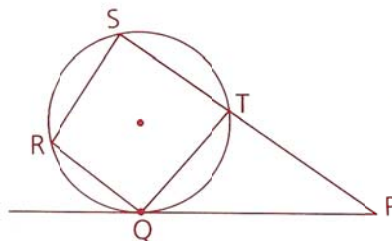
- 13** A rectangle with dimensions 18 by 24 is inscribed in a circle.
Find the radius of the circle.
- 14** A square is inscribed in a circle with a radius of 10. Find the
length of a side of the square.
- 15** Quadrilateral ABCD is inscribed in circle O. $AB = 12$, $BC = 16$,
 $CD = 10$, and $\angle ABC$ is a right angle. Find the measure of \widehat{AD} in
simplified radical form.

- 16** Circles O and P are tangent at F. \overline{AC} and
 \overline{CE} are tangent to $\odot P$ at B and D. If
 $\widehat{DFB} = 223^\circ$, find \widehat{AE} .



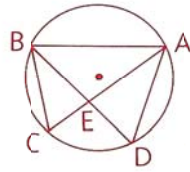
- 17** Given: $\angle S = 88^\circ$, $\widehat{QT} = 104^\circ$, $\widehat{ST} = 94^\circ$,
tangent \overline{PQ}

Find: **a** $\angle P$
b $\angle STQ$



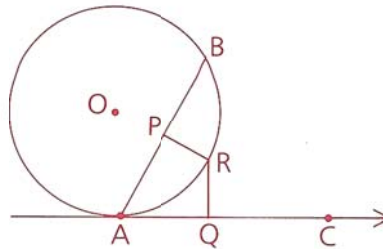
18 Given: $\widehat{BC} \cong \widehat{CD}$

Conclusion: $\triangle ABC \sim \triangle AED$

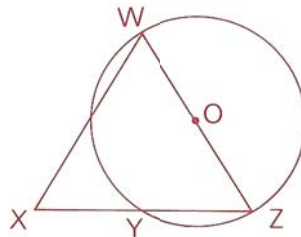


19 Given: \overleftrightarrow{AC} is tangent at A. $\angle APR$ and $\angle AQR$ are right \angle s. R is the mid-point of \widehat{AB} .

Conclusion: $\overline{PR} \cong \overline{RQ}$ (Hint: Draw \overline{AR} .)



- 20** Given: $\triangle WXZ$ is isosceles, with $\overline{WX} \cong \overline{WZ}$.
 \overline{WZ} is a diameter of $\odot O$.
 Prove: Y is the midpoint of \overline{XZ} .
 (Hint: Draw \overline{WY} .)



- 21** Given: \overline{AC} is tangent to $\odot O$ at A .
 Conclusion: $\triangle ADC \sim \triangle BDA$

