AMDG

NAME Student
Adv Geo - 5

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F 26 Apr 2013

10-6: More Angle-Arc Theorems

Review

If the vertex of the angle is ____ the circle	Then use this formula to find the angle's measure:
IN	$\angle = \frac{\frown + \frown}{2}$
ON	$\angle = \frac{\frown}{2}$
OUT	$\angle = \frac{\frown - \frown}{2}$

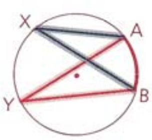
Objectives

After studying this section, you will be able to

- Recognize congruent inscribed and tangent-chord angles ← vertex on
- Determine the measure of an angle inscribed in a semicircle
- Apply the relationship between the measures of a tangent-tangent angle and its minor arc

Theorem 89 *If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.*

Given: X and Y are inscribed angles intercepting arc AB.
Conclusion: $\angle X \cong \angle Y$



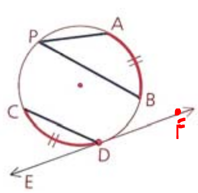
$\angle X \rightarrow \overset{\frown}{AB}$

$\angle Y \rightarrow \overset{\frown}{AB}$

$\angle X \cong \angle Y$

Theorem 90 *If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.*

If $\overleftrightarrow{ED}$ is the tangent at D and $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, we may conclude that $\angle P \cong \angle CDE$.



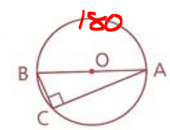
$\angle P \rightarrow \overset{\frown}{AB}$

$\angle CDE \rightarrow \overset{\frown}{CD}$

$\overset{\frown}{AB} \cong \overset{\frown}{CD} \rightarrow \angle P \cong \angle CDE$

Theorem 91 *An angle inscribed in a semicircle is a right angle.*

Given: $\overline{AB}$ is a diameter of ⊙O.
Prove: $\angle C$ is a right angle.



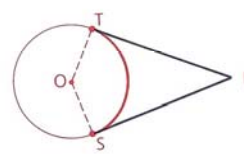
$\angle = \frac{180}{2} = 90°$

carpenter's trick

t-square

Theorem 92 *The sum of the measures of a tangent-tangent angle and its minor arc is 180.*

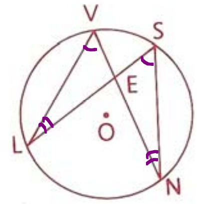
Given: $\overline{PT}$ and $\overline{PS}$ are tangent to circle O.
Prove: $m\angle P + m\overset{\frown}{TS} = 180$



Problem 1

Given: $\odot O$

Conclusion: $\triangle LVE \sim \triangle NSE$,
$EV \cdot EN = EL \cdot SE$



$\angle V \rightarrow \overset{\frown}{LN}$
$\angle S \rightarrow \overset{\frown}{LN}$

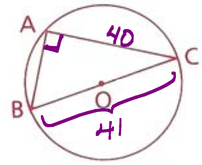
$\angle L \rightarrow \overset{\frown}{VS}$
$\angle N \rightarrow \overset{\frown}{VS}$

Proof

1 $\odot O$	1 Given
2 $\angle V \cong \angle S$	2 2 inscribed $\angle$s make same arc $\Rightarrow \cong \angle$s
3 $\angle L \cong \angle N$	3 same as 2
4 $\triangle LVE \sim \triangle NSE$	4 AA
5 $\dfrac{EV}{SE} = \dfrac{EL}{EN}$	5 $\sim \triangle$s $\Rightarrow$ corr sds prop
6 $EV \cdot EN = EL \cdot SE$	6 means extremes product

Problem 2

In circle O, $\overline{BC}$ is a diameter and the radius of the circle is 20.5 mm. Chord $\overline{AC}$ has a length of 40 mm. Find AB.



Solution

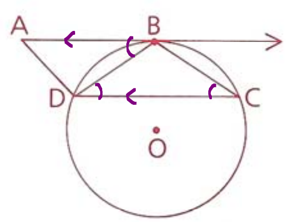
Since $\angle A$ is inscribed in a semicircle, it is a right angle. By the Pythagorean Theorem,

special $\rightarrow$ 9 mm

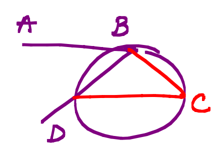
Problem 3

Given: $\odot O$ with $\overleftrightarrow{AB}$ tangent at B, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Prove: $\angle C \cong \angle BDC$



$\angle C \rightarrow \overset{\frown}{DB}$
$\angle ABD \rightarrow \overset{\frown}{DB}$



Proof

1 $\overleftrightarrow{AB}$ is tangent to $\odot O$.	1 Given
2 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	2 Given
3 $\angle ABD \cong \angle BDC$	3 $\parallel \Rightarrow$ ALT INT $\angle$s $\cong$
4 $\angle C \cong \angle ABD$	4 2 inscribed $\angle$s, same arc $\Rightarrow \cong \angle$
5 $\angle C \cong \angle BDC$	5 trans

$\cancel{\angle ABD = \angle BDC}$
$\cancel{\angle C = \angle ABD}$

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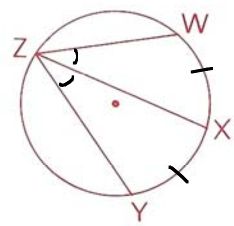
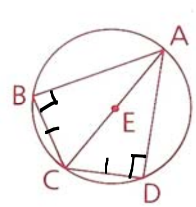
1 Given: X is the midpt. of $\widehat{WY}$.
Prove: $\overrightarrow{ZX}$ bisects $\angle WZY$.



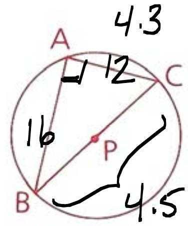
Statements	Reasons
1. X mdpt $\widehat{WY}$	1. Given
2. $\widehat{WX} \cong \widehat{XY}$	2. mdpt $\Rightarrow \cong \frown$s
3. $\angle WZX \cong \angle YZX$	3. $\cong \frown$s $\rightarrow$ 2 $\cong$ inscribed $\angle$s
4. $\overrightarrow{ZX}$ bis $\angle WZY$	4. 2 $\cong \angle$s $\Rightarrow$ bis

2 Given: $\odot E$ with diameter $\overline{AC}$, $\overline{BC} \cong \overline{CD}$
Conclusion: $\triangle ABC \cong \triangle ADC$



HL

3 In $\odot P$, $\overline{BC}$ is a diameter, AC = 12 mm, and BA = 16 mm. Find the radius of the circle.



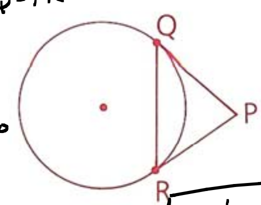
4(3, 4, 5)

$\therefore$ rad = 10 mm

4 Given: $\overline{PQ}$ and $\overline{PR}$ are tangent segments. → $PQ = PR$
$\widehat{QR} = 163°$

Find: **a** $\angle P$ + $\widehat{QR} = 180$, $\angle P + 163 = 180$, $\angle P = 17°$

b $\angle PQR$



$\boxed{b}$ $\overline{PQ}$ & $\overline{PR}$ are tan → $\overline{PQ} = \overline{PR}$ → $\angle Q = \angle R$

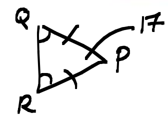
$\angle$s of $\Delta = 180°$

$\angle Q + \angle R + \angle P = 180$

$x + x + 17 = 180$

$2x = 163$

$x = 81.5°$



$\boxed{\text{or } b}$ $\frac{\widehat{QR}}{2} = \angle PQR$

$\frac{163}{2} = \angle PQR$

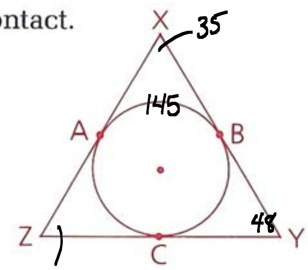
$81.5 = \angle PQR$

5 Given: A, B, and C are points of contact.
$\widehat{AB} = 145°$, $\angle Y = 48°$

Find: $\angle Z$

$X + Y + Z = 180$

$180 - (48 + 35) = 97°$

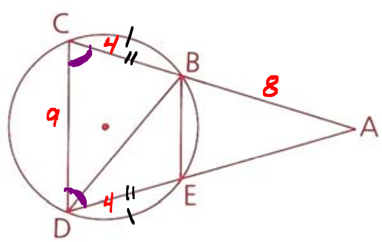


6 Given: $\widehat{BC} \cong \widehat{ED}$, AB = 8,
BC = 4, CD = 9

a Are $\overline{BE}$ and $\overline{CD}$ parallel? sidesplit ∴ yes

b Find BE.

c Is $\triangle ACD$ scalene? Isos Δ



$\angle BCD \rightarrow \widehat{BED} = \widehat{BE} + \widehat{ED}$

$\angle CDE \rightarrow \widehat{CBE} = \widehat{BE} + \widehat{CB}$

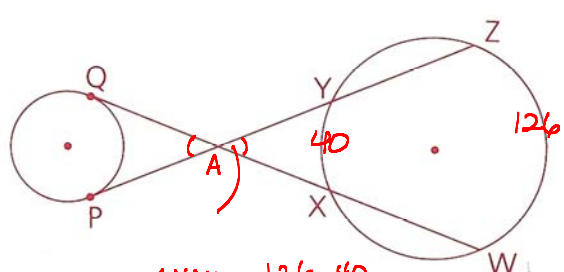
add ref given≅

$\triangle ABE \sim \triangle ACD$

$\frac{BE}{AB} = \frac{CD}{AC} \rightarrow \frac{BE}{8} = \frac{9}{12} \rightarrow BE = \frac{72}{12} = 6$

7 Given: $\overleftrightarrow{PY}$ and $\overleftrightarrow{QW}$ are tangents.
$\widehat{WZ} = 126°$, $\widehat{XY} = 40°$

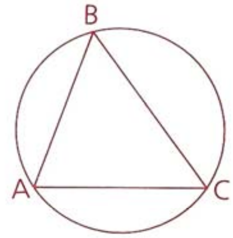
Find: $\widehat{PQ}$



$\angle QAP + \widehat{QP} = 180$

$\angle YAX = \frac{126 - 40}{2} =$

8 If $\triangle ABC$ is inscribed in a circle and $\widehat{AC} \cong \widehat{AB}$, tell whether each of the following must be true, could be true, or cannot be true.



a $\overline{AB} \cong \overline{AC}$

b $\overline{AC} \cong \overline{BC}$

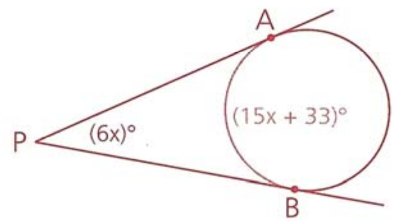
c $\overline{AB}$ and $\overline{AC}$ are equidistant from the center of the circle.

d $\angle B \cong \angle C$

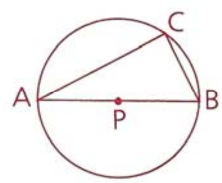
e $\angle BAC$ is a right angle.

f $\angle ABC$ is a right angle.

9 In the figure shown, find m$\angle$P.



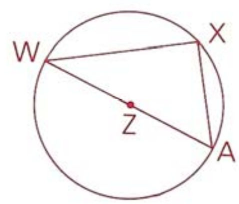
10 If $\overline{AB}$ is a diameter of $\odot$P, CB = 1.5 m, and CA = 2 m, find the radius of $\odot$P.



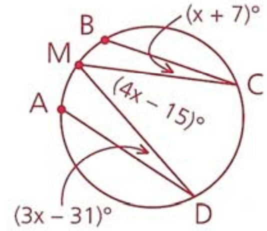
11 The radius of $\odot$Z is 6 cm and $\widehat{WX} = 120°$.

Find: **a** AX

b The perimeter of $\triangle$WAX



12 M is the midpoint of $\widehat{AB}$. Find m$\widehat{CD}$.

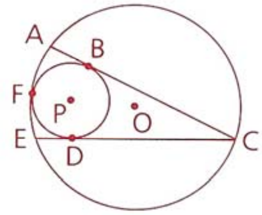


13 A rectangle with dimensions 18 by 24 is inscribed in a circle. Find the radius of the circle.

14 A square is inscribed in a circle with a radius of 10. Find the length of a side of the square.

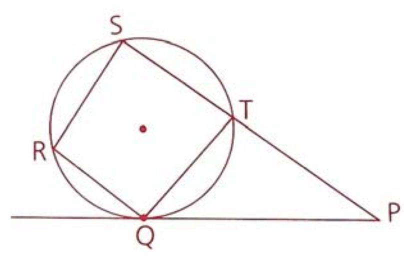
15 Quadrilateral ABCD is inscribed in circle O. AB = 12, BC = 16, CD = 10, and ∠ABC is a right angle. Find the measure of $\overline{AD}$ in simplified radical form.

16 Circles O and P are tangent at F. $\overline{AC}$ and $\overline{CE}$ are tangent to ⊙P at B and D. If $\overset{\frown}{DFB} = 223°$, find $\overset{\frown}{AE}$.

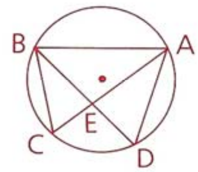


17 Given: ∠S = 88°, $\overset{\frown}{QT} = 104°$, $\overset{\frown}{ST} = 94°$, tangent $\overline{PQ}$

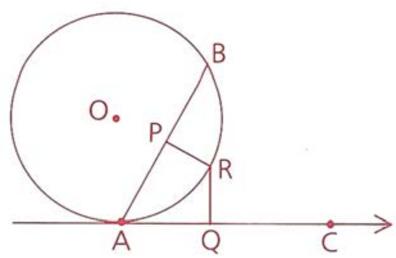
Find: **a** ∠P
b ∠STQ



18 Given: $\widehat{BC} \cong \widehat{CD}$
Conclusion: $\triangle ABC \sim \triangle AED$

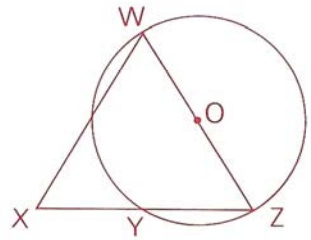


19 Given: $\overleftrightarrow{AC}$ is tangent at A. $\angle APR$ and $\angle AQR$ are right $\angle$s. R is the midpoint of $\widehat{AB}$.
Conclusion: $\overline{PR} \cong \overline{RQ}$ (Hint: Draw $\overline{AR}$.)



20 Given: $\triangle WXZ$ is isosceles, with $\overline{WX} \cong \overline{WZ}$.
$\overline{WZ}$ is a diameter of $\odot O$.

Prove: Y is the midpoint of $\overline{XZ}$.
(Hint: Draw $\overline{WY}$.)



21 Given: $\overline{AC}$ is tangent to $\odot O$ at A.

Conclusion: $\triangle ADC \sim \triangle BDA$