AMDG

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Adv Geo - 2

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F 26 Apr 2013

10-6: More Angle-Arc Theorems

Review

If the vertex of the angle is ____ the circle	Then use this formula to find the angle's measure:
IN	$\angle = \frac{\frown + \frown}{2}$
ON inscribed $\angle$	$\angle = \frac{\frown}{2}$
OUT	$\frac{\frown - \frown}{2} = \angle$

Objectives

After studying this section, you will be able to

- Recognize congruent inscribed and tangent-chord angles
- Determine the measure of an angle inscribed in a semicircle
- Apply the relationship between the measures of a tangent-tangent angle and its minor arc

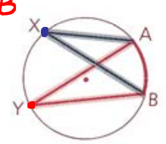
Theorem 89 *If two inscribed (vertex on) or tangent-chord angles intercept the same arc, then they are congruent.*

$\angle AXB$ makes $\widehat{AB}$

$\angle AYB$ make $\widehat{AB}$

Given: X and Y are inscribed angles intercepting arc AB.

Conclusion: $\angle X \cong \angle Y$



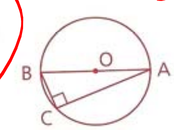
2 inscribed $\angle$s form same $\frown$ $\Rightarrow \cong \angle$s

Theorem 91 *An angle inscribed in a semicircle is a right angle.*

$\frac{1}{2}\odot = 180°$

Given: $\overline{AB}$ is a diameter of $\odot O$.

Prove: $\angle C$ is a right angle.



$\angle = \frac{\frown}{2}$

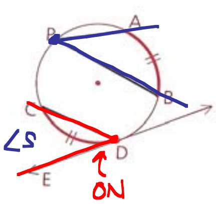
$\angle = \frac{180}{2}$

$\angle = 90°$

Theorem 90 *If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.*

If $\overleftrightarrow{ED}$ is the tangent at D and $\widehat{AB} \cong \widehat{CD}$, we may conclude that $\angle P \cong \angle CDE$.

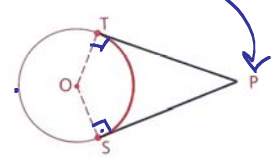
$\cong \frown$s $\Rightarrow \cong$ inscr $\angle$s



Theorem 92 *The sum of the measures of a tangent-tangent angle and its minor arc is 180.*

Given: $\overline{PT}$ and $\overline{PS}$ are tangent to circle O.

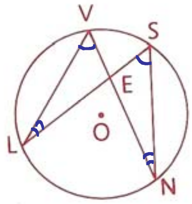
Prove: $m\angle P + m\widehat{TS} = 180$



$\widehat{ST} + \angle P = 180°$

Problem 1 Given: $\odot O$

Conclusion: $\triangle LVE \sim \triangle NSE$,
$EV \cdot EN = EL \cdot SE$

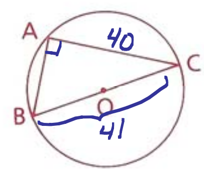


Proof

1 $\odot O$	1 Given
2 $\angle V \cong \angle S$	2 2 inscr $\angle$s make same $\frown \Rightarrow \cong \angle$s
3 $\angle L \cong \angle N$	3 same as 2
4 $\triangle LVE \sim \triangle NSE$	4 AA
5 $\dfrac{EV}{SE} = \dfrac{EL}{EN}$	5 $\sim\triangle$s $\Rightarrow$ corr. sds. prop.
6 $EV \cdot EN = EL \cdot SE$	6 means - extremes product

$\angle V$ makes $\overset{\frown}{LN}$
$\angle S$ makes $\overset{\frown}{LN}$

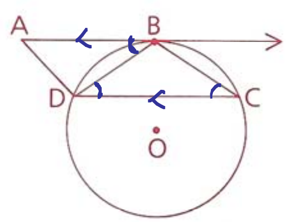
Problem 2 *In circle O, $\overline{BC}$ is a diameter and the radius of the circle is 20.5 mm. Chord $\overline{AC}$ has a length of 40 mm. Find AB.*

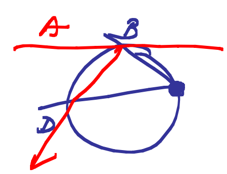


Solution Since $\angle A$ is inscribed in a semicircle, it is a right angle. By the Pythagorean Theorem,

$AB^2 + AC^2 = CB^2$
$AB^2 + 40^2 = 41^2$
This is a special $\rightarrow$ $AB = 9mm$!

Problem 3 *Given: $\odot O$ with $\overleftrightarrow{AB}$ tangent at B, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$*
Prove: $\angle C \cong \angle BDC$





Proof

1 $\overleftrightarrow{AB}$ is tangent to $\odot O$.	1 Given
2 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	2 Given
3 $\angle ABD \cong \angle BDC$	3 $\parallel \Rightarrow$ ALT INT $\angle$s $\cong$
4 $\angle C \cong \angle ABD$	4 2 inscrib $\angle$s make same arc $\Rightarrow \cong \angle$s
5 $\angle C \cong \angle BDC$	5 trans.

$\cancel{\angle ABD} = \angle BDC$
$\angle C = \angle \cancel{ABD}$

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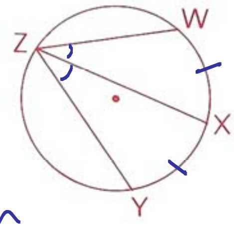
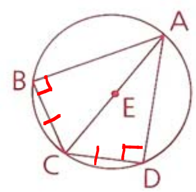
1 Given: X is the midpt. of $\widehat{WY}$.
Prove: $\overrightarrow{ZX}$ bisects $\angle WZY$.



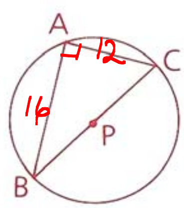
1. X mdpt $\widehat{WY}$	1. Given
2. $\widehat{WX} \cong \widehat{XY}$	2. mdpt $\Rightarrow \cong$ arcs
3. $\angle WZX \cong \angle YZX$	3. 2 $\cong$ ⌒s $\Rightarrow$ 2 $\cong$ inscribed $\angle$s
4. $\overrightarrow{ZX}$ bis $\angle WZY$	4. 2 $\cong \angle$s $\Rightarrow$ bis.

2 Given: $\odot$E with diameter $\overline{AC}$, $\overline{BC} \cong \overline{CD}$
Conclusion: $\triangle ABC \cong \triangle ADC$



HL

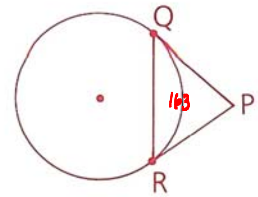
3 In $\odot$P, $\overline{BC}$ is a diameter, AC = 12 mm, and BA = 16 mm. Find the radius of the circle.



$AB^2 + AC^2 = BC^2$
$12^2 + 16^2 = BC^2$
(12, 16, —)
4(3, 4,) → BC = 20
∴ BP = 10

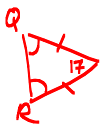
4 Given: $\overline{PQ}$ and $\overline{PR}$ are tangent segments.
$\overset{\frown}{QR} = 163°$

Find: **a** $\angle P$
b $\angle PQR$



a) minor arc + ext ∠ = 180°
$163 + \angle P = 180$
$\angle P = 17°$

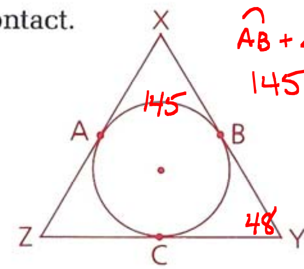
b)



QP = PR (tan ⇒ ≅)
∠Q = ∠R (≅△ = ≅△)
$180 = 17 + 2x$ (∠s of △ = 180)
$163 = 2x$
$81.5 = x$

5 Given: A, B, and C are points of contact.
$\overset{\frown}{AB} = 145°$, $\angle Y = 48°$

Find: $\angle Z$



$\overset{\frown}{AB} + \angle X = 180$
$145 + \angle X = 180$
$\angle X = 35$

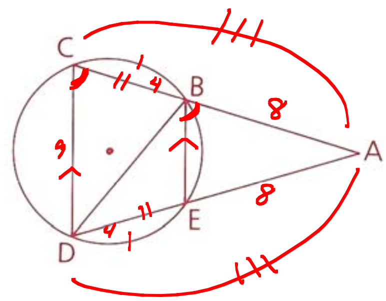
$\angle X + \angle Y + \angle Z = 180$
$35 + 48 + \angle Z = 180°$
$\angle Z = 180 - 83$
$\angle Z = 97°$

6 Given: $\overset{\frown}{BC} \cong \overset{\frown}{ED}$, AB = 8,
BC = 4, CD = 9

a Are $\overline{BE}$ and $\overline{CD}$ parallel? Yes side-split

b Find BE. 6

c Is △ACD scalene? No, it's Isos.



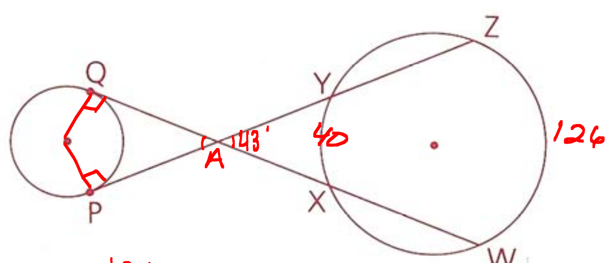
∠C ⇒ $\overset{\frown}{BED}$
∠D ⇒ $\overset{\frown}{CBE}$
∠C ≅ ∠D

b) △ACD ~ △ABE

$\frac{BE}{CD} = \frac{AB}{AC} \rightarrow \frac{BE}{9} = \frac{8}{12} \rightarrow BE = \frac{\cancel{9} \cdot \cancel{8}}{\cancel{4} \cdot \cancel{3}} = 6$

7 Given: $\overleftrightarrow{PY}$ and $\overleftrightarrow{QW}$ are tangents.
$\overset{\frown}{WZ} = 126°$, $\overset{\frown}{XY} = 40°$

Find: $\overset{\frown}{PQ}$



$\angle YAX = \frac{\overset{\frown}{WZ} - \overset{\frown}{YX}}{2}$

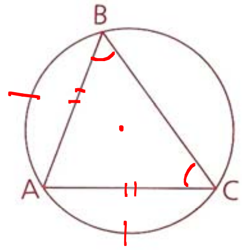
$\angle YAX = \frac{126 - 40}{2} = \frac{86}{2} = 43°$

$126 - 40$

$\angle QAP + \overset{\frown}{PQ} = 180$
$43 + \overset{\frown}{PQ} = 180$
$PQ = 180 - 43 = 137°$

8 If $\triangle ABC$ is inscribed in a circle and $\overset{\frown}{AC} \cong \overset{\frown}{AB}$, tell whether each of the following must be true, could be true, or cannot be true.

N A S



a $\overline{AB} \cong \overline{AC}$ A

b $\overline{AC} \cong \overline{BC}$ S

c $\overline{AB}$ and $\overline{AC}$ are equidistant from the center of the circle. A

d $\angle B \cong \angle C$ $\cong\angle s \Rightarrow \cong\triangle s \rightarrow$ A

e $\angle BAC$ is a right angle. S

f $\angle ABC$ is a right angle. N

can't have 2 rt∠s in a △

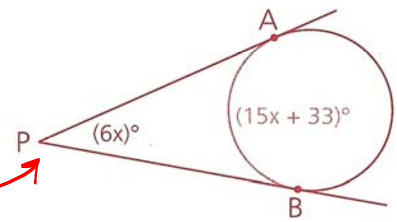
9 In the figure shown, find m∠P.

$6x + 15x + 33 = 180$
$\quad\quad\quad\quad -33 \quad -33$

$\frac{21x}{21} = \frac{147}{21}$

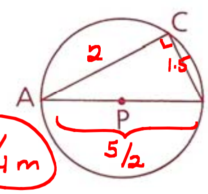
$x = 7$

then $6x = 42°$



10 If $\overline{AB}$ is a diameter of ⊙P, CB = 1.5 m, and CA = 2 m, find the radius of ⊙P.

$\frac{5}{2} \cdot \frac{1}{2} = \frac{5}{4}$ m



(1.5, 2, __)

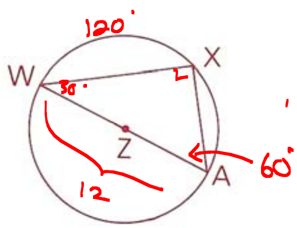
$\frac{1}{2}$(3, 4, __)

11 The radius of ⊙Z is 6 cm and $\overset{\frown}{WX} = 120°$.

Find: **a** AX = 6 cm

b The perimeter of $\triangle WAX$

$\boxed{18 + 6\sqrt{3} \text{ cm}}$

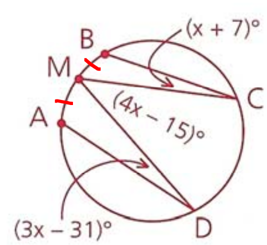


30	60	90
x	$x\sqrt{3}$	2x
6	$6\sqrt{3}$	12

12 M is the midpoint of $\overset{\frown}{AB}$. Find $m\overset{\frown}{CD}$.

$\overset{\frown}{Am} = \overset{\frown}{mB} \rightarrow \angle C = \angle D$

$x + 7 = 3x - 31$

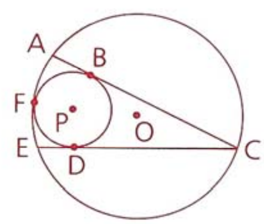


13 A rectangle with dimensions 18 by 24 is inscribed in a circle. Find the radius of the circle.

14 A square is inscribed in a circle with a radius of 10. Find the length of a side of the square.

15 Quadrilateral ABCD is inscribed in circle O. AB = 12, BC = 16, CD = 10, and $\angle ABC$ is a right angle. Find the measure of $\overline{AD}$ in simplified radical form.

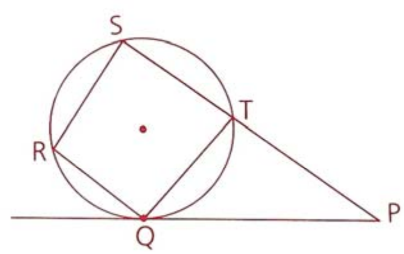
16 Circles O and P are tangent at F. $\overline{AC}$ and $\overline{CE}$ are tangent to $\odot P$ at B and D. If $\overset{\frown}{DFB} = 223°$, find $\overset{\frown}{AE}$.



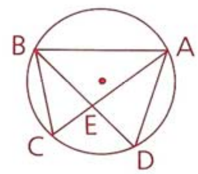
17 Given: $\angle S = 88°$, $\overset{\frown}{QT} = 104°$, $\overset{\frown}{ST} = 94°$, tangent $\overline{PQ}$

Find: **a** $\angle P$

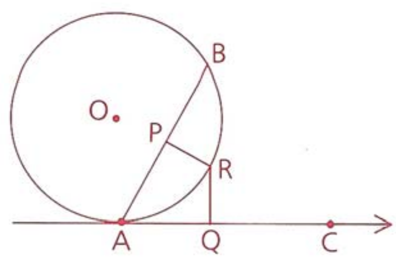
b $\angle STQ$



18 Given: $\widehat{BC} \cong \widehat{CD}$
Conclusion: $\triangle ABC \sim \triangle AED$

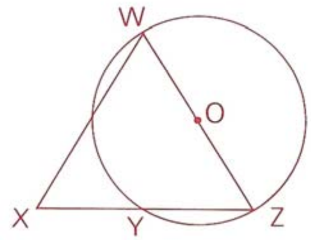


19 Given: $\overleftrightarrow{AC}$ is tangent at A. $\angle APR$ and $\angle AQR$ are right $\angle$s. R is the midpoint of $\widehat{AB}$.
Conclusion: $\overline{PR} \cong \overline{RQ}$ (Hint: Draw $\overline{AR}$.)



20 Given: $\triangle WXZ$ is isosceles, with $\overline{WX} \cong \overline{WZ}$.
$\overline{WZ}$ is a diameter of $\odot O$.

Prove: Y is the midpoint of $\overline{XZ}$.
(Hint: Draw $\overline{WY}$.)



21 Given: $\overline{AC}$ is tangent to $\odot O$ at A.

Conclusion: $\triangle ADC \sim \triangle BDA$