AMDG

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Adv Geo - 2

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F 26 Apr 2013

10-6: More Angle-Arc Theorems

Review

If the vertex of the angle is ____ the circle	Then use this formula to find the angle's measure:
IN	$\angle = \frac{\frown + \frown}{2}$
ON unscribed ∠	$\angle = \frac{\frown}{2}$
OUT	$\frac{\frown - \frown}{2} = \angle$

Objectives

After studying this section, you will be able to

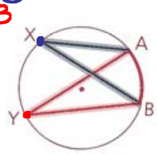
- Recognize congruent inscribed and tangent-chord angles
- Determine the measure of an angle inscribed in a semicircle
- Apply the relationship between the measures of a tangent-tangent angle and its minor arc

Theorem 89 *If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.*

vertex on

$\angle AXB$ makes $\overset{\frown}{AB}$
$\angle AYB$ make $\overset{\frown}{AB}$

Given: X and Y are inscribed angles intercepting arc AB.
Conclusion: $\angle X \cong \angle Y$

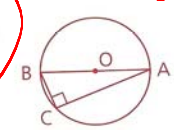


2 inscribed ∠s form same ⌒
⇒ ≅ ∠s

Theorem 91 *An angle inscribed in a semicircle is a right angle.*

$\frac{1}{2}\odot = 180°$

Given: $\overline{AB}$ is a diameter of ⊙O.
Prove: ∠C is a right angle.



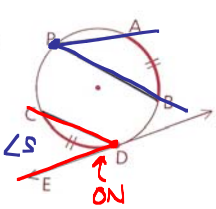
$\angle = \frac{\frown}{2}$

$\angle = \frac{180}{2}$

$\angle = 90°$

Theorem 90 *If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.*

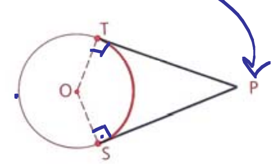
If $\overleftrightarrow{ED}$ is the tangent at D and $\widehat{AB} \cong \widehat{CD}$, we may conclude that $\angle P \cong \angle CDE$.



≅ ⌒s ⇒ ≅ unscr ∠s

Theorem 92 *The sum of the measures of a tangent-tangent angle and its minor arc is 180.*

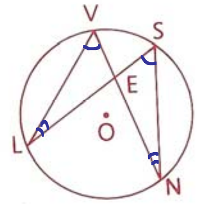
Given: $\overline{PT}$ and $\overline{PS}$ are tangent to circle O.
Prove: $m\angle P + m\widehat{TS} = 180$



$\overset{\frown}{ST} + \angle P = 180°$

Problem 1 Given: $\odot O$

Conclusion: $\triangle LVE \sim \triangle NSE$,
$EV \cdot EN = EL \cdot SE$

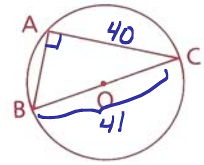


Proof

$\angle V$ makes $\overset{\frown}{LN}$
$\angle S$ makes $\overset{\frown}{LN}$

1 $\odot O$	1 Given
2 $\angle V \cong \angle S$	2 2 inscr $\angle$s make same $\frown$ $\Rightarrow$ $\cong$ $\angle$s
3 $\angle L \cong \angle N$	3 same as 2
4 $\triangle LVE \sim \triangle NSE$	4 AA
5 $\dfrac{EV}{SE} = \dfrac{EL}{EN}$	5 $\sim\triangle$s $\Rightarrow$ corr. sds. prop.
6 $EV \cdot EN = EL \cdot SE$	6 means - extremes product

Problem 2 *In circle O, $\overline{BC}$ is a diameter and the radius of the circle is 20.5 mm. Chord $\overline{AC}$ has a length of 40 mm. Find AB.*



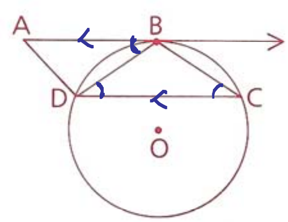
Solution Since $\angle A$ is inscribed in a semicircle, it is a right angle. By the Pythagorean Theorem,

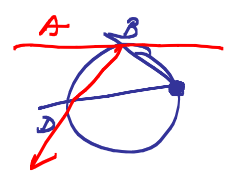
$$AB^2 + AC^2 = CB^2$$
$$AB^2 + 40^2 = 41^2$$

This is a special → $AB = 9$ mm !

Problem 3 *Given: $\odot O$ with $\overleftrightarrow{AB}$ tangent at B, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$*

Prove: $\angle C \cong \angle BDC$





Proof

$\cancel{\angle ABD} = \angle BDC$
$\angle C = \cancel{\angle ABD}$

1 $\overleftrightarrow{AB}$ is tangent to $\odot O$.	1 Given
2 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	2 Given
3 $\angle ABD \cong \angle BDC$	3 $\parallel \Rightarrow$ ALT INT $\angle$s $\cong$
4 $\angle C \cong \angle ABD$	4 2 inscrib $\angle$s make same arc $\Rightarrow \cong \angle$s
5 $\angle C \cong \angle BDC$	5 trans.

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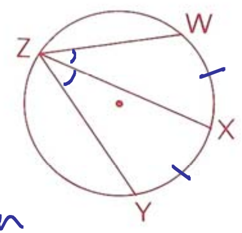
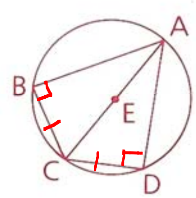
10-6: More Angle-Arc Theorems

1 Given: X is the midpt. of $\widehat{WY}$.
Prove: $\overrightarrow{ZX}$ bisects $\angle WZY$.



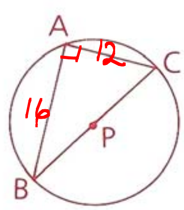
1. X mdpt $\widehat{WY}$	1. Given
2 $\widehat{WX} \cong \widehat{XY}$	2. mdpt $\Rightarrow$ $\cong$ arcs
3. $\angle WZX \cong \angle YZX$	3. 2 $\cong$ ⌒s $\Rightarrow$ 2 $\cong$ inscribed $\angle$s
4. $\overrightarrow{ZX}$ bis $\angle WZY$	4. 2 $\cong \angle$s $\Rightarrow$ bis.

2 Given: $\odot$E with diameter $\overline{AC}$, $\overline{BC} \cong \overline{CD}$
Conclusion: $\triangle ABC \cong \triangle ADC$



HL

3 In $\odot$P, $\overline{BC}$ is a diameter, AC = 12 mm, and BA = 16 mm. Find the radius of the circle.



$AB^2 + AC^2 = BC^2$

$12^2 + 16^2 = BC^2$

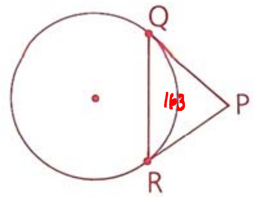
(12, 16, —)

4(3, 4,) $\rightarrow$ BC = 20

$\therefore$ BP = 10

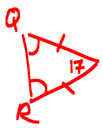
4 Given: $\overline{PQ}$ and $\overline{PR}$ are tangent segments.
$\overparen{QR} = 163°$

Find: **a** $\angle P$
b $\angle PQR$



a) minor arc + ext $\angle$ = 180°
$163 + \angle P = 180$
$\angle P = 17°$

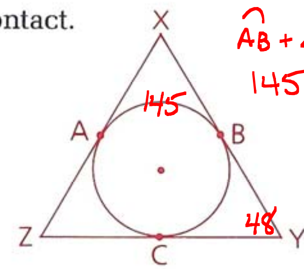
b)



QP = PR (tan ⇒ ≅)
$\angle Q = \angle R$ (≅ sides = ≅ angles)
$180 = 17 + 2x$ ($\angle$s of $\triangle$ = 180)
$163 = 2x$
$81.5 = x$

5 Given: A, B, and C are points of contact.
$\overparen{AB} = 145°$, $\angle Y = 48°$

Find: $\angle Z$

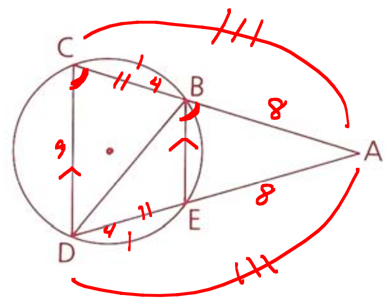


$\overparen{AB} + \angle X = 180$
$145 + \angle X = 180$
$\angle X = 35$

$\angle X + \angle Y + \angle Z = 180$
$35 + 48 + \angle Z = 180°$
$\angle Z = 180 - 83$
$\angle Z = 97°$

6 Given: $\overparen{BC} \cong \overparen{ED}$, AB = 8,
BC = 4, CD = 9

a Are $\overline{BE}$ and $\overline{CD}$ parallel? Yes side-split
b Find BE. 6
c Is $\triangle ACD$ scalene? No, it's Isos.



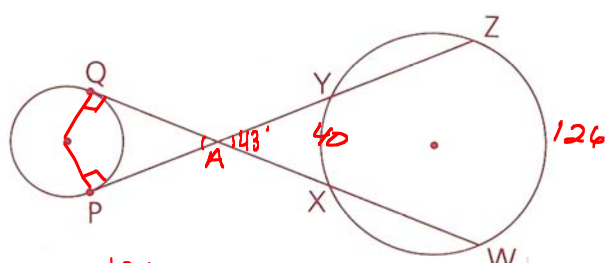
$\angle C \Rightarrow \overparen{BED}$
$\angle D \Rightarrow \overparen{CBE}$
$\angle C \cong \angle D$

$\boxed{b}$ $\triangle ACD \sim \triangle ABE$

$\frac{BE}{CD} = \frac{AB}{AC} \rightarrow \frac{BE}{9} = \frac{8}{12} \rightarrow BE = \frac{\cancel{9} \cdot \cancel{8}}{\cancel{4} \cdot \cancel{3}} = 6$

7 Given: $\overleftrightarrow{PY}$ and $\overleftrightarrow{QW}$ are tangents.
$\overparen{WZ} = 126°$, $\overparen{XY} = 40°$

Find: $\overparen{PQ}$



$\angle YAX = \frac{\overparen{WZ} - \overparen{YX}}{2}$

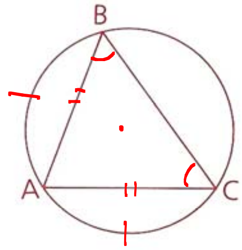
$\angle YAX = \frac{126 - 40}{2} = \frac{86}{2} = 43°$

$126 - 40$

$\angle QAP + \overparen{PQ} = 180$
$43 + \overparen{PQ} = 180$
$PQ = 180 - 43 = 137°$

8 If $\triangle ABC$ is inscribed in a circle and $\widehat{AC} \cong \widehat{AB}$, tell whether each of the following must be true, could be true, or cannot be true.

A S N



a $\overline{AB} \cong \overline{AC}$ A

b $\overline{AC} \cong \overline{BC}$ S

c $\overline{AB}$ and $\overline{AC}$ are equidistant from the center of the circle. A

d $\angle B \cong \angle C$ $\cong$s $\Rightarrow$ $\triangle$ $\rightarrow$ A

e $\angle BAC$ is a right angle. S

f $\angle ABC$ is a right angle. N

can't have 2 rt∠s in a $\triangle$

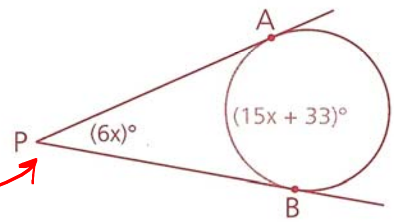
9 In the figure shown, find m∠P.

$6x + 15x + 33 = 180$
$\quad -33 \qquad -33$

$\frac{21x}{21} = \frac{147}{21}$

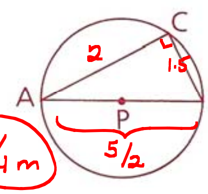
$x = 7$

then $6x = 42°$



10 If $\overline{AB}$ is a diameter of ⊙P, CB = 1.5 m, and CA = 2 m, find the radius of ⊙P.

$\frac{5}{2} \cdot \frac{1}{2} = \frac{5}{4}$ m



(1.5, 2, __)

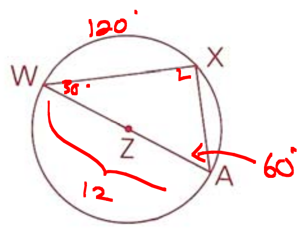
$\frac{1}{2}$(3, 4, __)

11 The radius of ⊙Z is 6 cm and $\widehat{WX} = 120°$.

Find: **a** AX = 6 cm

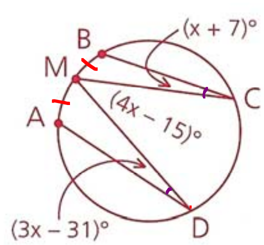
b The perimeter of $\triangle WAX$

$\boxed{18 + 6\sqrt{3} \text{ cm}}$



30	60	90
x	$x\sqrt{3}$	$2x$
6	$6\sqrt{3}$	12

12 M is the midpoint of $\widehat{AB}$. Find $m\widehat{CD}$.



$\widehat{AM} = \widehat{MB} \rightarrow \angle C = \angle D$

$x + 7 = 3x - 31$
$-x \ +31 \quad -x \ +31$

$38 = 2x$

$19 = x \longrightarrow m\angle M = 4(19) - 15$

$4(20 - 1) - 15$

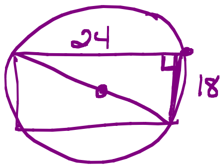
$80 - 4 - 15 = 61°$

$2m\angle M = \widehat{CD}$

$2(61) = \widehat{CD}$

$122° = \widehat{CD}$

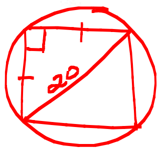
13 A rectangle with dimensions 18 by 24 is inscribed in a circle. Find the radius of the circle.



(18, 24, __)

6(3, 4, 5) ∴ diam = 30 so radius = 15.

14 A square is inscribed in a circle with a radius of 10. Find the length of a side of the square.

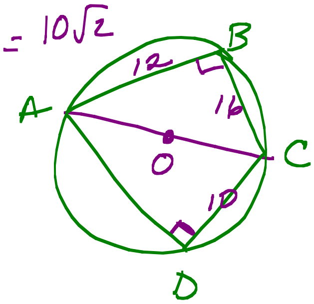


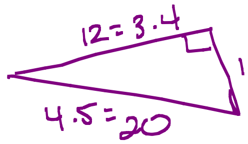
45	45	90
x	x	$x\sqrt{2}$
$10\sqrt{2}$		20

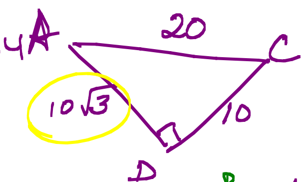
If $x\sqrt{2} = 20$

$x = \frac{20}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{20\sqrt{2}}{2} = 10\sqrt{2}$

15 Quadrilateral ABCD is inscribed in circle O. AB = 12, BC = 16, CD = 10, and ∠ABC is a right angle. Find the measure of $\overline{AD}$ in simplified radical form.







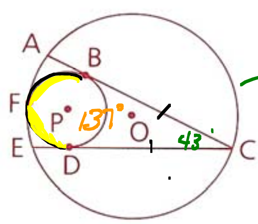
(?, 10, 20)

10(?, 1, 2) → $?^2 + 1^2 = 2^2$

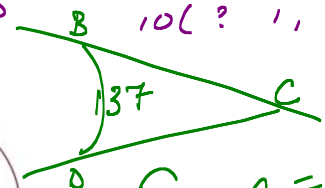
$?^2 = 3$

$? = \sqrt{3}$

16 Circles O and P are tangent at F. $\overline{AC}$ and $\overline{CE}$ are tangent to ⊙P at B and D. If $\widehat{DFB}$ = 223°, find $\widehat{AE}$.



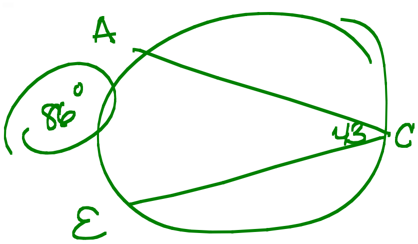
360
−223
37



$\widehat{BD} + \angle C = 180$

$137 + \angle C = 180$

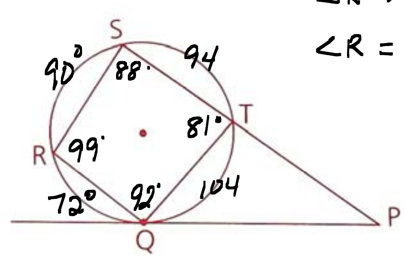
$\angle C = 43°$



17 Given: ∠S = 88°, $\widehat{QT}$ = 104°, $\widehat{ST}$ = 94°, tangent $\overline{PQ}$

Find: **a** ∠P

b ∠STQ



$\angle R \rightarrow \widehat{STQ} = 198$

$\angle R = 99$

$\angle S = \frac{1}{2}(\widehat{QT} + \widehat{QR})$

$88 = \frac{1}{2}(104 + \widehat{QR})$

$178 = 104 + \widehat{QR}$

$-104 \quad -104$

$72 = \widehat{QR}$

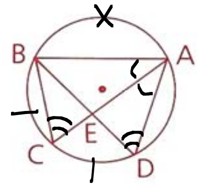
$\angle Q = 180 - 88 =$

$\angle P = \frac{(90+72) - 104}{2} = \frac{162 - 104}{2} = \frac{58}{2} = 29°$

$\angle STQ = 180 - 99 = 81°$

18 Given: $\overparen{BC} \cong \overparen{CD}$

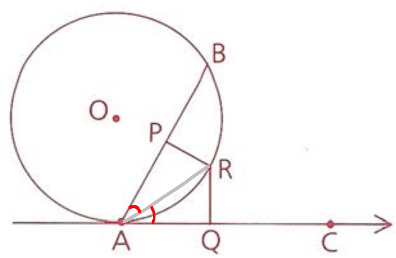
Conclusion: $\triangle ABC \sim \triangle AED$



1. $\overparen{BC} \cong \overparen{CD}$	1. Given
2. $\angle BAC \cong \angle CAD$	2. $\cong$ arcs $\Rightarrow$ inscribed $\angle$s $\cong$
3. $\angle BCA \cong \angle BDA$	3. same arc $\Rightarrow$ inscribed $\angle$s $\cong$
4. $\triangle ABC \sim \triangle AED$	4. AA$\sim$

19 Given: $\overleftrightarrow{AC}$ is tangent at A. $\angle APR$ and $\angle AQR$ are right $\angle$s. R is the midpoint of $\overparen{AB}$.

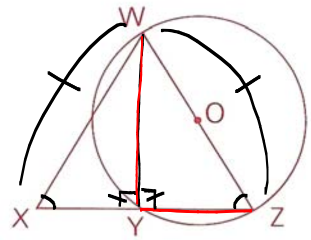
Conclusion: $\overline{PR} \cong \overline{RQ}$ (Hint: Draw $\overline{AR}$.)



1. $\overline{AC}$ tan A $\angle APR$ & $\angle AQR$ rt$\angle$s R mdpt $\overparen{AB}$	1. given
2. $\overparen{BR} \cong \overparen{RA}$	2.
3. DRAW $\overline{AR}$	3.
4. $m\angle RAQ = \frac{1}{2}\overparen{AR}$	4. m tan-chd $\angle$ is ½ arc
5. $m\angle PAR = \frac{1}{2}\overparen{BR}$	5. m inscrib $\angle$ is ½ arc
6. $\angle RAQ = \angle PAR$	6.
7. $\overline{RA} \cong \overline{RA}$	7.
8. $\triangle RAQ \cong \triangle PAR$	8.
9. $\overline{PR} \cong \overline{RQ}$	9.

20 Given: $\triangle WXZ$ is isosceles, with $\overline{WX} \cong \overline{WZ}$.
$\overline{WZ}$ is a diameter of $\odot O$.

Prove: Y is the midpoint of $\overline{XZ}$.
(Hint: Draw $\overline{WY}$.)

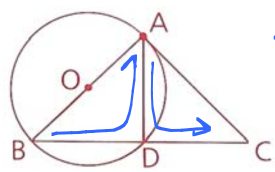


$\overline{WZ}$ diam $\odot O$

1. $\triangle WXZ$ isos, $\overline{XW} \cong \overline{WZ}$	1. given
2. Draw $\overline{WY}$	2. 2 pts det line
3. $\angle X \cong \angle Z$	3. $\cong$ sides $\rightarrow \cong \angle$s
4. $\angle WYZ$ rt$\angle$	4. diam $\Rightarrow$ inscrib $\angle$ is rt $\angle$
5. $\angle WYX$ rt$\angle$	5. subtract
6. $\angle WYZ \cong \angle WYX$	6. rt$\angle$s $\Rightarrow \cong \angle$s
7. $\triangle WYX \cong \triangle WYZ$	7. AAS
8. $\overline{XY} \cong \overline{YZ}$	8. cpctc
9. Y mdpt $\overline{XZ}$	9. $\cong$ segs $\Rightarrow$ mdpt

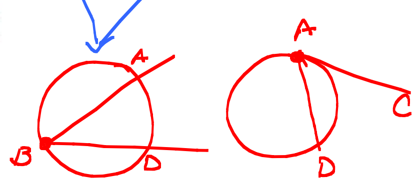
or HL

21 Given: $\overline{AC}$ is tangent to $\odot O$ at A.
Conclusion: $\triangle ADC \sim \triangle BDA$



$\triangle ADC \sim \triangle BDA$

AA~



1. AC tan $\odot O$ @ A	1. given
2. $\angle ABD \cong \angle CAD$	2. 2 inscrib $\angle$s form same arc $\Rightarrow \cong \angle$s
3. $\angle ADB$ rt$\angle$	3. diam $\rightarrow$ inscr $\angle$ is rt
4. $\angle BDC$ st$\angle$	4. Assumed
5. $\angle ADC$ rt $\angle$	5. subtract
6. $\angle ADB \cong \angle ADC$	6. rt$\angle \Rightarrow \cong \angle$s
7. $\triangle ADC \sim \triangle BDA$	7. AA~