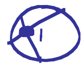
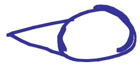
AMDG

NAME StuDent
Adv Geo - 1

Ms. Kresovic
F 26 Apr 2013

10-6: More Angle-Arc Theorems

Review

If the vertex of the angle is ____ the circle	Then use this formula to find the angle's measure:
IN 	$\angle 1 = \frac{\frown + \frown}{2}$
ON 	$\angle = \frac{\frown}{2}$
OUT 	$\angle = \frac{\frown - \frown}{2}$

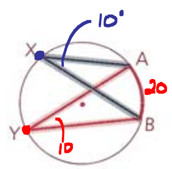
Objectives

After studying this section, you will be able to

- Recognize congruent inscribed and tangent-chord angles
- Determine the measure of an angle inscribed in a semicircle
- Apply the relationship between the measures of a tangent-tangent angle and its minor arc

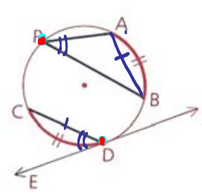
Theorem 89 ***If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.***

Given: X and Y are inscribed angles intercepting arc AB.
Conclusion: $\angle X \cong \angle Y$



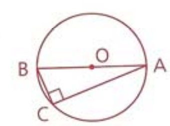
Theorem 90 ***If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.***

If $\overleftrightarrow{ED}$ is the tangent at D and $\widehat{AB} \cong \widehat{CD}$, we may conclude that $\angle P \cong \angle CDE$.



Theorem 91 ***An angle inscribed in a semicircle is a right angle.***

Given: $\overline{AB}$ is a diameter of $\odot$O.
Prove: $\angle C$ is a right angle.



CARPENTER'S TRICK

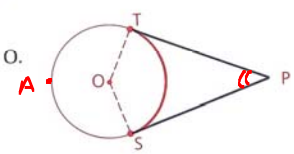
$\angle = \frac{\frown}{2}$

$90° = \frac{\frown}{2}$

$180 = \frown$

Theorem 92 ***The sum of the measures of a tangent-tangent angle and its minor arc is 180.***

Given: $\overline{PT}$ and $\overline{PS}$ are tangent to circle O.
Prove: $m\angle P + m\widehat{TS} = 180$

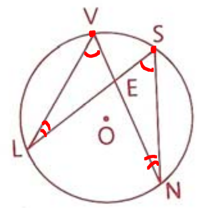


$\frac{\widehat{SAT} - \widehat{ST}}{2} = \angle P$

$\widehat{SAT} - \widehat{ST} = 2(\angle P)$

$\widehat{SAT} = 2(\angle P) + \widehat{ST}$ Alex says there's no point bc this is what we've been doing.

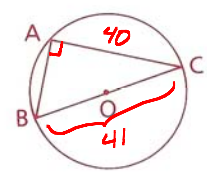
Problem 1 Given: ⊙O
Conclusion: $\triangle LVE \sim \triangle NSE$,
$EV \cdot EN = EL \cdot SE$



Proof

1 ⊙O	1 Given
2 $\angle V \cong \angle S$	2 If 2 inscribed ∠s make same arc then ∠s ≅.
3 $\angle L \cong \angle N$	3 Same as 2
4 $\triangle LVE \sim \triangle NSE$	4 AA
5 $\frac{EV}{SE} = \frac{EL}{EN}$	5 ~△s ⇒ corr. sds. prop.
6 $EV \cdot EN = EL \cdot SE$	6 means-extremes

Problem 2 *In circle O, $\overline{BC}$ is a* <u>diameter</u> *and the radius of the circle is 20.5 mm.*
Chord $\overline{AC}$ has a length of 40 mm. Find AB.

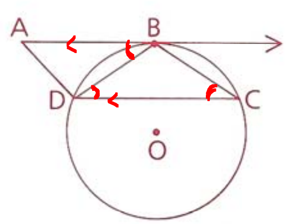


Solution Since ∠A is inscribed in a semicircle, it is a right angle. By the Pythagorean Theorem,

$AB^2 + AC^2 = BC^2$
$AB^2 + 40^2 = 41^2$
It's a special!

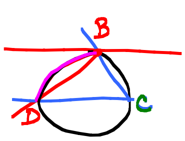
$AB = 9$ mm

Problem 3 *Given:* ⊙O *with $\overleftrightarrow{AB}$ tangent at B, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$*
Prove: $\angle C \cong \angle BDC$



Proof

1 $\overleftrightarrow{AB}$ is tangent to ⊙O.	1 Given
2 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	2 Given
3 $\angle ABD \cong \angle BDC$	3 ∥ ⇒ alt int ∠s ≅
4 $\angle C \cong \angle ABD$	4 If 2 inscribed ∠s make same arc then ≅ ∠s.
5 $\angle C \cong \angle BDC$	5 trans.



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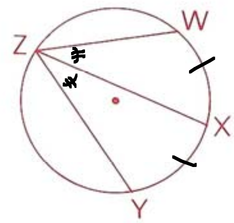
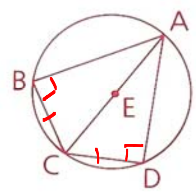
10-6: More Angle-Arc Theorems

1 Given: X is the midpt. of $\widehat{WY}$.
Prove: $\overrightarrow{ZX}$ bisects $\angle WZY$.



Statements	Reasons
1. X mdpt $\widehat{WY}$	1. Given
2. $\widehat{WX} \cong \widehat{XY}$	2. mdpt $\Rightarrow \cong \cap$s
3. $\angle WZX \cong \angle YXZ$	3. $\cong$ arcs $\Rightarrow$ $\cong$ inscribed $\angle$s
4. $\overrightarrow{ZX}$ bis $\angle WZY$	4. $\cong \angle$s $\Rightarrow$ bis

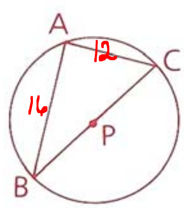
2 Given: $\odot E$ with diameter $\overline{AC}$, $\overline{BC} \cong \overline{CD}$
Conclusion: $\triangle ABC \cong \triangle ADC$



HL

3 In $\odot P$, $\overline{BC}$ is a diameter, AC = 12 mm, and BA = 16 mm. Find the radius of the circle.

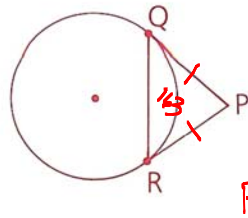
$\Rightarrow \angle A =$ right



$AC^2 + AB^2 = BC^2$
$12^2 + 16^2 = BC^2$
$(12, 16, —)$
$4(3, 4, 5) \Rightarrow BC = 20$mm
$\therefore BP = 10$mm

4 Given: $\overline{PQ}$ and $\overline{PR}$ are tangent segments.
$\widehat{QR} = 163°$

Find: **a** $\angle P + \widehat{QR} = 180° \rightarrow \angle P = 180 - 163 = 17°$
b $\angle PQR$

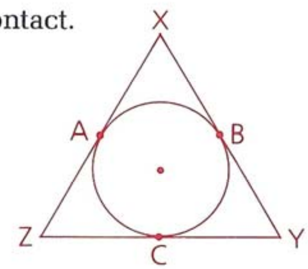


$\angle P = 17°$ → $PQ = PR$ (2tan ⇒ ≅ seg)
$\angle RQP = \angle QRP$ (△ ⇒ △)

$17 + x + x = 180$ (∠s triangle = 180°)
$2x = 163$
$x = 81.5°$

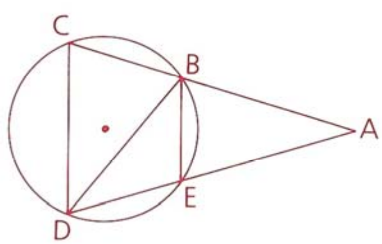
5 Given: A, B, and C are points of contact.
$\widehat{AB} = 145°$, $\angle Y = 48°$

Find: $\angle Z$



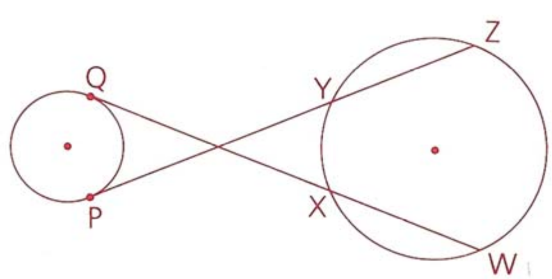
6 Given: $\widehat{BC} \cong \widehat{ED}$, AB = 8,
BC = 4, CD = 9

a Are $\overline{BE}$ and $\overline{CD}$ parallel?
b Find BE.
c Is △ACD scalene?

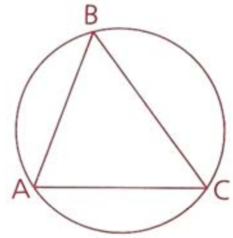


7 Given: $\overleftrightarrow{PY}$ and $\overleftrightarrow{QW}$ are tangents.
$\widehat{WZ} = 126°$, $\widehat{XY} = 40°$

Find: $\widehat{PQ}$



8 If $\triangle ABC$ is inscribed in a circle and $\overset{\frown}{AC} \cong \overset{\frown}{AB}$, tell whether each of the following must be true, could be true, or cannot be true.



a $\overline{AB} \cong \overline{AC}$

b $\overline{AC} \cong \overline{BC}$

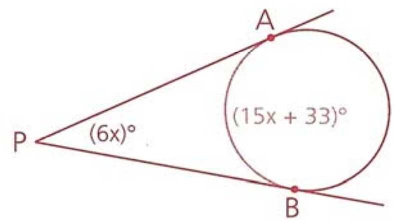
c $\overline{AB}$ and $\overline{AC}$ are equidistant from the center of the circle.

d $\angle B \cong \angle C$

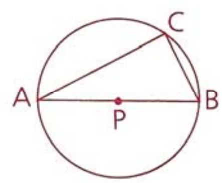
e $\angle BAC$ is a right angle.

f $\angle ABC$ is a right angle.

9 In the figure shown, find m$\angle P$.



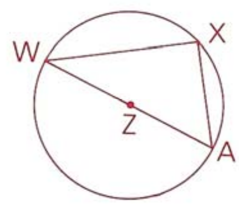
10 If $\overline{AB}$ is a diameter of $\odot P$, CB = 1.5 m, and CA = 2 m, find the radius of $\odot P$.



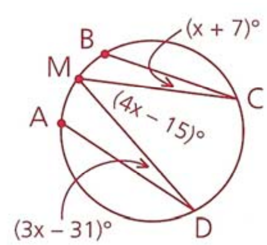
11 The radius of $\odot Z$ is 6 cm and $\overset{\frown}{WX} = 120°$.

Find: **a** AX

b The perimeter of $\triangle WAX$



12 M is the midpoint of $\overset{\frown}{AB}$. Find m$\overset{\frown}{CD}$.

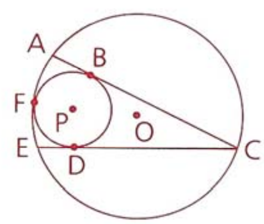


13 A rectangle with dimensions 18 by 24 is inscribed in a circle. Find the radius of the circle.

14 A square is inscribed in a circle with a radius of 10. Find the length of a side of the square.

15 Quadrilateral ABCD is inscribed in circle O. AB = 12, BC = 16, CD = 10, and $\angle ABC$ is a right angle. Find the measure of $\overline{AD}$ in simplified radical form.

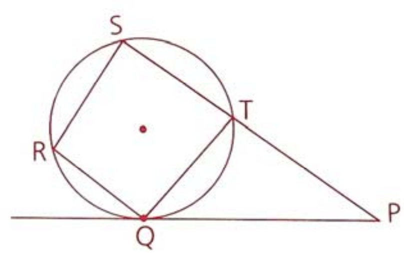
16 Circles O and P are tangent at F. $\overline{AC}$ and $\overline{CE}$ are tangent to $\odot P$ at B and D. If $\overset{\frown}{DFB} = 223°$, find $\overset{\frown}{AE}$.



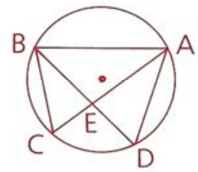
17 Given: $\angle S = 88°$, $\overset{\frown}{QT} = 104°$, $\overset{\frown}{ST} = 94°$, tangent $\overline{PQ}$

Find: **a** $\angle P$

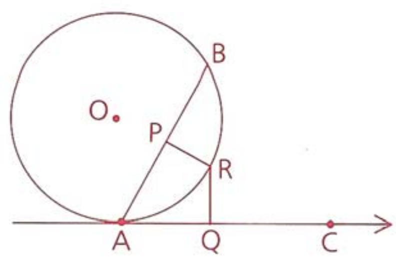
b $\angle STQ$



18 Given: $\overset{\frown}{BC} \cong \overset{\frown}{CD}$
Conclusion: $\triangle ABC \sim \triangle AED$

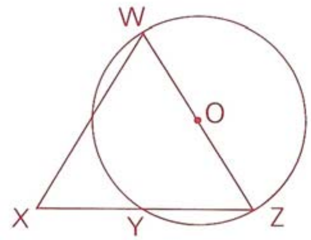


19 Given: $\overleftrightarrow{AC}$ is tangent at A. $\angle APR$ and $\angle AQR$ are right $\angle$s. R is the midpoint of $\overset{\frown}{AB}$.
Conclusion: $\overline{PR} \cong \overline{RQ}$ (Hint: Draw $\overline{AR}$.)



20 Given: $\triangle WXZ$ is isosceles, with $\overline{WX} \cong \overline{WZ}$.
$\overline{WZ}$ is a diameter of $\odot O$.

Prove: Y is the midpoint of $\overline{XZ}$.
(Hint: Draw $\overline{WY}$.)



21 Given: $\overline{AC}$ is tangent to $\odot O$ at A.

Conclusion: $\triangle ADC \sim \triangle BDA$