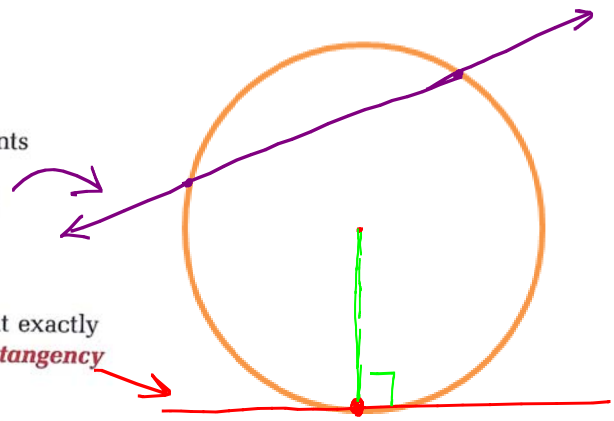
Name ________________________ Ms. Kresovic

Adv Geo – period _____ F 12 Apr 213

10.4 Secants and Tangents

Objectives

After studying this section, you will be able to
- Identify secant and tangent lines
- Identify secant and tangent segments
- Distinguish between two types of tangent circles
- Recognize common internal and common external tangents



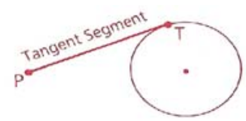
Definition A ***secant*** is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.)

Definition A ***tangent*** is a line that intersects a circle at exactly one point. This point is called the ***point of tangency*** or ***point of contact***.

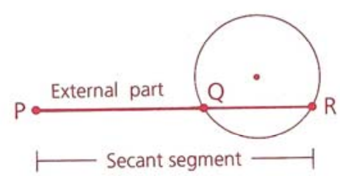
Postulate ***A tangent line is perpendicular to the radius drawn to the point of contact.***

Postulate ***If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.***

Definition A ***tangent segment*** is the part of a tangent line between the point of contact and a point outside the circle.



Definition A ***secant segment*** is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

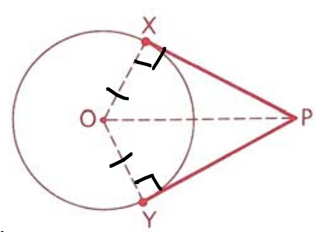


Definition The ***external part*** of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.

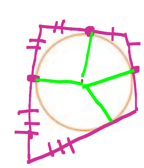
Theorem 85 ***If two tangent segments are drawn to a circle from an external point, then those segments are congruent. (Two-Tangent Theorem)***

Given: ⊙O;
$\overline{PX}$ and $\overline{PY}$ are tangent segments.

Prove: $\overline{PX} \cong \overline{PY}$



Statements	Reasons
1. ⊙O, $\overline{PX}$ & $\overline{PY}$ tan	1. Given
2. ∠OXP & ∠OYP rt∠s	2. tan ⇒ rt∠s
3. $\overline{OX} \cong \overline{OY}$	3. ⊙ ⇒ rad ≅
4 $\overline{OP} \cong \overline{OP}$	4. Ref
5. △POX ≅ △POY	5. HL
6. $\overline{PX} \cong \overline{PY}$	6. CPCTC



2 tan ⇒ 2 ≅ segs

Tangent Circles

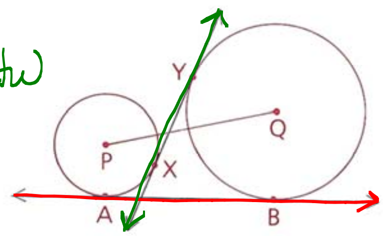
Definition ***Tangent circles*** are circles that intersect each other at exactly one point.

Common Tangents

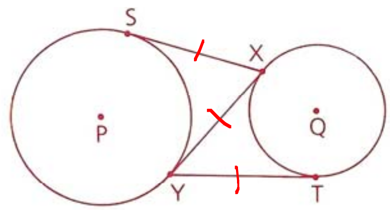
$\overleftrightarrow{PQ}$ is the line of centers.

$\overleftrightarrow{XY}$ is a ***common internal tangent.*** betw

$\overleftrightarrow{AB}$ is a ***common external tangent.***



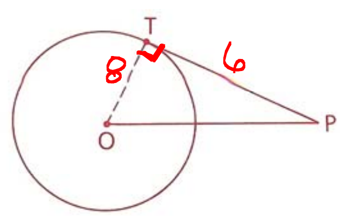
Problem 1 *Given:* $\overline{XY}$ *is a common internal tangent to* ⓢ P *and* Q *at* X *and* Y.
$\overline{XS}$ *is tangent to* ⊙P *at* S.
$\overline{YT}$ *is tangent to* ⊙Q *at* T.
Conclusion: $\overline{XS} \cong \overline{YT}$



Proof

1 $\overline{XS}$ is tangent to ⊙P. $\overline{YT}$ is tangent to ⊙Q.	1 Given
2 $\overline{XY}$ is tangent to ⓢ P and Q.	2 Given
3 $\overline{XS} \cong \overline{XY}$	3 2 tan ⇒ ≅ segs
4 $\overline{XY} \cong \overline{YT}$	4 2 tan ⇒ ≅ segs
5 $\overline{XS} \cong \overline{YT}$	5 trans.

Problem 2 $\overleftrightarrow{TP}$ *is tangent to circle* O *at* T.
The radius of circle O *is* 8 *mm.*
Tangent segment $\overline{TP}$ *is* 6 *mm long.*
Find the length of $\overline{OP}$.



Solution

6, 8, —

2(3, 4, 5) ⇒ 10

Common-Tangent Procedure

1 Draw the segment joining the centers.

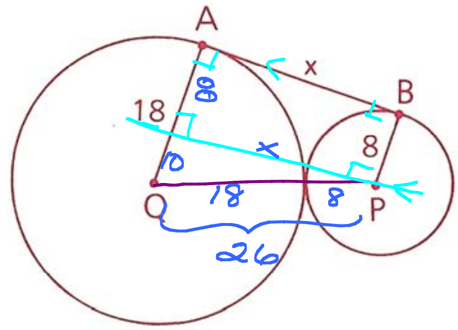
2 Draw the radii to the points of contact.

3 Through the center of the smaller circle, draw a line parallel to the common tangent.

4 Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.

5 Use the Pythagorean Theorem and properties of a rectangle.

Problem 3 *A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a common external tangent.*



Solution

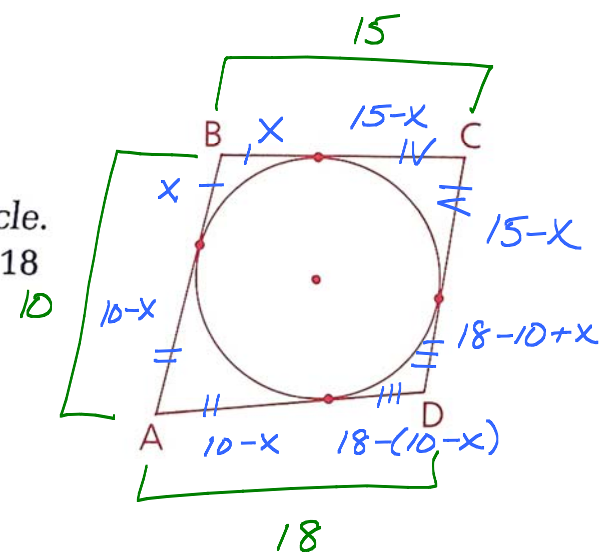
$x^2+10^2=26^2$

(10, –, 26)

2(5, 12, 13) → 24

Problem 4 *A walk-around problem:*
Given: Each side of quadrilateral ABCD is tangent to the circle.
AB = 10, BC = 15, AD = 18
Find: CD

Test problem



2 tan ⇒ 2 ≅ segs

$CD = 15 - x + 18 - 10 + x$

$CD = 15 + 18 - 10$

$CD = 15 + 8 = 23$

AMDG

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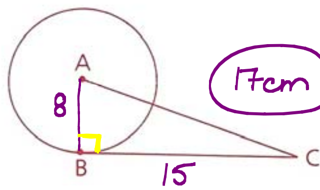
Homework 10.4 Secants and Tangents

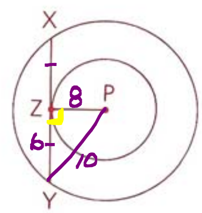
1 The radius of ⊙A is 8 cm.
Tangent segment $\overline{BC}$ is 15 cm long.
Find the length of $\overline{AC}$.



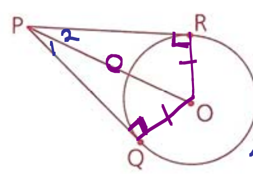
17cm

2 Concentric circles with radii 8 and 10 have center P.
$\overline{XY}$ is a tangent to the inner circle and is a chord of the outer circle.
Find $\overline{XY}$. (Hint: Draw $\overline{PX}$ and $\overline{PY}$.)

$2(6) = \boxed{12}$

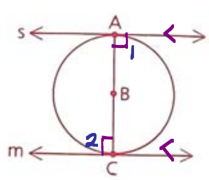


3 Given: $\overline{PR}$ and $\overline{PQ}$ are tangents to ⊙O at R and Q.
Prove: $\overrightarrow{PO}$ bisects ∠RPQ. (Hint: Draw $\overline{RO}$ and $\overline{OQ}$.)



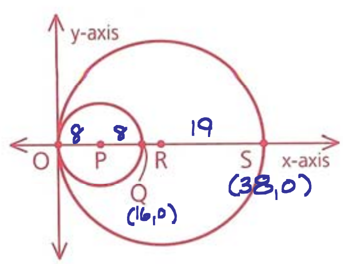
Statements	Reasons
1. $\overline{PR}$ & $\overline{PQ}$ tan ⊙O @ R & Q	1. Given
2. $\overline{PQ} \perp \overline{QO}$ & $\overline{PR} \perp \overline{RO}$	2. tan ⇒ ⊥
3. ∠PQO & ∠PRO rt∠s	3. ⊥ ⇒ rt∠s
4. $\overline{PO} \cong \overline{PO}$	4. Ref
5. $\overline{RO} \cong \overline{QO}$	5. ⊙ ⇒ ≅ rad
6. △PRO ≅ △PQO	6. HL
7. ∠1 ≅ ∠2	7. CPCTC
8. $\overrightarrow{PO}$ bis ∠RPQ	8. ≅∠s ⇒ bis

4 Given: $\overline{AC}$ is a diameter of ⊙B.
Lines s and m are tangents to the ⊙ at A and C.
Conclusion: s ∥ m

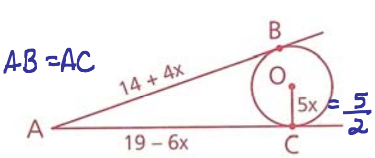


1. AC diameter ⊙B, s & m tan ⊙ @ A & C	1. Given
2. s ⊥ $\overline{AC}$ & m ⊥ $\overline{AC}$	2. tan ⇒ ⊥
3. ∠1 & ∠2 rt∠s	3. ⊥ ⇒ rt∠
4. ∠1 ≅ ∠2	4. rt∠ ≅
5. s ∥ m	5. Alt int ∠s ≅ ⇒ ∥

5 ⊙P and ⊙R are internally tangent at O.
P is at (8, 0) and R is at (19, 0).
a Find the coordinates of Q and S.
b Find the length of $\overline{QR}$. 19 − 16 = 3



6 $\overline{AB}$ and $\overline{AC}$ are tangents to ⊙O, ⇒ AB = AC
and OC = 5x. Find OC.

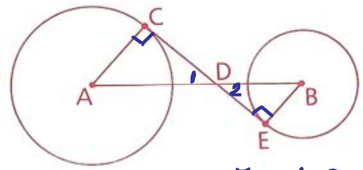


$14 + 4x = 19 - 6x$
$-14 + 6x \quad -14 + 6x$
$10x = 5$
$x = 1/2$

7 Given: $\overline{CE}$ is a common internal tangent to circles A and B at C and E.

Prove: **a** $\angle A \cong \angle B$

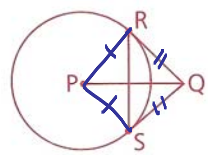
b $\frac{AD}{BD} = \frac{CD}{DE}$



1. $\overline{CE}$ comm int tan to ⊙s A&B @ C&E.	1. Given
2. $\overline{AC} \perp \overline{CE}$ & $\overline{BE} \perp \overline{CE}$	2. tan ⇒ ⊥
3. ∠ACE & ∠BED rt∠s	3. ⊥ ⇒ rt∠
4. ∠ACE ≅ ∠BED	4. rt∠ ⇒ ≅∠s
5. ∠1 ≅ ∠2	5. Vert ∠s
6. ∠A ≅ ∠B	6. No Choice
7. △ACD ~ △BED	7. AA~
8. $\frac{AD}{BD} = \frac{CD}{DE}$	8. ~△ ⇒ prop sds

8 Given: $\overline{QR}$ and $\overline{QS}$ are tangent to ⊙P at points R and S.

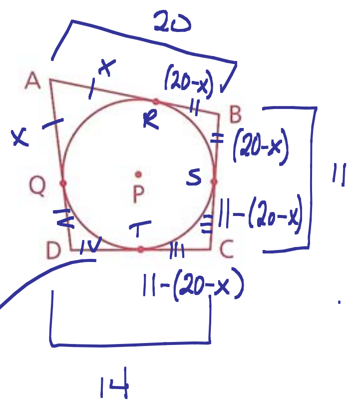
Prove: $\overline{PQ} \perp \overline{RS}$ (Hint: This can be proved in just a few steps.)



1. $\overline{QR}$ + $\overline{QS}$ tan ⊙P at R&S	1. Given
2. Draw $\overline{PR}$ & $\overline{PS}$	2. aux lines
3. $\overline{PR} \cong \overline{PS}$	3. ⊙ ⇒ ≅ radii
4. $\overline{QR} \cong \overline{QS}$	4. 2tan ⇒ ≅ segs
5. $\overline{PQ} \perp \overline{RS}$	5. equidist ⇒ ⊥bis

10 ⊙P is tangent to each side of ABCD. AB = 20, BC = 11, and DC = 14. Let AQ = x and find AD.

2tan ⇒ ≅ segs



$AD = AQ + QD$

$= x + 14 - [11 - (20 - x)]$

$[11 - 20 + x]$

$= x + 14 - [-9 + x]$

$= \boxed{x} + 14 + 9 \boxed{-x}$

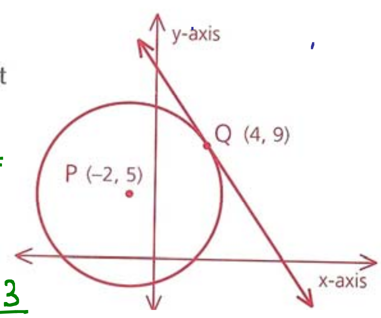
$14 - [11 - (20 - x)]$

$AD = 23$

11 **a** Find the radius of ⊙P.

Qu13

b Find the slope of the tangent to ⊙P at point Q.



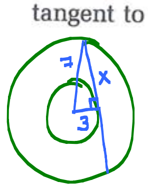
a] $PQ = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(-2-4)^2 + (5-9)^2} = \sqrt{36 + 16} =$

$= \sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$

b] tan ⇒ ⊥ → slopes opp/recip:

$m\ PQ = \frac{\Delta y}{\Delta x} = \frac{9-5}{4+2} = \frac{4}{6} = \frac{2}{3}$ & opp recip $= -\frac{3}{2}$

12 Two concentric circles have radii 3 and 7. Find, to the nearest hundredth, the length of a chord of the larger circle that is tangent to the smaller circle. (See problem 2 for a diagram.)



PYTH: $3^2 + x^2 = 7^2$

$x^2 = 49 - 9$

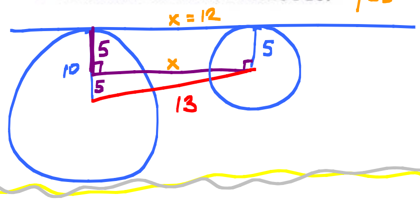
$x = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$

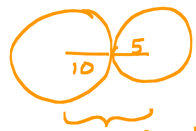
So whole chord is $2(2\sqrt{10}) = 4\sqrt{10}$.

13 The centers of two circles of radii 10 cm and 5 cm are 13 cm apart.

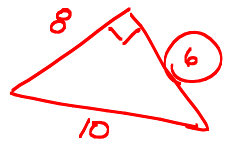
a Find the length of a common external tangent. (Hint: Use the common-tangent procedure.) 12

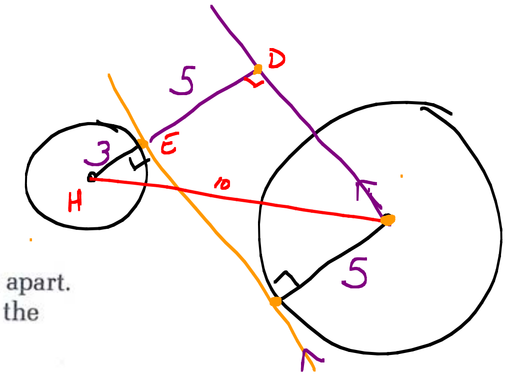
b Do the circles intersect? Yes (13<15)





15 if tan ③

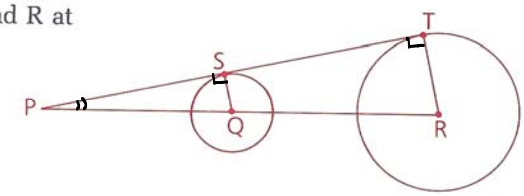




14 The centers of two circles with radii 3 and 5 are 10 units apart. Find the length of a common internal tangent. (Hint: Use the common-tangent procedure.)

15 Given: $\overline{PT}$ is tangent to ⓢ Q and R at points S and T.

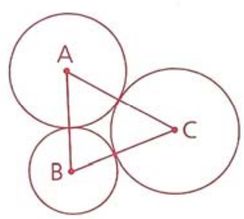
Conclusion: $\frac{PQ}{PR} = \frac{SQ}{TR}$



1. $\overline{PT}$ tan ⊙Q & R @ S & T	1. Given
2. $\overline{QS}$ & $\overline{RT}$ ⊥ PT	2. tan ⇒ ⊥
3. ∠QSP & ∠RTP rt∠s	3. ⊥ ⇒ rt∠
4. ∠QSP ≅ ∠RTP	4. rt∠s ≅ ∠s
5. ∠P ≅ ∠P	5. Ref
6. △QSP ~ △RTP	6. AA~
7. $\frac{PQ}{PR} = \frac{SQ}{TR}$	7. ~△s ⇒ prop sds

16 Given: Tangent ⓢ A, B, and C, AB = 8, BC = 13, AC = 11

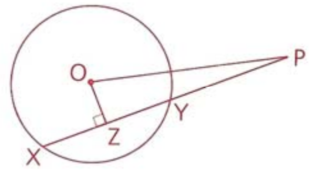
Find: The radii of the three ⓢ (Hint: This is a walk-around problem.)



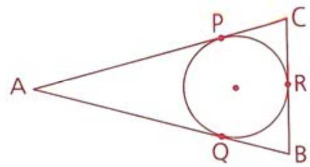
17 The radius of ⊙O is 10.
The secant segment $\overline{PX}$ measures 21 and is 8 units from the center of the ⊙.

a Find the external part (PY) of the secant segment.

b Find OP.



18 Given: △ABC is isosceles, with base $\overline{BC}$.
Conclusion: $\overline{BR} \cong \overline{RC}$



19 If two of the seven circles are chosen at random, what is the probability that the chosen pair are

a Internally tangent?

b Externally tangent?

c Not tangent?