AMDG

Name ___________________________ Ms. Kresovic

Adv Geo – period _____ F 12 Apr 213

10.3 Homework Check: Any talking will result in a 0 grade. Use your homework. You are not provided time to complete the exercises. Copy the work from your homework. You have 5 minutes. Each problem is ¼ point, totaling 3 points.

2 Given: Two concentric circles with center O; $\angle BOC$ is acute.

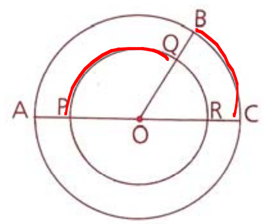
a Name a major arc of the smaller circle.

b Name a minor arc of the larger circle.

c What is $m\widehat{BC} + m\widehat{PQ}$? → degrees

d Which is greater, $m\widehat{BC}$ or $m\widehat{PQ}$?

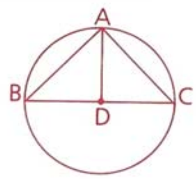
e Is $\widehat{BC}$ congruent to $\widehat{QR}$? NOT ≅ ⊙s



2a	$\widehat{QPR}$ or $\widehat{QRP}$
2b	$\widehat{BC}$ or $\widehat{AB}$
2c	180°
2d	$m\widehat{PQ}$
2e	No

6 Given: ⊙D, $\angle B \cong \angle C$

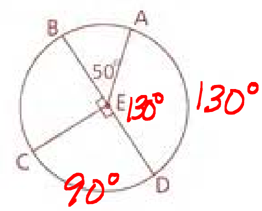
Conclusion: $\widehat{AB} \cong \widehat{AC}$



Statements	Reasons
1. ⊙D, $\angle B \cong \angle C$	1. Given
2. $\overline{AB} \cong \overline{AC}$	2. △ ⇒ △
3. $\widehat{AB} \cong \widehat{AC}$	3. ≅ chds ⇒ ≅ arcs

3 In circle E, find each of the following.

b $m\widehat{AD}$ **e** $m\widehat{ADC}$



3b	130°
3e	220°

9b What fractional part of a circle is an arc that measures 240? $\frac{240}{360} = \frac{2}{3}$

10a Find the measure of an arc that is 3/5 of its circle. 216°

$\frac{3}{5} = \frac{?}{360}$

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2 Given: Two concentric circles with center O; $\angle BOC$ is acute.

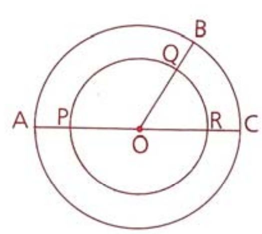
a Name a major arc of the smaller circle.

b Name a minor arc of the larger circle.

c What is $m\widehat{BC} + m\widehat{PQ}$?

d Which is greater, $m\widehat{BC}$ or $m\widehat{PQ}$?

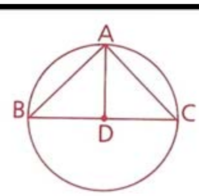
e Is $\widehat{BC}$ congruent to $\widehat{QR}$?



2a	
2b	
2c	
2d	
2e	

6 Given: ⊙D, $\angle B \cong \angle C$

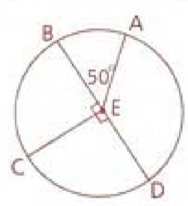
Conclusion: $\widehat{AB} \cong \widehat{AC}$



Statements	Reasons
1. ⊙D, $\angle B \cong \angle C$	3.
2. $\overline{AB} \cong \overline{AC}$	4.
3. $\widehat{AB} \cong \widehat{AC}$	3.

3 In circle E, find each of the following.

b $m\widehat{AD}$ **e** $m\widehat{ADC}$



3b	
3e	

9b What fractional part of a circle is an arc that measures 240?

10a Find the measure of an arc that is 3/5 of its circle.

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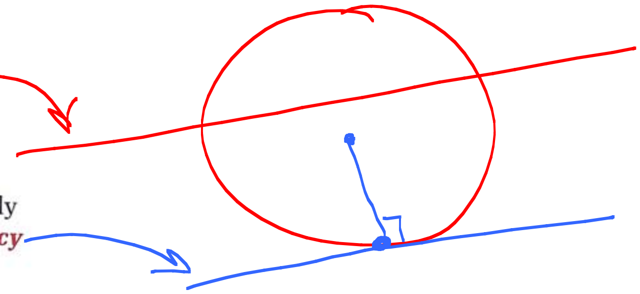
Adv Geo – period __2__ F 12 Apr 213

10.4 Secants and Tangents

Objectives

After studying this section, you will be able to

- Identify secant and tangent lines
- Identify secant and tangent segments
- Distinguish between two types of tangent circles
- Recognize common internal and common external tangents



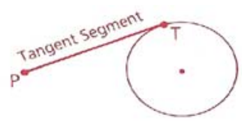
Definition A ***secant*** is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.)

Definition A ***tangent*** is a line that intersects a circle at exactly one point. This point is called the ***point of tangency*** or ***point of contact***.

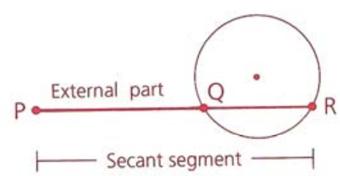
Postulate ***A tangent line is perpendicular to the radius drawn to the point of contact.***

Postulate ***If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.***

Definition A ***tangent segment*** is the part of a tangent line between the point of contact and a point outside the circle.



Definition A ***secant segment*** is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

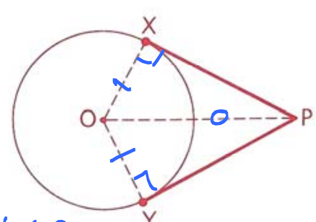


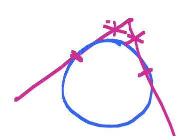
Definition The ***external part*** of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.

Theorem 85 ***If two tangent segments are drawn to a circle from an external point, then those segments are congruent.*** (*Two-Tangent Theorem*)

Given: $\odot O$;
$\overline{PX}$ and $\overline{PY}$ are tangent segments.

Prove: $\overline{PX} \cong \overline{PY}$



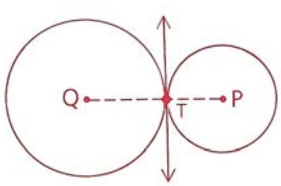


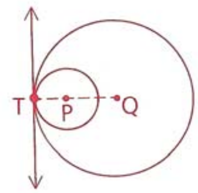
1. $\odot O$, $\overline{PX}$ & $\overline{PY}$ tangent	1. given
2. $\angle OXP$ & $\angle OYP$ rt$\angle$s	2. tan $\Rightarrow$ rt$\angle$s
3. $\overline{OX} \cong \overline{OY}$	3. $\odot \Rightarrow \cong$ rad
4. $\overline{OP} \cong \overline{OP}$	4. Ref
5. $\triangle OXP \cong \triangle OYP$	5. HL
6. $\overline{PX} \cong \overline{PY}$	6. CPCTC

2 tan $\Rightarrow \cong$ seg

Tangent Circles

Definition *Tangent circles* are circles that intersect each other at exactly one point.



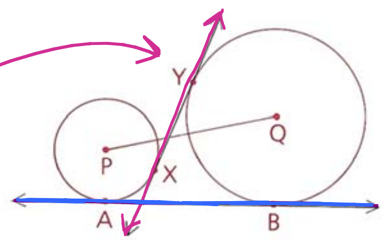


Common Tangents

$\overleftrightarrow{PQ}$ is the line of centers. betw

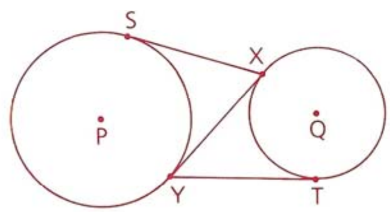
$\overleftrightarrow{XY}$ is a ***common internal tangent.***

$\overleftrightarrow{AB}$ is a ***common external tangent.***



Problem 1 *Given:* $\overline{XY}$ *is a common internal tangent to* Ⓢ P *and* Q *at* X *and* Y. $\overline{XS}$ *is tangent to* ⊙P *at* S. $\overline{YT}$ *is tangent to* ⊙Q *at* T.

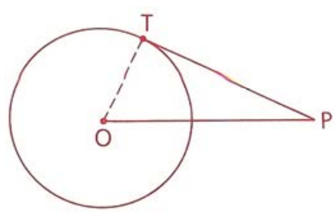
Conclusion: $\overline{XS} \cong \overline{YT}$



Proof

1 $\overline{XS}$ is tangent to ⊙P. $\overline{YT}$ is tangent to ⊙Q.	1 Given
2 $\overline{XY}$ is tangent to Ⓢ P and Q.	2 Given
3 $\overline{XS} \cong \overline{XY}$	3
4 $\overline{XY} \cong \overline{YT}$	4
5 $\overline{XS} \cong \overline{YT}$	5

Problem 2 $\overleftrightarrow{TP}$ *is tangent to circle* O *at* T. *The radius of circle* O *is* 8 *mm. Tangent segment* $\overline{TP}$ *is* 6 *mm long. Find the length of* $\overline{OP}$.



Solution

Common-Tangent Procedure

1 Draw the segment joining the centers.

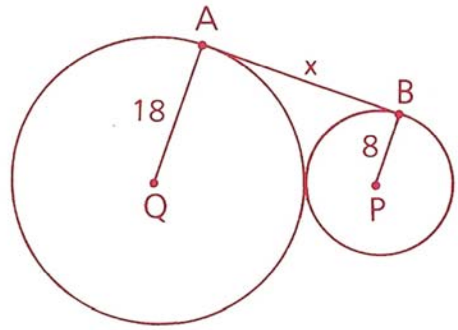
2 Draw the radii to the points of contact.

3 Through the center of the smaller circle, draw a line parallel to the common tangent.

4 Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.

5 Use the Pythagorean Theorem and properties of a rectangle.

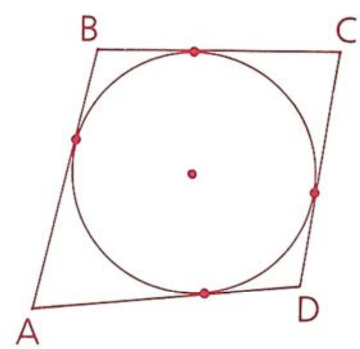
Problem 3 *A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a common external tangent.*



Solution

Problem 4 *A walk-around problem:*
Given: Each side of quadrilateral ABCD is tangent to the circle.
AB = 10, BC = 15, AD = 18

Find: CD



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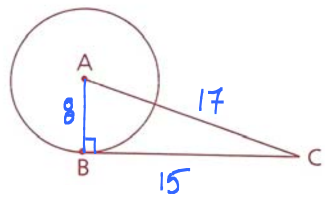
Homework 10.4 Secants and Tangents

1 The radius of ⊙A is 8 cm.
Tangent segment $\overline{BC}$ is 15 cm long.
Find the length of $\overline{AC}$.

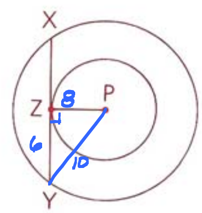
tan ⇒ ⊥



2 Concentric circles with radii 8 and 10 have center P.
$\overline{XY}$ is a tangent to the inner circle and is a chord of the outer circle.
Find $\overline{XY}$. (Hint: Draw $\overline{PX}$ and $\overline{PY}$.)

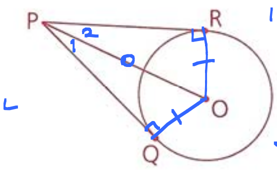
(_8, 10) = 2(3,4,5) → 6

XY = 12



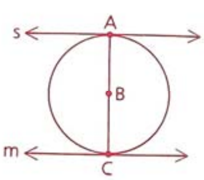
3 Given: $\overline{PR}$ and $\overline{PQ}$ are tangents to ⊙O at R and Q.
Prove: $\overrightarrow{PO}$ bisects ∠RPQ. (Hint: Draw $\overline{RO}$ and $\overline{OQ}$.)

HL



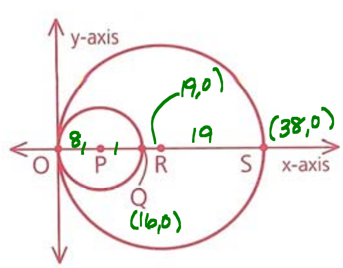
Statements	Reason
1. $\overline{PR}$ & $\overline{PQ}$ tan R&Q	1. Given
2. $\overline{OR} \perp \overline{PR}$, $\overline{OQ} \perp \overline{PQ}$	2. tan ⇒ ⊥
3. $\overline{OR} \cong \overline{OQ}$	3. ⊙ ⇒ ≅ radii
4. $\overline{OP} \cong \overline{OP}$	4. Ref
5. △POR ≅ △POQ	5. HL
6. ∠1 ≅ ∠2	6. CPCTC
7. $\overrightarrow{PO}$ bis ∠RPQ	7. ≅∠s ⇒ bis

4 Given: $\overline{AC}$ is a diameter of ⊙B.
Lines s and m are tangents to the ⊙ at A and C.
Conclusion: s ∥ m



1. $\overline{AC}$ diameter ⊙B, s & m tan ⊙ A & C	1. Given
2. $\overline{AB} \perp s$, $\overline{AB} \perp m$	2. tan ⇒ ⊥
3. s ∥ m	3. 2 lines ⊥ same line ⇒ ∥

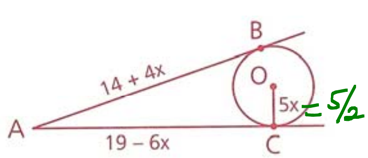
5 ⊙P and ⊙R are internally tangent at O.
P is at (8, 0) and R is at (19, 0).
a Find the coordinates of Q and S.
b Find the length of $\overline{QR}$. 19 − 16 = 3



6 $\overline{AB}$ and $\overline{AC}$ are tangents to ⊙O, and OC = 5x. Find OC.

→ AC = AB

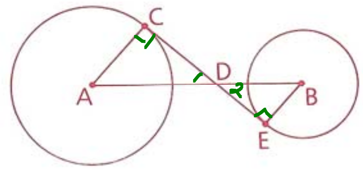
14 + 4x = 19 − 6x
−14 +6x −14 + 6x
10x = 5
x = 1/2



7 Given: $\overline{CE}$ is a common internal tangent to circles A and B at C and E.

Prove: **a** $\angle A \cong \angle B$ ← no choice

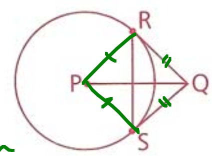
b $\dfrac{AD}{BD} = \dfrac{CD}{DE}$



1. $\overline{CE}$ comm int tan ⊙A+B @ C & E	1. Given
2. $\overline{AC} \perp \overline{CE}$ & $\overline{BE} \perp \overline{CE}$	2. tan ⇒ ⊥
3. ∠ACE & ∠BED rt∠s	3. ⊥ ⇒ rt∠
4. ∠ACE ≅ ∠BED	4. rt∠s ⇒ ≅∠s
5. ∠1 ≅ ∠2	5. Vert∠s
6. △ACD ~ △BED	6. AA~
7. ∠A ≅ ∠B	7. ~△ ⇒ ≅∠s
8. $\dfrac{AD}{BD} = \dfrac{CD}{DE}$	8. ~△ ⇒ prop sds

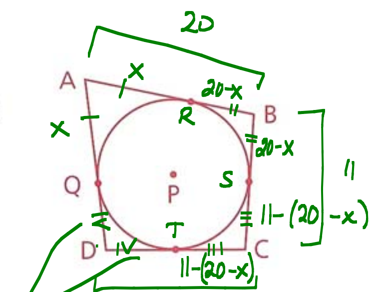
8 Given: $\overline{QR}$ and $\overline{QS}$ are tangent to ⊙P at points R and S.

Prove: $\overline{PQ} \perp \overline{RS}$ (Hint: This can be proved in just a few steps.)



1. QR + QS tan ⊙P @ R & S	1. Given
2. $\overline{PR} \cong \overline{PS}$	2. ⊙ ⇒ ≅ rad
3. $\overline{RQ} \cong \overline{QS}$	3. 2 tan ⇒ ≅ seg
4. $\overline{PQ} \perp \overline{RS}$	4. 2 segs equidist ⇒ ⊥ bis

10 ⊙P is tangent to each side of ABCD. AB = 20, BC = 11, and DC = 14. Let AQ = x and find AD.



$14 - [11 - (20 - x)]$

$AD = x + 14 - [11 - (20 - x)]$

$x + 14 - [11 - 20 + x]$

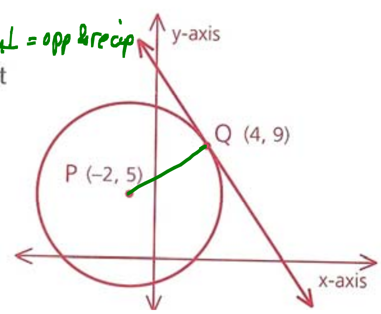
$x + 14 - [-9 + x]$

$x + 14 + 9 - x$

$\boxed{AD = 23}$

11 **a** Find the radius of ⊙P.

b Find the slope of the tangent to ⊙P at point Q. → ⊥ & ⊥ = opp & recip

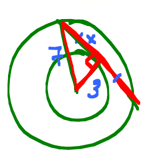


$PQ = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(4+2)^2 + (9-5)^2} = \sqrt{36+16}$

$= \sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$

$m\,PQ = \dfrac{\Delta y}{\Delta x} = \dfrac{9-5}{4+2} = \dfrac{4}{6} = \dfrac{2}{3}$ ∴ opp, recip = $-\dfrac{3}{2}$

12 Two concentric circles have radii 3 and 7. Find, to the nearest hundredth, the length of a chord of the larger circle that is tangent to the smaller circle. (See problem 2 for a diagram.)



$x^2 + 3^2 = 7^2$ (Pyth)

$x^2 = 49 - 9$

$x^2 = 40$

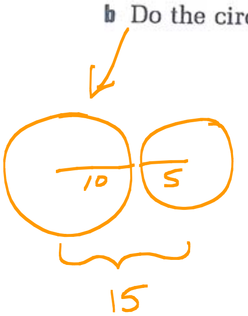
$x = \sqrt{4 \cdot 10} = 2\sqrt{10}$

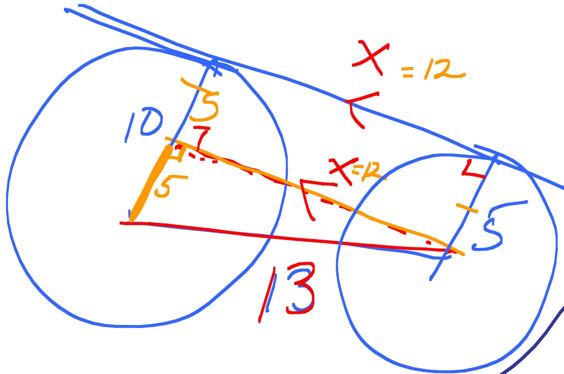
x is ½ chord ∴ chord $2(2\sqrt{10}) = 4\sqrt{10}$

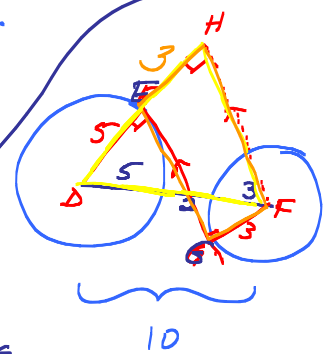
13 The centers of two circles of radii 10 cm and 5 cm are 13 cm apart.

a Find the length of a common external tangent. (Hint: Use the common-tangent procedure.) 12

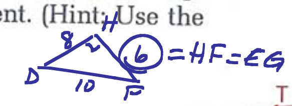
b Do the circles intersect? yes





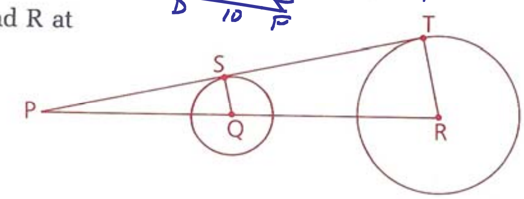


14 The centers of two circles with radii 3 and 5 are 10 units apart. Find the length of a common internal tangent. (Hint: Use the common-tangent procedure.)



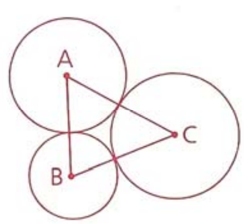
15 Given: $\overline{PT}$ is tangent to ⓢ Q and R at points S and T.

Conclusion: $\frac{PQ}{PR} = \frac{SQ}{TR}$



16 Given: Tangent ⓢ A, B, and C, AB = 8, BC = 13, AC = 11

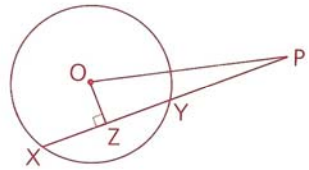
Find: The radii of the three ⓢ (Hint: This is a walk-around problem.)



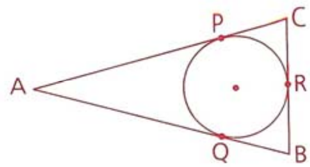
17 The radius of ⊙O is 10.
The secant segment $\overline{PX}$ measures 21 and is 8 units from the center of the ⊙.

a Find the external part (PY) of the secant segment.

b Find OP.



18 Given: △ABC is isosceles, with base $\overline{BC}$.
Conclusion: $\overline{BR} \cong \overline{RC}$



19 If two of the seven circles are chosen at random, what is the probability that the chosen pair are

a Internally tangent?

b Externally tangent?

c Not tangent?