AMDG

Name ___________________________ Ms. Kresovic

Adv Geo – period _____ F 12 Apr 213

10.3 Homework Check: Any talking will result in a 0 grade. Use your homework. You are not provided time to complete the exercises. Copy the work from your homework. You have 5 minutes. Each problem is ¼ point, totaling 3 points.

2 Given: Two concentric circles with center O; $\angle BOC$ is acute.

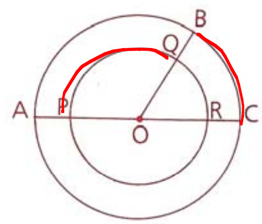
a Name a major arc of the smaller circle.

b Name a minor arc of the larger circle.

c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$?

d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$?

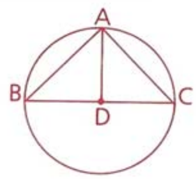
e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$?



2a	
2b	
2c	180°
2d	$\overset{\frown}{PQ}$
2e	No

6 Given: $\odot D$, $\angle B \cong \angle C$

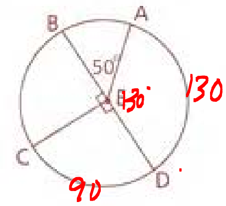
Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$



Statements	Reasons
1. $\odot D$, $\angle B \cong \angle C$	1. Given
2. $\overline{AB} \cong \overline{AC}$	2. △ → ∡
3. $\overset{\frown}{AB} \cong \overset{\frown}{AC}$	3. ≅ chds ⇒ ≅ ⌒s

3 In circle E, find each of the following.

b $m\overset{\frown}{AD}$ **e** $m\overset{\frown}{ADC}$



3b	130°
3e	220°

9b What fractional part of a circle is an arc that measures 240? $\frac{240}{360}$ 2/3

10a Find the measure of an arc that is 3/5 of its circle. $\frac{3}{5} = \frac{216°}{360}$

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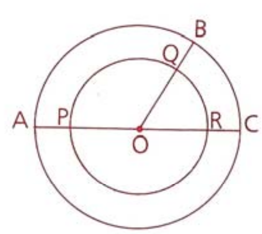
b Name a minor arc of the larger circle.

c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$?

d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$?

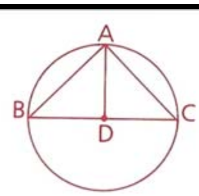
e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$?



2a	
2b	
2c	
2d	
2e	

6 Given: $\odot D$, $\angle B \cong \angle C$

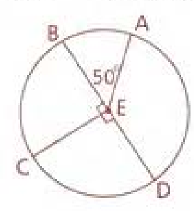
Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$



Statements	Reasons
1. $\odot D$, $\angle B \cong \angle C$	3.
2. $\overline{AB} \cong \overline{AC}$	4.
3. $\overset{\frown}{AB} \cong \overset{\frown}{AC}$	3.

3 In circle E, find each of the following.

b $m\overset{\frown}{AD}$ **e** $m\overset{\frown}{ADC}$



3b	
3e	

9b What fractional part of a circle is an arc that measures 240?

10a Find the measure of an arc that is 3/5 of its circle.

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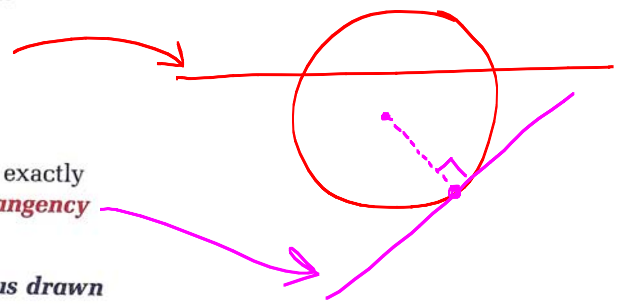
10.4 Secants and Tangents

Objectives

After studying this section, you will be able to
- Identify secant and tangent lines
- Identify secant and tangent segments
- Distinguish between two types of tangent circles
- Recognize common internal and common external tangents

Definition A ***secant*** is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.)

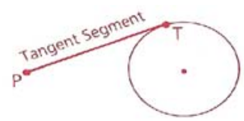


Definition A ***tangent*** is a line that intersects a circle at exactly one point. This point is called the ***point of tangency*** or ***point of contact***.

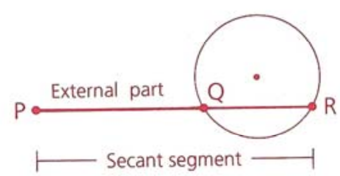
Postulate ***A tangent line is perpendicular to the radius drawn to the point of contact.***

Postulate ***If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.***

Definition A ***tangent segment*** is the part of a tangent line between the point of contact and a point outside the circle.



Definition A ***secant segment*** is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

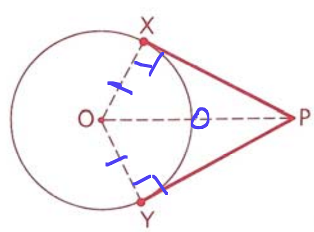


Definition The ***external part*** of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.

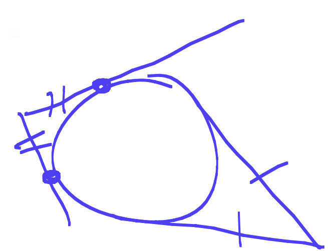
Theorem 85 ***If two tangent segments are drawn to a circle from an external point, then those segments are congruent. (Two-Tangent Theorem)***

Given: ⊙O;
$\overline{PX}$ and $\overline{PY}$ are tangent segments.

Prove: $\overline{PX} \cong \overline{PY}$



Statements	Reasons
1. ⊙O, $\overline{PX}$ & $\overline{PY}$ tangents	1. Given
2. ∠OYP & ∠OXP rt∠s	2. tan ⇒ rt∠s
3. $\overline{XO} \cong \overline{YO}$	3. ⊙ ⇒ ≅ rad
4. $\overline{OP} \cong \overline{OP}$	4. Ref
5. △POX ≅ △POY	5. HL
6. $\overline{PX} \cong \overline{PY}$	6. CPCTC

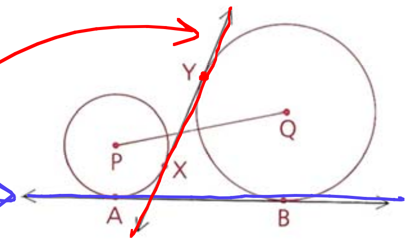


Tangent Circles

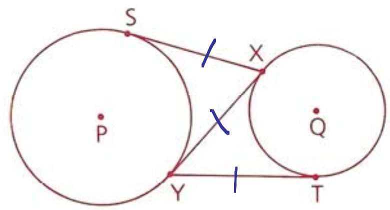
Definition ***Tangent circles*** are circles that intersect each other at exactly one point.

Common Tangents

$\overleftrightarrow{PQ}$ is the line of centers.
$\overleftrightarrow{XY}$ is a ***common internal tangent.***
$\overleftrightarrow{AB}$ is a ***common external tangent.***



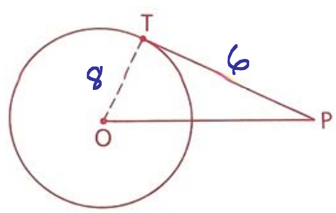
Problem 1 *Given:* $\overline{XY}$ *is a common internal tangent to ⓢ P and Q at X and Y.*
$\overline{XS}$ *is tangent to ⊙P at S.*
$\overline{YT}$ *is tangent to ⊙Q at T.*
Conclusion: $\overline{XS} \cong \overline{YT}$



Proof

1 $\overline{XS}$ is tangent to ⊙P. $\overline{YT}$ is tangent to ⊙Q.	1 Given
2 $\overline{XY}$ is tangent to ⓢ P and Q.	2 Given
3 $\overline{XS} \cong \overline{XY}$	3 two-tan $\Rightarrow \cong$ seg
4 $\overline{XY} \cong \overline{YT}$	4 same as 3
5 $\overline{XS} \cong \overline{YT}$	5 transitive

Problem 2 *$\overleftrightarrow{TP}$ is tangent to circle O at T. The radius of circle O is 8 mm. Tangent segment $\overline{TP}$ is 6 mm long. Find the length of $\overline{OP}$.*



Solution

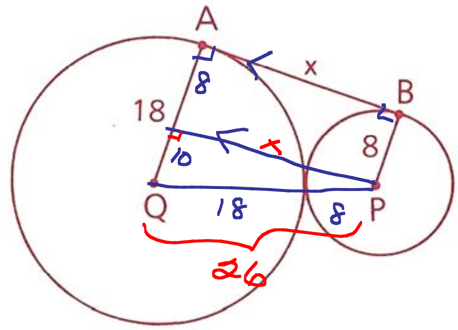
6, 8, —

2(3, 4, 5) = 10

Common-Tangent Procedure

1. Draw the segment joining the centers.
2. Draw the radii to the points of contact.
3. Through the center of the smaller circle, draw a line parallel to the common tangent.
4. Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.
5. Use the Pythagorean Theorem and properties of a rectangle.

Problem 3 *A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a common external tangent.*



Solution

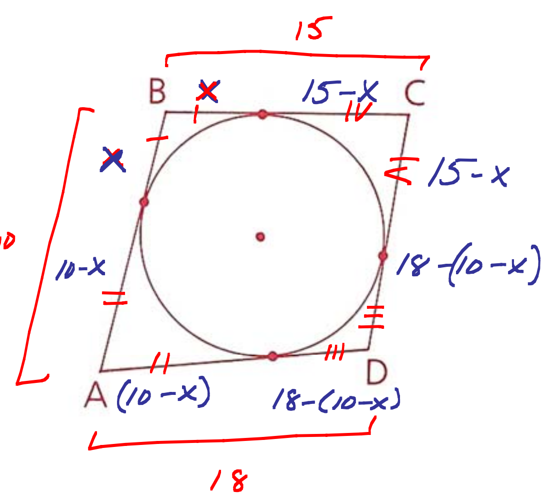
$x^2 + 10^2 = 26^2$

$2(5, 12, 13)$

24 cm

Problem 4 *A walk-around problem:*
Given: Each side of quadrilateral ABCD is tangent to the circle.
AB = 10, BC = 15, AD = 18
Find: CD

Test problem :)



2 tan ⇒ ≅ seg

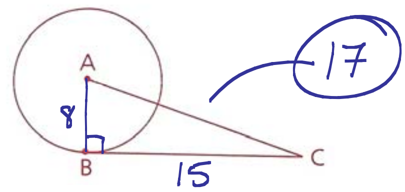
$$CD = (15 - x) + (18 - (10 - x))$$
$$= 15 - x + 18 - 10 + x$$
$$= 15 + 8$$
$$= 23$$

Name ________________________ Ms. Kresovic

Adv Geo – period __/__ F 12 Apr 213

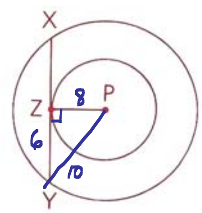
Homework 10.4 Secants and Tangents

1 The radius of ⊙A is 8 cm.
Tangent segment $\overline{BC}$ is 15 cm long.
Find the length of $\overline{AC}$.

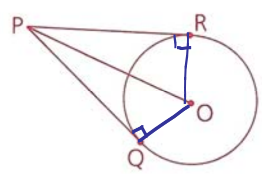


2 Concentric circles with radii 8 and 10 have center P.
$\overline{XY}$ is a tangent to the inner circle and is a chord of the outer circle.
Find $\overline{XY}$. (Hint: Draw $\overline{PX}$ and $\overline{PY}$.)

→ 2(6) = 12

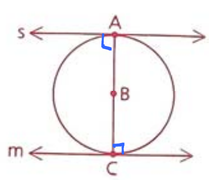


3 Given: $\overline{PR}$ and $\overline{PQ}$ are tangents to ⊙O at R and Q.
Prove: $\overrightarrow{PO}$ bisects ∠RPQ. (Hint: Draw $\overline{RO}$ and $\overline{OQ}$.)



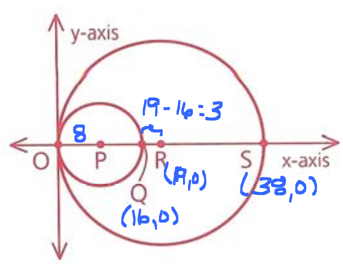
Statements	Reasons
1. $\overline{PR}$ & $\overline{PQ}$ tan in ⊙O at R & Q	1. given
2. Draw $\overline{OR}$ & $\overline{OQ}$	2. Aux
3. $\overline{OR} \cong \overline{OQ}$	3. ⊙ ⇒ ≅ rad
4. ∠PRO & ∠PQO rt ∠s	4. tan ⇒ rt ∠

4 Given: $\overline{AC}$ is a diameter of ⊙B.
Lines s and m are tangents to the ⊙ at A and C.
Conclusion: s ∥ m



1. $\overline{AC}$ diameter of ⊙B, s & m tan to ⊙A & ⊙C	1. Given
2. AC ⊥ s & AC ⊥ m	2. tan ⇒ ⊥
3. s ∥ m	3. 2 lines ⊥ same line ⇒ ∥

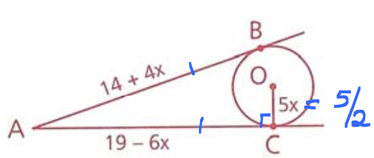
5 ⊙P and ⊙R are internally tangent at O.
P is at (8, 0) and R is at (19, 0).
a Find the coordinates of Q and S.
b Find the length of $\overline{QR}$.



6 $\overline{AB}$ and $\overline{AC}$ are tangents to ⊙O, and OC = 5x. Find OC.

2 tan ⇒ AB = AC

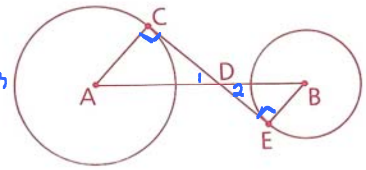
$14 + 4x = 19 - 6x$
$-14 \quad +6x \quad -14 \quad +6x$
$10x = 5$
$x = 1/2$



7 Given: $\overline{CE}$ is a common internal tangent to circles A and B at C and E.

Prove: **a** $\angle A \cong \angle B$ ← no choice if stopping here

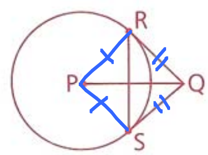
b $\frac{AD}{BD} = \frac{CD}{DE}$



1. $\overline{CE}$ comm int tan to ⊙A & B at C & E.	1. Given
2. $\overline{AC} \perp \overline{CE}$ & $\overline{BE} \perp \overline{CE}$	2. tan ⇒ ⊥
3. ∠ACE & ∠BEC rt∠s	3. ⊥ ⇒ rt∠s
4. ∠ACE ≅ ∠BEC	4. rt∠s ≅
5. ∠1 ≅ ∠2	5. Vert∠s ⇒ ≅∠s
6. △ACD ~ △BED	6. AA~
7. ∠A ≅ ∠B	7. Corr ∠s of ~△ ≅
8. $\frac{AD}{BD} = \frac{CD}{DE}$	8. Corr sds ~△ prop.

8 Given: $\overline{QR}$ and $\overline{QS}$ are tangent to ⊙P at points R and S.

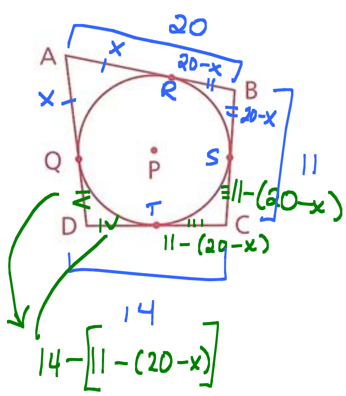
Prove: $\overline{PQ} \perp \overline{RS}$ (Hint: This can be proved in just a few steps.)



1. $\overline{QR}$ + $\overline{QS}$ tan ⊙P @ R & S	1. given
2. $\overline{PR} \perp \overline{RQ}$, $\overline{PS} \perp \overline{QS}$	2. tan ⇒ ⊥
3. $\overline{PR} \cong \overline{PS}$	3. ⊙ ⇒ ≅ rad
4. $\overline{RQ} \cong \overline{SQ}$	4. 2 tan ⇒ equidist
5. $\overline{PQ} \perp \overline{RS}$	5. 2 seg equidist ⇒ ⊥ bis

10 ⊙P is tangent to each side of ABCD. AB = 20, BC = 11, and DC = 14. Let AQ = x and find AD.

2tan ⇒ ≅ seg



$AD = x + 14 - [11 - (20 - x)]$

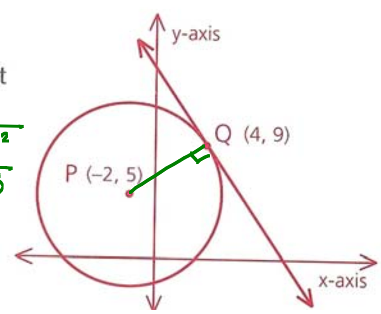
$x + 14 - [11 - 20 + x]$

$x + 14 - 11 + 20 - x$

$\boxed{23}$

11 **a** Find the radius of ⊙P.

b Find the slope of the tangent to ⊙P at point Q.



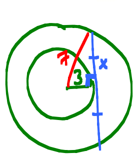
$PQ = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(4+2)^2 + (9-5)^2} = \sqrt{6^2 + 4^2}$

$= \sqrt{36 + 16} = \sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$

m tan at Q → (⊥ seg opp recip m)

$mPQ = \frac{\Delta Y}{\Delta X} = \frac{9-5}{4+2} = \frac{4}{6} = \frac{2}{3}$ So $m_{\perp} = -\frac{3}{2}$

12 Two concentric circles have radii 3 and 7. Find, to the nearest hundredth, the length of a chord of the larger circle that is tangent to the smaller circle. (See problem 2 for a diagram.)



$3^2 + x^2 = 7^2$

$x^2 = 49 - 9$

$x^2 = 40$

$x = \sqrt{4} \cdot \sqrt{10}$

$x = 2\sqrt{10}$ + we need 2 ∴ $2 \cdot 2\sqrt{10} = 4\sqrt{10}$

13 The centers of two circles of radii 10 cm and 5 cm are 13 cm apart.

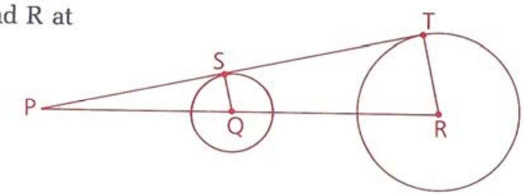
a Find the length of a common external tangent. (Hint: Use the common-tangent procedure.)

b Do the circles intersect?

14 The centers of two circles with radii 3 and 5 are 10 units apart. Find the length of a common internal tangent. (Hint: Use the common-tangent procedure.)

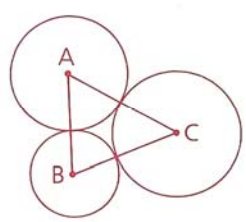
15 Given: $\overline{PT}$ is tangent to ⓢ Q and R at points S and T.

Conclusion: $\frac{PQ}{PR} = \frac{SQ}{TR}$



16 Given: Tangent ⓢ A, B, and C, $AB = 8$, $BC = 13$, $AC = 11$

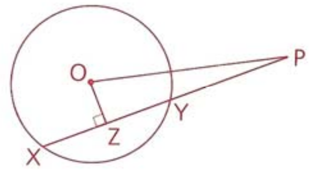
Find: The radii of the three ⓢ (Hint: This is a walk-around problem.)



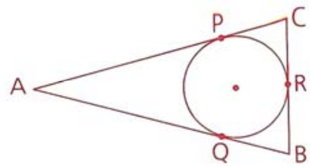
17 The radius of $\odot O$ is 10.
The secant segment $\overline{PX}$ measures 21 and is 8 units from the center of the $\odot$.

a Find the external part (PY) of the secant segment.

b Find OP.



18 Given: $\triangle ABC$ is isosceles, with base $\overline{BC}$.
Conclusion: $\overline{BR} \cong \overline{RC}$



19 If two of the seven circles are chosen at random, what is the probability that the chosen pair are

a Internally tangent?

b Externally tangent?

c Not tangent?