Name
Adv Geo -
10-3: Arcs of a Circle

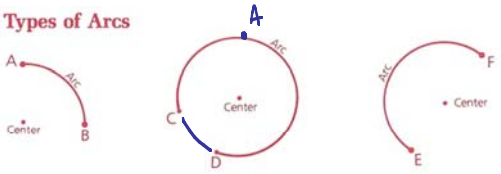
Ms. Kresovic
R 11 Apr 13

Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles

Types of Arcs

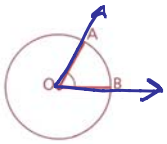


MAJOR ARC DAC
MINOR ARC CB

Definition An ***arc*** consists of two points on a circle and all points on the circle needed to connect the points by a single path.

Definition The center of an arc is the center of the circle of which the arc is a part.

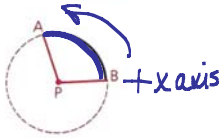
Definition A ***central angle*** is an angle whose vertex is at the center of a circle.



Radii $\overline{OA}$ and $\overline{OB}$ determine central angle AOB.

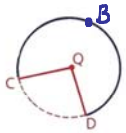
Definition A ***minor arc*** is an arc whose points are on or between the sides of a central angle.

BA or AB



Central angle APB determines minor arc AB.

Definition A ***major arc*** is an arc whose points are on or outside of a central angle.

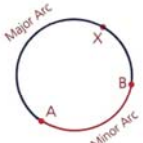


Central angle CQD determines major arc CD.

Definition A ***semicircle*** is an arc whose endpoints are the endpoints of a diameter.

Arc EF is a semicircle.

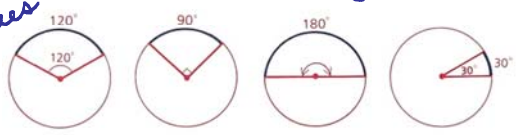
The symbol $\frown$ is used to label arcs. The minor arc joining A and B is called $\overset{\frown}{AB}$. The major arc joining A and B is called $\overset{\frown}{AXB}$. (The extra point, X, is named to make it clear that we are referring to the arc from A to B *by way of* point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)



≅ ∠s ⇒ ≅ s
≅ chds ≅ ∠s
≅ chds ⇒ ≅ s

The Measure of an Arc (not length of arc)

in degrees

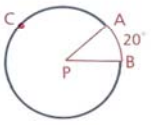


Definition The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

Definition The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

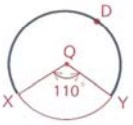
Example

a *Given:* $m\overset{\frown}{AB} = 20$
Find: $m\overset{\frown}{ACB}$



360
− 20
340° = m$\overset{\frown}{ACB}$

b *Given:* $m\angle XQY = 110$
Find: $m\overset{\frown}{XDY}$

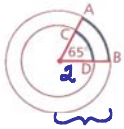


360
− 110
250° = m$\overset{\frown}{XDY}$

Congruent Arcs

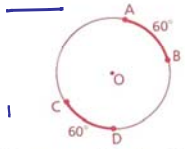
Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\overset{\frown}{AB} = 65$ and $m\overset{\frown}{CD} = 65$, but $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are *not* congruent. Under what conditions, do you think, will two arcs be congruent?

~ arcs



Definition Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.

same ° & length

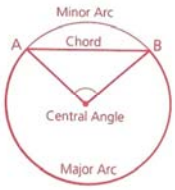


We may conclude that $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

If $\odot P \cong \odot Q$, we may conclude that $\overset{\frown}{EF} \cong \overset{\frown}{GH}$.

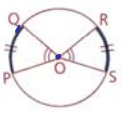
Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).



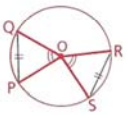
You can readily prove the following theorems.

Theorem 79 ***If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.***



Theorem 80 ***If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.***

Theorem 81 ***If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.***

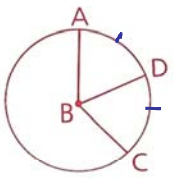


Theorem 82 ***If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.***

Theorem 83 ***If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.***

Theorem 84 ***If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.***

Problem 1 Given: ⊙B;
D *is the midpt. of* $\overset{\frown}{AC}$.
Conclusion: $\overrightarrow{BD}$ *bisects* ∠ABC.

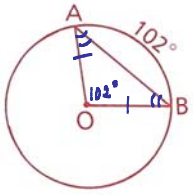


Proof

1 ⊙B; D is the midpt of $\overset{\frown}{AC}$.	1 Given
2 $\overset{\frown}{AD} \cong \overset{\frown}{DC}$	2 midpt ⇒ ≅ arcs
3 ∠ABD ≅ ∠DBC	3 ≅ arcs ⇒ ≅ central ∠s
4 $\overrightarrow{BD}$ bisects ∠ABC.	4 ≅ ∠s ⇒ bis

Problem 2 *If* $m\overset{\frown}{AB} = 102$ *in* ⊙O, *find* m∠A *and* m∠B *in* △AOB.

Solution $\overset{\frown}{AB} = 102°$



180 ← triangle
−102°
78° = 39° = m∠A = m∠B
2

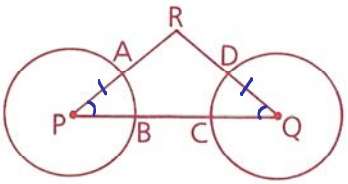
Problem 3
a *What fractional part of a circle is an arc of* 36°? *Of* 200°?
b *Find the measure of an arc that is* $\frac{7}{12}$ *of its circle.*

Solution There are 360° in a whole ⊙.

a $\frac{\text{PART}}{\text{WHOLE}} : \frac{36°}{360°} = \frac{1}{10}$

b $\frac{(30)\,7}{(30)\,12} = \frac{x}{360} = 210°$

Problem 4 *Given:* Ⓢ P *and* Q,
∠P ≅ ∠Q, $\overline{AR} \cong \overline{RD}$
Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ (Hint: First prove that ⊙P ≅ ⊙Q.)



Proof

1 Ⓢ P and Q	1 Given
2 ∠P ≅ ∠Q	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3 △ → △
4 $\overline{AR} \cong \overline{RD}$	4 Given
5 $\overline{AP} \cong \overline{DQ}$	5 Subtract
6 ⊙P ≅ ⊙Q	6 ≅ rad ⇒ ≅ Ⓢ
7 $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	7 ≅ central ∠s ⇒ ≅ arcs

AMDG

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10-3 Homework: Arcs of a Circle

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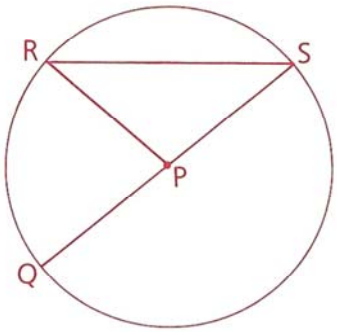
1 Match each item in the left column with the correct term in the right column.

a $\overset{\frown}{QRS}$	**1** Radius
b $\overline{QS}$	**2** Diameter
c $\overset{\frown}{RQS}$	**3** Chord
d $\overset{\frown}{RS}$	**4** Minor arc
e $\overline{RS}$	**5** Major arc
f $\angle RPQ$	**6** Semicircle
g $\overline{PS}$	**7** Central angle



2 Given: Two concentric circles with center O;
$\angle BOC$ is acute.

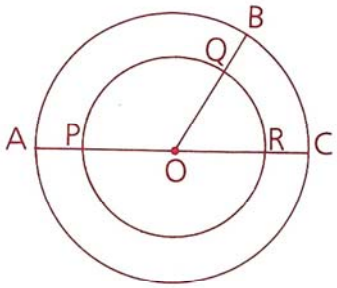
a Name a major arc of the smaller circle.

b Name a minor arc of the larger circle.

c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$?

d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$?

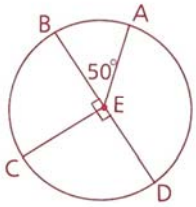
e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$?



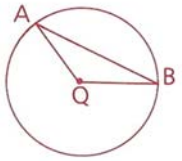
3 In circle E, find each of the following.

a $m\overset{\frown}{BC}$ **c** $m\overset{\frown}{ACD}$ **e** $m\overset{\frown}{ADC}$

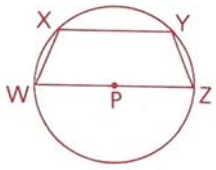
b $m\overset{\frown}{AD}$ **d** $m\overset{\frown}{BAD}$



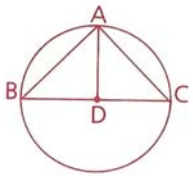
4 Given: $\odot Q$, $\angle A = 25°$
Find: $m\overset{\frown}{AB}$



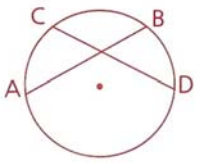
5 Given: $\odot P$,
$\overset{\frown}{WY} \cong \overset{\frown}{XZ}$
Conclusion: $\overline{WX} \cong \overline{YZ}$



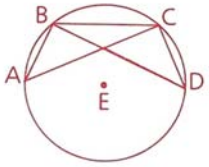
6 Given: $\odot D$, $\angle B \cong \angle C$
Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$



7 Given: $\overline{AB} \cong \overline{CD}$
Conclusion: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$



8 Given: $\odot E$,
$\overline{AB} \cong \overline{CD}$
Prove: $\overline{BD} \cong \overline{AC}$



9 What fractional part of a circle is an arc that measures

a 8

b 240

c 144

d 315

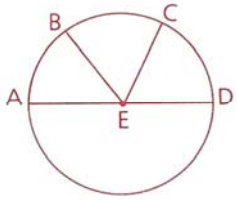
10 Find the measure of an arc that is

a $\frac{3}{5}$ of its circle

b $\frac{5}{9}$ of its circle

c 70% of its circle

11 Given: $\overline{AD}$ is a diameter of $\odot E$.
C is the midpoint of $\overset{\frown}{BD}$.
$m\overset{\frown}{AB} = 9x + 30$,
$m\overset{\frown}{CD} = 54 - x$
Find: $m\angle AEC$



12 Find the length of a chord that cuts off an arc measuring 60 in a circle with a radius of 12.

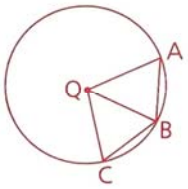
13 Find the length of each arc described. (The length is a fractional part of the circumference.)

a An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12

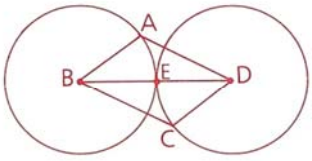
b An arc that has a measure of 270 and is part of a circle with radius 12

14 $\overline{AB}$ is a chord of circle E, and C is the midpoint of $\overset{\frown}{AB}$. Prove that $\overleftrightarrow{EC}$ is the perpendicular bisector of chord $\overline{AB}$.

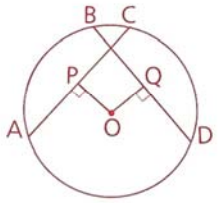
15 Given: $\odot Q$;
B is the midpt. of $\overset{\frown}{AC}$.
Conclusion: $\angle A \cong \angle C$



16 Given: $\odot B \cong \odot D$,
$\overset{\frown}{AE} \cong \overset{\frown}{CE}$
Prove: ABCD is a $\square$.



17 Given: $\odot O$,
$\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$,
$\overline{OP} \cong \overline{OQ}$
Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$



18 A polygon is *inscribed in* a $\odot$ if all its vertices lie on the $\odot$. Find the measure of the arc cut off by a side of each of the following inscribed polygons.

a A regular hexagon

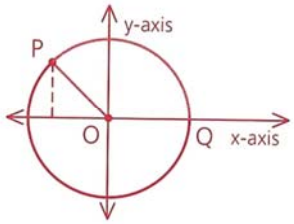
b A regular pentagon

c A regular octagon

19 Point P is located at $(-5, 5)$.

a Find the radius of $\odot O$.

b Find the measure of $\overset{\frown}{PQ}$.



20 Given: $\odot P \cong \odot Q$,
$\overline{BC} \cong \overline{CD}$
Conclusion: $\angle A \cong \angle E$