

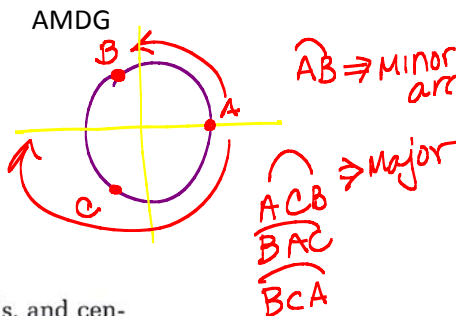
Name _____
 Adv Geo -
 10-3: Arcs of a Circle

Ms. Kresovic
 R 11 Apr 13

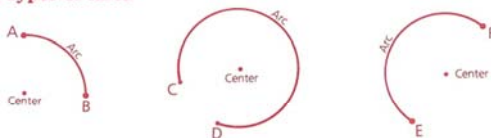
Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles



Types of Arcs



Definition An **arc** consists of two points on a circle and all points on the circle needed to connect the points by a single path.

Definition The center of an arc is the center of the circle of which the arc is a part.

Definition A **central angle** is an angle whose vertex is at the center of a circle.

Radii \overline{OA} and \overline{OB} determine central angle AOB.

Definition A **minor arc** is an arc whose points are on or between the sides of a central angle.

Central angle APB determines minor arc AB.

Definition A **major arc** is an arc whose points are on or outside of a central angle.

Central angle CQD determines major arc CD.

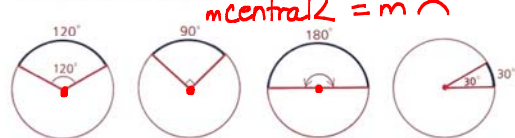
Definition A **semicircle** is an arc whose endpoints are the endpoints of a diameter.

Arc EF is a semicircle.

The symbol \frown is used to label arcs. The minor arc joining A and B is called \widehat{AB} . The major arc joining A and B is called \widehat{AXB} . (The extra point, X, is named to make it clear that we are referring to the arc from A to B by way of point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)

Size of Arc \rightarrow measure \rightarrow degrees
 \rightarrow Length \rightarrow distance (eg cm, in, mi, km)

The Measure of an Arc

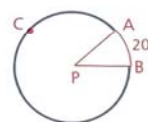


Definition The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

Definition The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

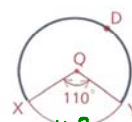
Example

a Given: $m\widehat{AB} = 20$
 Find: $m\widehat{ACB}$



$$\begin{array}{r} 360 \\ - 20 \\ \hline 340 \end{array} = m\widehat{ACB}$$

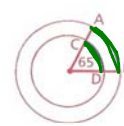
b Given: $m\angle XQY = 110$
 Find: $m\widehat{XDY}$



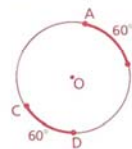
$$\begin{array}{r} 360 \\ - 110 \\ \hline 250 \end{array} = m\widehat{XDY}$$

Congruent Arcs $\Rightarrow \cong$

Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\widehat{AB} = 65$ and $m\widehat{CD} = 65$, but \widehat{AB} and \widehat{CD} are not congruent. Under what conditions, do you think, will two arcs be congruent?



Definition Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.



We may conclude that $\widehat{AB} \cong \widehat{CD}$.



If $\odot P \cong \odot Q$, we may conclude that $\widehat{EF} \cong \widehat{GH}$.

Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).

You can readily prove the following theorems.



Theorem 79 If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent. $\cong \text{cent}\angle s \Rightarrow \cong \text{arcs}$

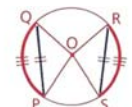
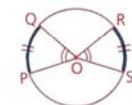
Theorem 80 If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent. $\cong \text{arcs} \Rightarrow \cong \text{cent}\angle s$

Theorem 81 If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent. $\cong \text{cent}\angle s \Rightarrow \cong \text{chords}$

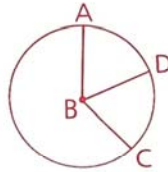
Theorem 82 If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent. $\cong \text{chords} \Rightarrow \cong \text{cent}\angle s$

Theorem 83 If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent. $\cong \text{arcs} \Rightarrow \cong \text{chords}$

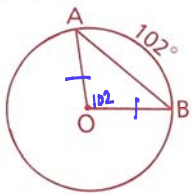
Theorem 84 If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent. $\cong \text{chords} \Rightarrow \cong \text{arcs}$



$$\cong \text{cent}\angle s \Rightarrow \cong \text{arcs} \Rightarrow \cong \text{chords}$$

Problem 1Given: $\odot B$;D is the midpt. of \widehat{AC} .Conclusion: \overrightarrow{BD} bisects $\angle ABC$.**Proof**

1 $\odot B$; D is the midpt. of \widehat{AC} .	1 Given
2 $\widehat{AD} \cong \widehat{DC}$	2 midpt $\Rightarrow \cong \widehat{s}$
3 $\angle ABD \cong \angle DBC$	3 $\cong \widehat{s} \Rightarrow \cong \text{cent } \angle s$
4 \overrightarrow{BD} bisects $\angle ABC$.	4 $\cong \angle s \Rightarrow \text{bis}$

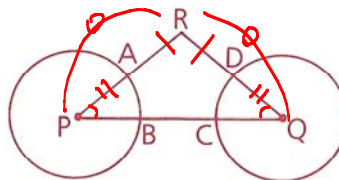
Problem 2If $m\widehat{AB} = 102^\circ$ in $\odot O$, find $m\angle A$ and $m\angle B$ in $\triangle AOB$.**Solution** $\widehat{AB} = 102^\circ$ 

$$\begin{aligned}
 m\widehat{AB} &= m\text{cent } \angle \therefore m\angle AOB = 102^\circ \\
 \odot &\Rightarrow \cong \text{rad} \therefore AO = BO \\
 \triangle &\Rightarrow \triangle \therefore m\angle A = m\angle B \\
 \text{triangle sum } \angle s &= \frac{180}{-102} \\
 &= 78 \\
 78/2 &= \boxed{39^\circ}
 \end{aligned}$$

Problem 3a What fractional part of a circle is an arc of 36° ? Of 200° ?b Find the measure of an arc that is $\frac{7}{12}$ of its circle.**Solution**There are 360° in a whole \odot .
$$\frac{\text{PART}}{\text{WHOLE}}$$

$$a \quad \frac{36^\circ}{360^\circ} = \frac{1}{10}$$

$$b \quad \frac{7}{12} = \frac{210^\circ}{360^\circ}$$

Problem 4Given: $\odot P$ and Q , $\angle P \cong \angle Q$, $\overline{AR} \cong \overline{RD}$ Prove: $\widehat{AB} \cong \widehat{CD}$ (Hint: First prove that $\odot P \cong \odot Q$.)**Proof**

1 $\odot P$ and Q	1 Given
2 $\angle P \cong \angle Q$	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3 $\triangle \Rightarrow \triangle$
4 $\overline{AR} \cong \overline{RD}$	4 Given
5 $\overline{AP} \cong \overline{DQ}$	5 Subtract
6 $\odot P \cong \odot Q$	6 $\cong \text{rad} \Rightarrow \cong \odot$
7 $\widehat{AB} \cong \widehat{CD}$	7 $\text{In } \cong \odot s, \cong \text{cent } \angle s \Rightarrow \cong \widehat{s}$

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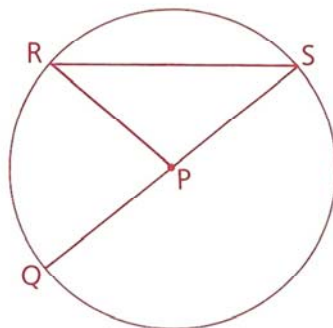
10-3 Homework: Arcs of a Circle

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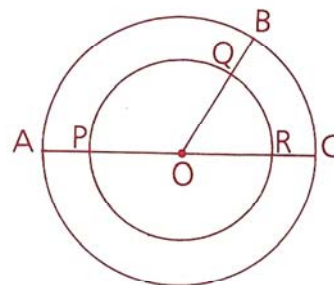
- 1 Match each item in the left column with the correct term in the right column.

- | | |
|-------------------|-----------------|
| a \widehat{QRS} | 1 Radius |
| b \overline{QS} | 2 Diameter |
| c \widehat{RQS} | 3 Chord |
| d \widehat{RS} | 4 Minor arc |
| e \overline{RS} | 5 Major arc |
| f $\angle RPQ$ | 6 Semicircle |
| g \overline{PS} | 7 Central angle |



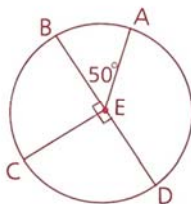
- 2 Given: Two concentric circles with center O;
 $\angle BOC$ is acute.

- Name a major arc of the smaller circle.
- Name a minor arc of the larger circle.
- What is $m\widehat{BC} + m\widehat{PQ}$?
- Which is greater, $m\widehat{BC}$ or $m\widehat{PQ}$?
- Is \widehat{BC} congruent to \widehat{QR} ?

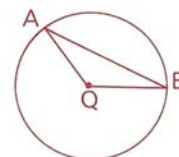


- 3 In circle E, find each of the following.

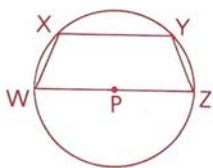
- | | | |
|-------------------|--------------------|--------------------|
| a $m\widehat{BC}$ | c $m\widehat{ACD}$ | e $m\widehat{ADC}$ |
| b $m\widehat{AD}$ | d $m\widehat{BAD}$ | |



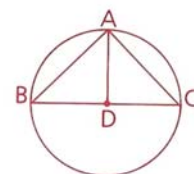
- 4 Given: $\odot Q$, $\angle A = 25^\circ$
Find: $m\widehat{AB}$



- 5 Given: $\odot P$,
 $\widehat{WY} \cong \widehat{XZ}$
Conclusion: $\overline{WX} \cong \overline{YZ}$



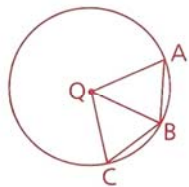
- 6 Given: $\odot D$, $\angle B \cong \angle C$
Conclusion: $\widehat{AB} \cong \widehat{AC}$



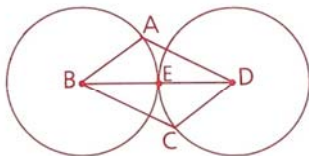
- 13** Find the length of each arc described. (The length is a fractional part of the circumference.)
- a** An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12
 - b** An arc that has a measure of 270 and is part of a circle with radius 12

- 14** \overleftrightarrow{AB} is a chord of circle E, and C is the midpoint of \widehat{AB} . Prove that \overleftrightarrow{EC} is the perpendicular bisector of chord \overleftrightarrow{AB} .

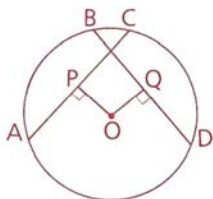
- 15** Given: $\odot Q$;
 B is the midpt. of \widehat{AC} .
 Conclusion: $\angle A \cong \angle C$



- 16** Given: $\odot B \cong \odot D$,
 $\widehat{AE} \cong \widehat{CE}$
 Prove: ABCD is a \square .



- 17 Given: $\odot O$,
 $\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$,
 $\overline{OP} \cong \overline{OQ}$
 Conclusion: $\widehat{AB} \cong \widehat{CD}$

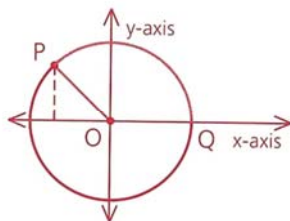


- 18 A polygon is inscribed in a \odot if all its vertices lie on the \odot . Find the measure of the arc cut off by a side of each of the following inscribed polygons.



- a A regular hexagon
- b A regular pentagon
- c A regular octagon

- 19 Point P is located at $(-5, 5)$.
 a Find the radius of $\odot O$.
 b Find the measure of \widehat{PQ} .



- 20 Given: $\odot P \cong \odot Q$,
 $\overline{BC} \cong \overline{CD}$
 Conclusion: $\angle A \cong \angle E$

