Name   Ms. Kresovic
Adv Geo   W 10 Apr 2013

10-2: Congruent Chords

Objective

After studying this section, you will be able to
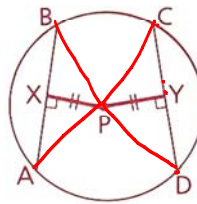- Apply the relationship between congruent chords of a circle

Theorem 77 *If two chords of a circle are equidistant from the center, then they are congruent.*

chds eqdist ⟹ ≅ chds

Given: $\odot P$, $\overline{PX} \perp \overline{AB}$, $\overline{PY} \perp \overline{CD}$, $\overline{PX} \cong \overline{PY}$
Prove: $\overline{AB} \cong \overline{CD}$

4 △'s ≅ by HL
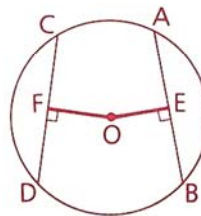
Theorem 78 *If two chords of a circle are congruent, then they are equidistant from the center of the circle.*
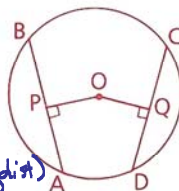
≅ chds ⟹ chds eqdist.

Given: $\odot O$, $\overline{AB} \cong \overline{CD}$, $\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$
Prove: $\overline{OE} \cong \overline{OF}$

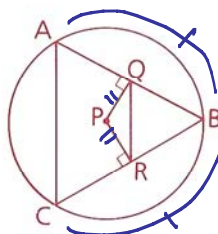**Problem 1**   Given: $\odot O$, $\overline{AB} \cong \overline{CD}$,
           $OP = 12x - 5$, $OQ = 4x + 19$
         Find: OP
           ↓
           not proof

Solution

$\odot O,\ AB = CD\ (g) \rightarrow OP = OQ$ ($\cong$ chds → chds eqdist)

$$12x - 5 = 4x + 19$$
$$-4x + 5 \quad\quad -4x + 5$$
$$8x = 24$$
$$x = 3$$

$$OP = 12(3) - 5 = \boxed{31}$$

Problem 2   Given: $\triangle ABC$ is isosceles, with base $\overline{AC}$.
           $\odot P$, $\overline{PQ} \perp \overline{AB}$, $\overline{PR} \perp \overline{CB}$
         Prove: $\triangle PQR$ is isosceles.

Proof

1 $\odot P$, $\overline{PQ} \perp \overline{AB}$, $\overline{PR} \perp \overline{CB}$	1 Given
2 $\triangle ABC$ is isosceles, with base $\overline{AC}$.	2 Given
3 $\overline{AB} \cong \overline{BC}$	3 isos → 2 sds $\cong$
4 $\overline{PQ} \cong \overline{PR}$	4 $\cong$ chds → chds eqdist
5 $\triangle PQR$ is isosceles.	5 2 sds $\cong$ isos

1 In a circle, chord $\overline{AB}$ is 325 cm long and chord $\overline{CD}$ is $3\frac{1}{4}$ m long. Which is closer to the center?
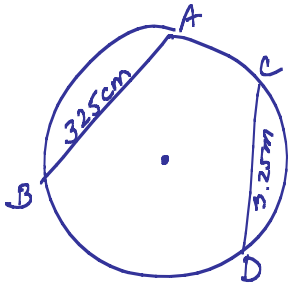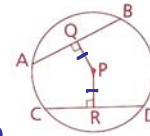
3.25m

2 ≅ chds ⇒ chds
equidist
from ctr.

2 Given: ⊙P, $\overline{PQ} \cong \overline{PR}$,
AB = 6x + 14,
CD = 4 − 4x
Find: AB

PQ=PR→AB=CD
(equids → ≅chds)

$6x+14 = 4 - 4x$
$+4x \quad -14 \quad -14 \quad +4x$

$10x = -10$
$x = -1$

$AB = 6(-1)+14 = \boxed{8}$

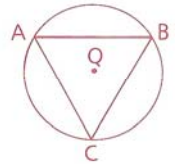**3** Given: ⊙P, $\overline{PR} \perp \overline{WX}$,
$\overline{PS} \perp \overline{XY}$, $\overline{PR} \cong \overline{PS}$
Conclusion: ∠W ≅ ∠Y

1. ⊙P, $\overline{PR} \perp \overline{WX}$, $\overline{PS} \perp \overline{XY}$ 1. given
2. $\overline{WX} \cong \overline{YX}$ 2. eqdist ⇒ ≅chds
3. △WXY isos, base WY   3. 2 ≅ sds ⇒ isos
4. ∠W ≅ ∠Y   4. △ ⇒ △
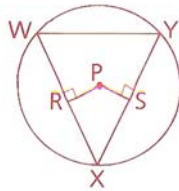
4 Given: Equilateral △ ABC is inscribed in ⊙Q.
Conclusion: $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are equidistant from the center.

5 Given: ⊙P;
P is the midpoint of $\overline{MN}$.
$\overline{MN} \perp \overline{AD}$, $\overline{MN} \perp \overline{BC}$
Conclusion: ABCD is a ▱.
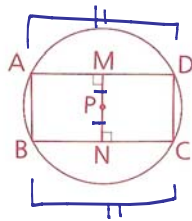
1. ⊙P, P mdpt $\overline{MN}$ 1. given
2. $\overline{PM} \cong \overline{NP}$ 2. mdpt ⇒ ≅ segs
3. $\overline{AD} \cong \overline{BC}$ 3. eqdist ⇒ ≅ chds
4. $\overline{MN} \perp \overline{AD}$ & $\overline{BC}$ 4. given
5. $\overline{AD} \parallel \overline{BC}$ 5. lines ⊥ to same line ⇒ ∥
6. ▱ ABCD   6. in quad, if 1 pr opp sds is both ≅ & ∥, then ▱

6 A fly is sitting at the midpoint of a wooden chord of a circular wheel. The wheel has a radius of 10 cm, and the chord has a length of 12 cm.
a How far from the hub (center) is the fly? 8 cm
b The wheel is spun. What is the path of the fly?

Circle

12

8

10
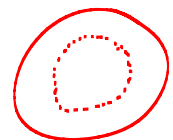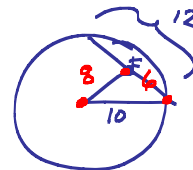
7 To the nearest hundredth, find
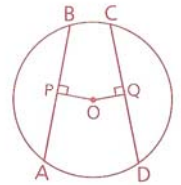 **a** The area of the circle
  b The circumference of the circle

a. $A = r^2 \pi = \left(\frac{19}{2}\right)^2 \pi =$
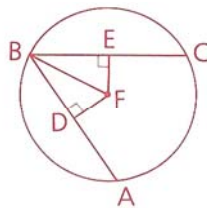
b. $C = d\pi = 19\pi$

11 In circle O, PB = 3x − 17, CD = 15 − x, and OQ = OP = 3.
  a Find AB.
  b Find the radius of ⊙O.

9 Given: ⊙F,
  $\overline{FE} \perp \overline{BC}, \overline{FD} \perp \overline{AB}$;
  $\overrightarrow{BF}$ bisects ∠ABC.
  Prove: $\overline{BC} \cong \overline{BA}$

12 A regular hexagon with a perimeter of 24 is inscribed in a circle. How far from the center is each side?